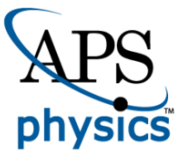

Floquet-Engineering Counterdiabatic Protocols in Quantum Many-Body Systems

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Floquet-engineering counterdiabatic protocols in quantum many-body systems

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Counterdiabatic (CD) driving presents a way of generating adiabatic dynamics at arbitrary pace, where excitations due to non-adiabaticity are exactly compensated by adding an auxiliary driving term to the Hamiltonian. While this CD term is theoretically known and given by the adiabatic gauge potential, obtaining and implementing this potential in many-body systems is a formidable task, requiring knowledge of the spectral properties of the instantaneous Hamiltonians and control of highly nonlocal multibody interactions. We show how an approximate gauge potential can be systematically built up as a series of nested commutators, remaining well-defined in the thermodynamic limit. Furthermore, the resulting CD driving protocols can be realized up to arbitrary order without leaving the available control space using tools from periodically-driven (Floquet) systems. This is illustrated on few- and many-body quantum systems, where the resulting Floquet protocols significantly suppress dissipation and provide a drastic increase in fidelity.

Introduction. – Adiabaticity presents one of the fundamental tools in physics, ranging from heat engines in thermodynamics to quantum state preparation and computation [1–4]. However, true adiabatic control can only be obtained using slow driving and asymptotically long timescales. While faster driving leads to diabatic excitations and resulting dissipative losses, the inevitable presence of decoherence and noise in realistic quantum systems limits the available timescales, preventing true adiabaticity. Various methods have been proposed in order to achieve so-called "Shortcuts to Adiabaticity" (STAs) both theoretically [5–8] and experimentally [9–18], mimicking adiabatic dynamics without requiring slow driving.

One way of circumventing this loss of fidelity at finite driving rates is through counterdiabatic (CD) or transitionless driving – a velocity-dependent term is added to the control Hamiltonian, exactly compensating the diabatic contributions to the Hamiltonian in the moving frame [19–22]. This term is known as the *adiabatic gauge potential* (or gauge connection), encoding the geometry of eigenstates [22]. However, while this potential may be exactly obtained in few-body systems, its construction in general requires diagonalization of the Hamiltonian in the full Hilbert space, prohibiting its use in general many-body systems. Furthermore, the resulting operator tends to involve highly nontrivial and nonlocal couplings not present in the control Hamiltonian, preventing its actual implementation [23–25]. While various applications of STA in many-body systems have been investigated, these generally impose restrictions on the studied system (for a recent review, see [26]) – either dynamic symmetries or scaling laws [27, 28], Born-Oppenheimer dynamics [29], underlying Lax pairs [30],... Various efforts have also been made to use STA to counteract the Kibble-Zurek mechanism in critical systems [31, 32].

Restricting driving to available (local) couplings led to the development of fast-forward (FF) protocols [33–36], which only follow the adiabatic path at the beginning and end of the driving. However, there exists no general way of constructing these for complex systems. One specific class of FF protocols is those where CD driving is realized through Floquet-engineering: high-frequency oscillations are added to the control so that the resulting Floquet Hamiltonian mimics the CD Hamiltonian. This has already been used for high-fidelity quantum state manipulation both theoretically, in closed [37–39] and open systems [40], and experimentally [41, 42].

We propose a method of (i) finding an efficient and controlled approximation to the gauge potential, remaining well-defined in many-body systems, which can then (ii) be systematically realized through Floquet-engineering by resonantly oscillating the instantaneous Hamiltonian with the driving term. Effectively, we propose a general strategy for designing fast adiabatic protocols, applicable both in small quantum systems to achieve high fidelity for state preparation and in large systems, quantum or classical, to suppress dissipative losses.

Methods. – Consider a control Hamiltonian $\mathcal{H}(\lambda)$ dependent on a control parameter λ. Our goal is to transport a stationary state or distribution, at an initial value of the control parameter λ_i, to one corresponding to a final value λ_f. In the standard approach, this is done by adiabatically changing $\lambda(t)$ from λ_i to λ_f, which is often impractical because of the necessary access to long timescales. The key idea of CD driving is to vary the parameter $\lambda(t)$ at a finite rate while simultaneously compensating the diabatic excitations by explicitly adding an auxiliary term as

$$\mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + \dot{\lambda}\mathcal{A}_\lambda. \tag{1}$$

Adiabatic control at arbitrary driving rates for arbitrary initial states is realized provided the adiabatic gauge potential $\mathcal{A}_\lambda$ [22] satisfies

$$\langle m|\mathcal{A}_\lambda|n\rangle = i\langle m|\partial_\lambda n\rangle = -i\frac{\langle m|\partial_\lambda \mathcal{H}|n\rangle}{\epsilon_m - \epsilon_n}, \tag{2}$$

where $|n\rangle$ and ϵ_n are the eigenstates and the energy spectrum of the instantaneous Hamiltonian, $\mathcal{H}(\lambda)\,|n\rangle = \epsilon_n\,|n\rangle$.

Eq. (2) highlights the issues with many-body CD driving: the gauge potential is defined in the eigenbasis of the instantaneous Hamiltonian, requiring exact diagonalization. Furthermore, for increasing system sizes the denominator $(\epsilon_m - \epsilon_n)$ can become exponentially small, leading to divergent matrix elements and an ill-defined gauge potential in the thermodynamic limit [22, 43]. Physically, at least in chaotic systems, the exact gauge potential also cannot be local [44].

We propose an approximate gauge potential defined as

$$\mathcal{A}_\lambda^{(\ell)} = i\sum_{k=1}^{\ell}\alpha_k\,[\underbrace{\mathcal{H},[\mathcal{H},\dots[\mathcal{H}}_{2k-1},\partial_\lambda\mathcal{H}]], \tag{3}$$

determined by a set of coefficients $\{\alpha_1,\alpha_2,\dots,\alpha_\ell\}$, where ℓ determines the order of the expansion. The exact gauge potential can be represented in this form in the limit $\ell\to\infty$ [45]. Instead we consider a finite value of ℓ and treat the expansion coefficients as variational parameters, which can be obtained by minimizing the action S_ℓ

$$S_\ell = \mathrm{Tr}\left[G_\ell^2\right], \qquad G_\ell = \partial_\lambda\mathcal{H} - i[\mathcal{H},\mathcal{A}_\lambda^{(\ell)}]. \tag{4}$$

The exact gauge potential is known to follow from the variational minimization of an action [46]. However, it is not a priori clear what (local) operators should be included in the variational basis. The total number of possible operators increases exponentially with their support, limiting brute-force minimization to highly local operators with restricted support. Furthermore, it is far from guaranteed that such operators will be experimentally realizable. The proposed ansatz tackles both problems simultaneously. (i) The number of variational coefficients can be kept small while still returning an accurate approximation to the exact gauge potential. As such, Eq. (3) can be seen as a variational ansatz including only the most important contributions with the maximum range of operators set by ℓ. (ii) In addition, this gauge potential can be engineered with a Floquet protocol. This is possible because the high-frequency expansion of the Floquet Hamiltonian shares the commutator structure of Eq. (3). This expansion exhibits the symmetries of the exact solution, and as additional bonus we remark that this ansatz has a well-defined classical limit, where even the local-operator basis becomes infinite-dimensional. In classical systems, the commutators in Eq. (3) only need to be replaced by Poisson brackets.

Since the action is the Hilbert-Schmidt norm of G_ℓ, this method has the clear advantage that the action can be calculated without explicitly constructing the operator matrix in the full Hilbert space. There are various ways of motivating Eq. (3) (see Supplementary Material [45]): it can be seen as an expansion in the Krylov subspace generated by the action of G_ℓ, or by noting that such commutators appear through the Baker-Campbell-Hausdorff expansion in the definition of a (properly regularized) gauge potential, or by noting that its matrix elements share the structure of those of the exact gauge potential. Namely, evaluating Eq. (3) in the eigenbasis of $\mathcal{H}$ returns

$$\begin{aligned}\langle m|\mathcal{A}_\lambda^{(\ell)}|n\rangle &= i\sum_{k=1}^{\ell}\alpha_k\,\langle m|\,[\underbrace{\mathcal{H},[\mathcal{H},\dots[\mathcal{H}}_{2k-1},\partial_\lambda\mathcal{H}]]|n\rangle \\ &= i\left[\sum_{k=1}^{\ell}\alpha_k(\epsilon_m-\epsilon_n)^{2k-1}\right]\langle m|\partial_\lambda\mathcal{H}|n\rangle\,. \end{aligned}\tag{5}$$

This can be compared to the exact expression (2), containing a state-dependent factor $\langle m|\partial_\lambda\mathcal{H}|n\rangle$ and a prefactor only dependent on the excitation frequency $\omega_{mn} = (\epsilon_m - \epsilon_n)$. The variational optimization can be seen as approximating the exact prefactor $1/\omega_{mn}$ by a power-series prefactor $a_\lambda^{(\ell)}(\omega_{mn}) \equiv \sum_{k=1}^{\ell}\alpha_k\omega_{mn}^{2k-1}$ for the range of relevant excitation frequencies set by $\langle m|\partial_\lambda\mathcal{H}|n\rangle$.

While such an approximation is generally impossible due to the divergence of $1/\omega_{mn}$ near $\omega_{mn}=0$ and the divergence of the power series for $\omega_{mn}\to\infty$, the approximation does not need to hold in these limits. First, for large ω_{mn} the matrix elements of local operators $\langle m|\partial_\lambda\mathcal{H}|n\rangle$ typically decay exponentially with ω_{mn} [44], leading to a negligible contribution to the gauge potential. Second, there are physical motivations for allowing transitions for small ω_{mn}. When speeding up adiabatic driving in the presence of an energy gap Δ, only transitions with $\omega_{mn}\geq\Delta$ need to be suppressed to achieve unit fidelity, and in more general gapless regimes corresponding to e.g. excited states the resulting excitations will be confined to a narrow energy shell, the width of which decreases with the order ℓ of the expansion.

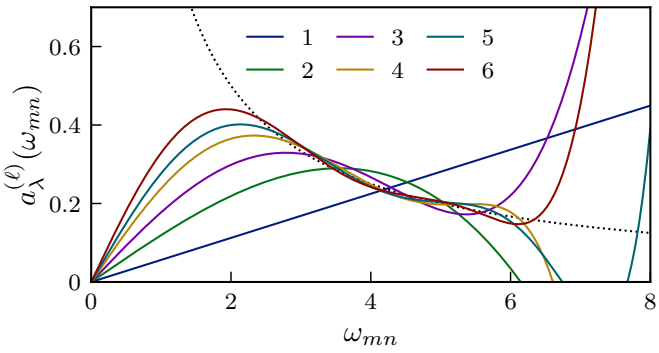


FIG. 1: Variationally-obtained power-series prefactor $a_\lambda^{(\ell)}(\omega_{mn})$ for Eq. (6). Dotted line corresponds to exact prefactor $1/\omega_{mn}$. Parameters $L=14$, $J=1$, $h_x=h_z=0.3$, $\lambda=1$.

We illustrate how this expansion works in Fig. 1, for a

non-integrable Ising chain with

$$\mathcal{H} = J\sum_{i=1}^{L}\sigma_i^z\sigma_{i+1}^z + \lambda\left(h_z\sum_{i=1}^{L}\sigma_i^z + h_x\sum_{i=1}^{L}\sigma_i^x\right), \quad (6)$$

where no exact gauge potential can be obtained in the thermodynamic limit. It is clear that the variational optimization returns a gauge potential optimized for a relevant window of excitation frequencies, where the approximation necessarily improves with increasing ℓ.

The resulting gauge potential can be used to reliably speed up adiabatic protocols taking $\mathcal{H}_{CD}^{(\ell)}(t) = \mathcal{H}(\lambda) + \dot{\lambda}\mathcal{A}_\lambda^{(\ell)}(\lambda)$. While this presents a guaranteed improvement in fidelity, it also requires access to interaction terms not necessarily available, where the only interactions that are generally present are those of $\mathcal{H}(\lambda)$ and $\partial_\lambda\mathcal{H}(\lambda)$. Remarkably, this CD Hamiltonian can be realized as an effective Floquet Hamiltonian by oscillating these two terms at high frequency. Consider

$$\mathcal{H}_{FE}(t) = \left[1 + \frac{\omega}{\omega_0}\cos(\omega t)\right]\mathcal{H}(\lambda) + \dot{\lambda}\left[\sum_{k=1}^{\infty}\beta_k \sin\left((2k-1)\omega t\right)\right]\partial_\lambda\mathcal{H}(\lambda), \quad (7)$$

with β_k the Fourier coefficients of the additional drive and ω_0 a reference frequency typically set by the excitation energy of the system, with both to be determined later. Floquet theory allows for the definition of a time-independent Floquet Hamiltonian reproducing time evolution over a single driving cycle (with $T = 2\pi/\omega$)

$$\exp\left(-i\mathcal{H}_F T\right) \equiv \mathcal{T}\exp\left(-i\int_t^{t+T}\mathcal{H}_{FE}(t')\,dt'\right). \quad (8)$$

The limit where the driving term scales with the frequency is known to give rise to non-trivial Floquet Hamiltonians $\mathcal{H}_F$ in various scenarios [47–51].

More specifically, the proposed series expansion for the adiabatic gauge potential can be implemented in the infinite-frequency limit $\omega \to \infty$, realizing (stroboscopic) CD driving. This Floquet Hamiltonian follows from the Magnus expansion, presenting a series expansion of $\mathcal{H}_F$ in powers of the inverse-frequency. Essentially, the $\omega \to \infty$ limit combined with the scaling of $\mathcal{H}$ with ω guarantees that only commutators of the form $[\mathcal{H}, \ldots, [\mathcal{H}, \partial_\lambda\mathcal{H}]]]$ survive in the Magnus expansion, which can then be found as $\mathcal{H}_F = \mathcal{H}(\lambda) + \dot{\lambda}\mathcal{A}_F$ [45], with

$$\langle m|\mathcal{A}_F|n\rangle = i\sum_{k=1}^{\infty}\beta_k\mathcal{J}_{2k-1}\left(\frac{\omega_{mn}}{\omega_0}\right)\langle m|\partial_\lambda\mathcal{H}|n\rangle, \quad (9)$$

where $\mathcal{J}_k$ are Bessel functions of the first kind. Again, this reproduces the correct structure of the gauge potential, where the frequency-dependent prefactor is now expressed in terms of $\mathcal{J}_k$. For small ω_{mn}/ω_0, $\mathcal{J}_k(\omega_{mn}/\omega_0) \propto \omega_{mn}^k$, which can be used to stroboscopically engineer the CD term by choosing the Fourier harmonics such that the Floquet prefactor reproduces the power series (5) in the relevant range of excitation frequencies. In first approximation, this can be done by restricting time-evolution to ℓ harmonics and setting

$$\sum_{k=1}^{\ell}\beta_k\mathcal{J}_{2k-1}\left(\frac{\omega_{mn}}{\omega_0}\right) = \sum_{k=1}^{\ell}\alpha_k\omega_{mn}^{2k-1} + \mathcal{O}(\omega_0^{-2}). \quad (10)$$

Analytic expressions can easily be obtained for matching the harmonics to the coefficients in the gauge potential up to arbitrary order and, if necessary, higher-order harmonics can be added to compensate the $\mathcal{O}(\omega_0^{-2})$ corrections order by order [45]. As an illustration, taking $\beta_1 = 2\alpha_1\omega_0$ and $\beta_2 = 2\omega_0(24\alpha_2\omega_0^2 + 3\alpha_1)$ for the expansion with two terms, the resulting protocol approximately reproduces the CD evolution at stroboscopic times $t = n\cdot T, n \in \mathbb{N}$. In finite systems, the exact gauge potential can always be obtained from a large enough ansatz, which can be reproduced as a Floquet Hamiltonian from a similarly large number of harmonics, such that exact counterdiabatic driving can always be realized through Floquet-engineering. However, while this protocol does not introduce new interactions in the Hamiltonian, the additional cost is that it requires high-frequency oscillations of both $\mathcal{H}$ and $\partial_\lambda\mathcal{H}$ rather than just $\partial_\lambda\mathcal{H}$.

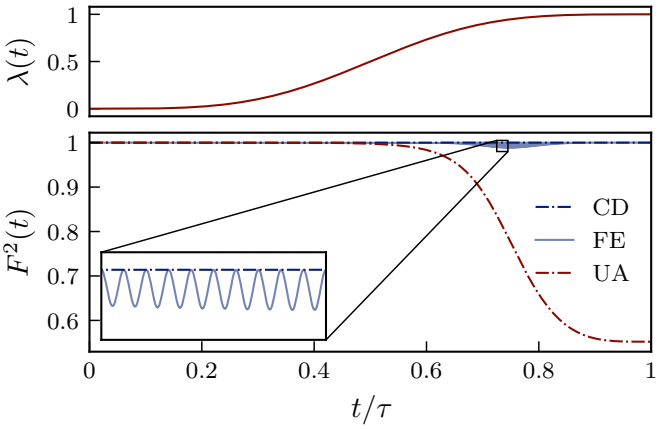


FIG. 2: Fidelity in the 2-qubit system (11) for UA, CD and FE protocol. Increasing ω further suppresses the Floquet oscillations. Parameters $J = -1$, $h_z = 5$, $\tau = 0.1$, $\omega_0 = 10\cdot 2\pi$ and $\omega = 250\cdot\omega_0$.

Applications. – This procedure can now be applied on various systems with increasing complexity. In all examples, we consider a specific driving protocol $\lambda(t) = \sin^2\left(\frac{\pi}{2}\sin^2\left(\frac{\pi t}{2\tau}\right)\right)$, ramping from $\lambda(0) = 0$ to $\lambda(\tau) = 1$ in such a way that $\dot{\lambda}$ and $\ddot{\lambda}$ vanish at the beginning and end of the protocol. λ behaves as an annealing parameter, and as first measure for the effectiveness of the protocol we initialize the system in the ground state for $\lambda = 0$ and

calculate the fidelity of the time-evolved state w.r.t. the instantaneous ground state $F^2(t) = |\langle\psi(t)|\psi_0(\lambda(t))\rangle|^2$.

First consider a two-qubit system, for which all calculations can be performed analytically [45],

$$\mathcal{H}(\lambda) = J\left(\sigma_1^x\sigma_2^x + \sigma_1^z\sigma_2^z\right) + h_z(\lambda-1)\left(\sigma_1^z + \sigma_2^z\right). \quad (11)$$

The first-order expansion leads to

$$\mathcal{A}_\lambda^{(1)} = -\frac{Jh_z}{2}\frac{(\sigma_1^y\sigma_2^x + \sigma_1^x\sigma_2^y)}{J^2 + 4(\lambda-1)^2h_z^2}. \quad (12)$$

Remarkably, this already returns the exact adiabatic gauge potential as presented in Ref. [38]. This can be understood either by noting that $[\mathcal{H},[\mathcal{H},[\mathcal{H},\partial_\lambda\mathcal{H}]] \propto [\mathcal{H},\partial_\lambda\mathcal{H}]$, such that the higher-order commutators do not introduce new operators in the expansion, $\mathcal{A}_\lambda^{(\ell)} \propto \mathcal{A}_\lambda^{(1)}$, and the variational approach can be seen as a resummation of all higher-order terms exactly determining the prefactor. Second, this system behaves as a two-level system since any instantaneous Hamiltonian only couples $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$, leading to a single excitation frequency which can be exactly cancelled by a single commutator.

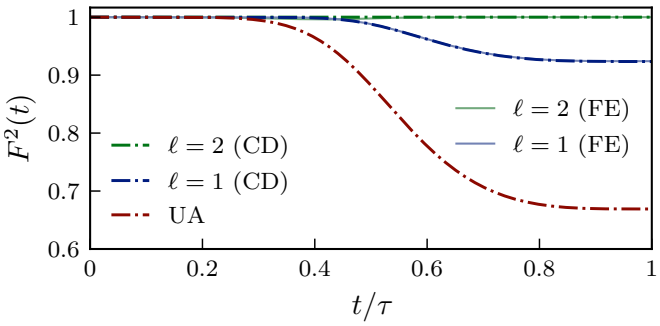


FIG. 3: Fidelity in the 2-qubit system (14) for the UA, CD and FE protocol with $\ell = 1, 2$. Parameters $\tau = 0.1$, $J = 1$, $h = 2$, $\omega_0 = 10\cdot 2\pi$ and $\omega = 2.5\cdot 10^{2\ell}\cdot\omega_0$.

The resulting CD driving can be realized up to $\mathcal{O}(\omega_0^{-2})$ using a single harmonic as

$$\mathcal{H}_{FE}(t) = \left[1 + \frac{\omega}{\omega_0}\cos(\omega t)\right]\mathcal{H}(\lambda(t)) - \dot{\lambda}\frac{2h_z\omega_0\sin(\omega t)}{4J^2 + 16(\lambda(t)-1)^2h_z^2}\left(\sigma_1^z + \sigma_2^z\right). \quad (13)$$

The results are illustrated in Fig. 2, where the duration of the protocol has been chosen in such a way that τ is too small for the unassisted (UA) protocol to accurately prepare the final Bell state $|\psi_0(\lambda=1)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$. Exact CD driving returns unit fidelity by definition, which can be well approximated (with a final error of the order 10^{-5}) using the proposed Floquet-engineered (FE) protocol.

Next, consider a two-qubit system behaving as a three-level system,

$$\mathcal{H}(\lambda) = -2J\sigma_1^z\sigma_2^z - h\left(\sigma_1^z + \sigma_2^z\right) + 2h\lambda\left(\sigma_1^x + \sigma_2^x\right), \quad (14)$$

where the total spin-0 state $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ decouples from the rest of the Hilbert space. Transitionless protocols in three-level systems have recently attracted experimental [52] and theoretical [53–55] interest, since exact protocols can no longer be trivially obtained. As shown in Fig. 3, the fidelity for the unassisted protocol is 67%, increasing to 92% for $\ell = 1$, before reaching approximate unit fidelity (up to an error 10^{-6}) for $\ell = 2$. Again, for $\ell = 2$ the variational approach returns the exact gauge potential, without any reference to exact diagonalization, since only two excitation frequencies are present. The FE protocol accurately reproduces the CD protocol.

Magnetic trap. – Moving to many-body systems, we consider the non-integrable Ising chain. Rather than changing the magnetic field uniformly, we will consider a more involved protocol where a local Gaussian magnetic trap is moved across the chain, similar to the 'optical tweezers' problem [56]. In this problem, a set of initially localized spins are to be moved across the model while minimizing dissipation. The Hamiltonian is given by

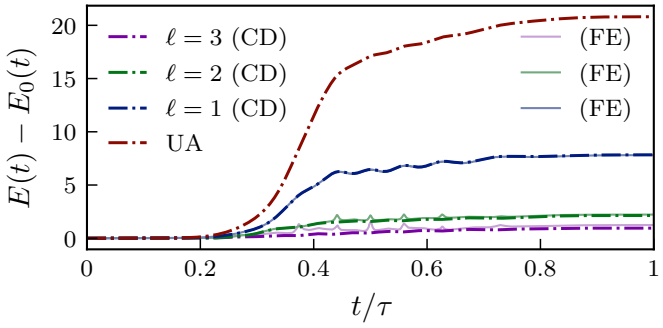
$$\mathcal{H}(\lambda) = \mathcal{H}_0 - h_t\sum_{i=1}^{L}\exp\left[-\frac{(i - c_t(\lambda))^2}{w_t^2}\right]\sigma_i^z, \quad (15)$$

$$\mathcal{H}_0 = J\sum_{i=1}^{L-1}\sigma_i^z\sigma_{i+1}^z + h_z\sum_{i=1}^{L}\sigma_i^z + h_x\sum_{i=1}^{L}\sigma_i^x, \quad (16)$$

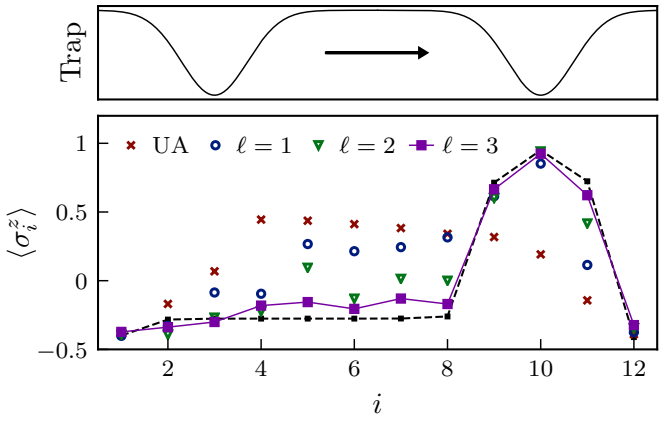
with $c_t(\lambda) = (1-\lambda)i_0 + \lambda i_f$. Tuning λ from 0 to 1 then drags the center of the trap $c_t(\lambda)$ with strength h_t and width w_t from site i_0 to i_f.

Rather than fidelity, we consider absorbed energy $E(t) - E_0(t) = \langle\psi(t)|\mathcal{H}(\lambda(t))|\psi(t)\rangle - \langle\psi_0(t)|\mathcal{H}(\lambda(t))|\psi_0(t)\rangle$ as a measure for dissipation, as shown in Fig. 4a for $\ell = 1, 2, 3$. It is clear that, for the given protocol duration, the UA protocol fails in reproducing the final state. This is remedied by including CD terms with $\ell = 1, 2, 3$, reducing dissipation and absorbed energy by a factor 20 [57]. The Floquet drive reproduces the CD results, with only minor deviations at intermediate times when $E_0(t)$ becomes extremal. The CD driving is crucial in reproducing the final spin profile σ_i^z (Fig. 4b). While the proposed method seems to work particularly well for this type of model, as also observed in the optical case [58], this is representative for more general many-body systems. Finally, note that it was not the derivation of the gauge potential and the Floquet drive that was the computational bottleneck, but rather the time evolution as validation of the protocol. The former remain applicable for arbitrary large system sizes and should similarly lead to significant suppression of energy losses.

Conclusion and outlook. – It was argued that the adiabatic gauge potential can be efficiently constructed as a series of variationally-optimized nested commutators. While constructions of the gauge potential and CD driving in complex systems generally rely on dy-



(a) Absorbed energy for the UA, CD and FE protocol with $\ell = 1, 2, 3$.



(b) Spin profile at time τ for the UA and the CD protocol with $\ell = 1, 2, 3$. Exact final profile is given in a dashed line.

FIG. 4: Moving the magnetic trap in time $\tau = 0.5$ from site $n_0 = 3$ to site $n_f = 10$ for an Ising model with parameters $L = 12$, $J = -1$, $h_x = 0.8$, $h_z = 0.9$, $h_t = 8$, $w_t = 1$. $\omega_0 = 10 \cdot 2\pi$ and $\omega = 10^4 \cdot \omega_0$.

namical symmetries or exactly-solvable models, the proposed expansion can be constructed without having to resort to exact diagonalization and remains well-defined in general (chaotic) many-body systems. Due to the similarity between this series and the Magnus expansion in periodically-driven systems, this potential is easily realized through Floquet-engineering, such that the resulting approximate counterdiabatic protocols can be realized via Floquet driving without introducing additional interactions. As illustrated on various few- and many-body systems, a small number of terms can result in a drastic increase in fidelity. This presents the usual trade-off in fast-forward protocols, where an increase in fidelity can be obtained provided precise control over the driving and access to large interaction strengths is available [59–61].

In practice, this protocol is expected to mainly be useful when no efficient CD protocol can be obtained or realized, as in ergodic systems, and when a few commutators already provide a large increase in fidelity (as when e.g. the induced gap is large). The number of necessary commutators is expected to increase with a decreasing gap, with an additional drawback being the high energetic cost of the oscillations (by now common in Floquet systems), where higher orders necessitate higher driving frequencies and access to higher harmonics. The presented method requires neither the presence of dynamical invariants, scaling laws, or the closing of the commutators under some Lie algebra, common requirements for CD driving in complex systems.

Future applications and extensions are plenty. Current simulations were performed on spin systems, but can immediately be extended towards bosonic or fermionic models. While the expansion of the gauge potential is particularly convenient for CD driving, the exact potential contains information about the geometry of all states, adiabatic deformations, integrability and its violations, approximate conservation laws,..., which also follow from the current approximation. This should allow for the construction of approximately-conserved operators and integrable gauge potentials analogous to integrable Floquet Hamiltonians [62].

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