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Photon pair condensation by engineered dissipation

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Dissipation can usually induce detrimental decoherence in a quantum system. However, engineered dissipation can be used to prepare and stabilize coherent quantum many-body states. Here, we show that by engineering dissipators containing photon pair operators, one can stabilize an exotic dark state, which is a condensate of photon pairs with a phase-nematic order. In this system, the usual superfluid order parameter, i.e. single-photon correlation, is absent, while the photon pair correlation exhibits long-range order. Although the dark state is not unique due to multiple parity sectors, we devise an additional type of dissipators to stabilize the dark state in a particular parity sector via a diffusive annihilation process which obeys Glauber dynamics in an Ising model. Furthermore, we propose an implementation of these photon-pair dissipators in circuit-QED architecture.

With the rapid development of quantum optical technology and quantum information platforms such as cavity/circuit quantum electrodynamics (QED) [1–3] and Rydberg polaritons [4], it is now possible to investigate strongly-correlated many-body physics of photons [3, 5–8]. While photons can have strong interactions in these platforms, they do not naturally thermalize, and one has to synthesize thermalization and a chemical potential to obtain many-body ground states [9–14]. Remarkably, dissipation induced by the environment, which is usually regarded as a noise source leading to decoherence of the states, can actually become a useful resource. If harnessed properly, dissipation can be used to autonomously prepare and stabilize an exotic many-body pure state as the steady/dark state of a system [15–28]. In the context of analogue quantum simulation, some well-known examples of dissipative engineering schemes include the autonomous preparation and stabilization of the Bose-Einstein condensate (BEC) state [16], the Majorana-fermion state [22] and the Chern insulator state [26], all of which can be thought as the ground states of non-interacting Hamiltonians. In a digital quantum simulation scheme [29–31], a class of jump operators has been realized [19, 32, 33], where the steady states correspond to the ground states of a specific class of interacting Hamiltonians.

Meanwhile, there have been significant experimental achievements in engineering analogue dissipators with higher-order photon jumps in small circuit-QED systems [34–40]. While these efforts have been motivated by autonomous error correction for a single- or two-sites system, it is interesting to investigate engineering many-body states, using these tools. Specifically, one can ask whether a strongly-correlated pure many-body state can be stabilized with an engineered analogue dissipator? In this Letter, we answer this question by proposing a type of two-photon jump operator which can dissipatively prepare and stabilize an exotic strongly correlated photon-pair condensate exhibiting phase-nematic order. Furthermore, we propose an analogue experimental realization with circuit-QED systems. The added benefit of this approach is that one does not require effective thermalization or generation of a chemical potential in a photonic system.

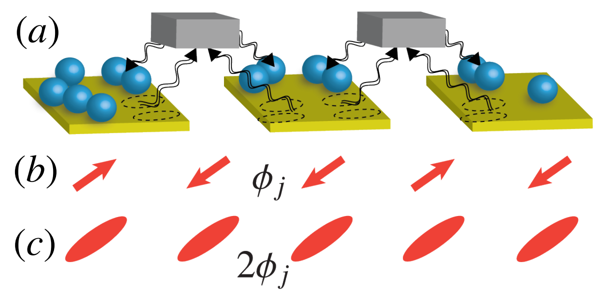

FIG. 1: (a) Illustration of the dissipative process described by the photon pair jump operator. (b) The U(1) phase variable ϕ_j (illustrated by the arrows) in the photon pair condensate is disordered due to the freedom to fluctuate by π. (c) Twice of the U(1) phase angle $2\phi_j$ (illustrated by the rod) is ordered, corresponding to a phase-nematic order.

In order to illustrate the key idea, we start with a canonical example of Ref. [16]. For an open quantum system with Markovian environment, the system dynamics can be described by the Lindblad master equation:

$$\frac{d}{dt}\rho = -i[H,\rho] + \mathcal{L}\rho, \tag{1}$$

where H is the Hamiltonian of the system and the Liouvillian $\mathcal{L}\rho = \sum_j \kappa_j (2 l_j \rho l_j^\dagger - l_j^\dagger l_j \rho - \rho l_j^\dagger l_j)$ describes the dissipation associated with the jump operator l_j with decay rate κ_j.

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Consider the dynamics of bosonic particles on 1-dimensional lattice with $H = 0$ and a number-conserving jump operator of the form $l_j=(a_j^\dagger + a_{j+1}^\dagger)(a_j - a_{j+1})$, connecting nearest neighbors in the lattice, where $a_j^\dagger$ is the boson creation operator on site j. These jump operators stabilize a BEC with fixed number of particles, i.e., a pure dark state $|D\rangle = (a_{k=0}^\dagger)^{N_{tot}}|0\rangle \propto (\sum_j a_j^\dagger)^{N_{tot}}|0\rangle$, where N_{tot} is the total number of bosons. A simple way to understand these jump operators is the mean-field picture in which $a_j \to \sqrt{\bar{n}}e^{i\phi_j}$, where ϕ_j represents the compact U(1) phase variable (mod 2π).

The dark-state condition $l_j|D\rangle = 0$ gives rise to the mean-field solution: $\phi_{j+1} - \phi_j = 0$ (mod 2π), suggesting a phase locking between neighboring sites. In this case, the dark state has long-range order, i.e., $\langle a_i^\dagger a_j\rangle \xrightarrow{|i-j|\to\infty} \langle a_i^\dagger\rangle\langle a_j\rangle = \bar{n}$, where we have introduced mean-field order parameter $\langle a_j\rangle \approx \sqrt{\bar{n}}\langle e^{i\phi}\rangle$ and ϕ is the uniform phase, after the spontaneous breaking of a $U(1)$ symmetry. While this order parameter is fragile in 1D, these ideas can be generalized to higher dimensions where the long-range order can become robust.

Pair jump operators.— In this work, we propose a quartic jump operator connecting site j and $j+1$ of the form

$$l_j = (a_j^{\dagger 2} + a_{j+1}^{\dagger 2})(a_j^2 - a_{j+1}^2), \tag{2}$$

in a 1D lattice, as shown in Fig. 1(a). This can be generalized to 2D and 3D by assigning a jump on each link of the lattice. Before deriving the exact form of the wave function, we consider a mean-field solution in which we take $a_j \to \sqrt{\bar{n}}e^{i\phi_j}$. The dark-state condition $l_j|D\rangle = 0$ hence gives rise to the mean-field solution: $2(\phi_{j+1} - \phi_j) = 0$ (mod 2π), suggesting a locking of twice the phase variables between neighboring sites. This leads to the mean-field order parameter $\langle a_j^2\rangle = \bar{n}\langle e^{2i\phi_j}\rangle$ rather than $\langle a_j\rangle$, as in the previous case. In fact, we now have $\langle a_j\rangle = 0$. For correlation functions, we get

$$\begin{aligned}&\langle a_i^{\dagger 2} a_j^2\rangle \xrightarrow{|i-j|\to\infty} \langle a_i^{\dagger 2}\rangle\langle a_j^2\rangle = \bar{n}^2\langle e^{i2(\phi_i-\phi_j)}\rangle = \bar{n}^2,\\ &\langle a_i^\dagger a_j\rangle = \bar{n}\langle e^{i(\phi_i-\phi_j)}\rangle = \bar{n}\langle e^{i(\phi_i-\phi_j)+\pi}\rangle = -\langle a_i^\dagger a_j\rangle = 0,\end{aligned} \tag{3}$$

for $i \neq j$. While $(2\phi_j)$ exhibits long-range order, ϕ_j does not since ϕ_j can flip by π and still satisfy the dark-state condition [c.f. Fig. 1(b,c)]. This photon pair condensate exhibits phase-nematic order. A similar state has been studied in the context of Josephson junction arrays [41],the symmetry breaking phase of a photon pair hopping Hamiltonian [42] and fragmented many-body state in the ultra-cold atomic system [43–45]. The oriented rods without an arrow head in Fig. 1(c) represent the local order parameter $\langle e^{i2\phi_j}\rangle$ for such a state, which does not differentiate the π-phase flip of ϕ_j and corresponds to the spontaneous breaking of a $U(1)/\mathbb{Z}_2$ symmetry.

Exact solutions.— For the system where the Hamiltonian $H = 0$ and the jump operator is described in Eq. (2).The steady state density matrix is given by $\rho_{ss} = |D\rangle\langle D|$ where $|D\rangle$ is annihilated by the jump operators in Eq. (2) satisfying $l_j|D\rangle = 0$.

We find that the dark state $|D_{2n}\rangle$ can be described as a condensate of n two-photon bound states:

$$|D_{2n}\rangle \propto A^{\dagger n}|0\rangle, \tag{4}$$

where $A^\dagger = \sum_j a_j^{\dagger 2}(n_j+1)^{-1}$ [58] is the creation operator of quasi-particles related to photon pair bound state and $n_j = a_j^\dagger a_j$ is the on-site number operator. Note that the extra normalization factor $(n_j+1)^{-1}$ in the definition creation operator $A^\dagger$ only affects the relative weights of different photon pair spatial configurations, but not the essence of the pair condensation.

One can easily see that the single-particle correlator $\langle a_i^\dagger a_j\rangle$ (for $i \neq j$) vanishes because $a_i|D_{2n}\rangle$ and $a_j|D_{2n}\rangle$ have zero overlap since the photon occupation on site i and j becomes odd respectively. On the other hand, the pair correlation $\langle a_i^{\dagger 2} a_j^2\rangle$ is flat since $a_i^2|D_{2n}\rangle = a_j^2|D_{2n}\rangle \propto |D_{2(n-1)}\rangle$ due to the fact that taking a pair out of the condensate at any site results in the same condensate with $n-1$ pairs of photons. This is just a manifestation of the definition of a pair condensate.

We numerically simulate the time-dependent master equation Eq. (1) for an open 1D chain via quantum trajectory method with time-evolving block decimation (TEBD) algorithm [46, 47], with results shown in Fig. 2. We start with a product Fock state $|2,0,2,0,...\rangle$, and the jump operator drives the system to the steady (dark) state. We see from Fig. 2(a) that the single-particle correlation function $\langle a_{L/4}^\dagger a_{L/4+10}\rangle$ remains zero at all times, while the pair correlation function $\langle a_{L/4}^{\dagger 2} a_{L/4+10}^2\rangle$ grows rapidly with an exponential saturation until reaching the steady state. The whole time evolution resembles a cooling process. The cooling time is independent of the system size, as seen in the plot where the total number of sites is varied as $L = 16, 24, 32$. The exponential saturation behavior and the cooling time is manifest on a logarithmic scale, see Fig. 2(b).

We also plot the pair correlators as a function of the distance between two sites, i.e., $\langle a_{L/4}^{\dagger 2} a_{L/4+j}^2\rangle$ versus time t, as shown in Fig. 2(c). We see that the state has almost flat correlation when reaching the dark state, consistent with the prediction from the analytical solution shown above. Before reaching the dark state, the correlator is not flat and decays with distance. This is due to the fact that correlation between more distant sites needs more time to be built up. Fig. 2(d) shows the equilibrium time T_{eq} as function of distance. The equilibrium time $T_{\rm eq}$ is defined as the time it takes for the correlator $\langle a_{L/4}^{\dagger 2} a_{L/4+j}^2\rangle$ to reach 80% of its steady state value. The spreading of the correlation function follows the Lieb-Robinson light cone beheavior. In addition, we have observed that the introduction of a Kerr non-linearity in the form $H = U a_j^{\dagger 2} a_j^2$ in the system Hamiltonian leads to an exponential decay of correlator (in 1D) as a function of the distance $j-1$. The decay becomes faster increasing U.

Parity sectors.—The above analytical and numerical analyses only consider a simplified situation where the initial condition has all even number of photons. We note that even for fixed total photon number, the dark-state subspace has extensive degeneracies 2^{L-1}. (L is the number of sites), labeled by the local parity $P_j = (-1)^{n_j}$ on each site. The exact wave function we wrote down above in Eq. (4) is only the exact wave function for the sector where the parities of all sites are all even, i.e., $P_j = 1$ for all j, which we call a "pure pair condensate". On top of that, there are odd-parity "defects", which

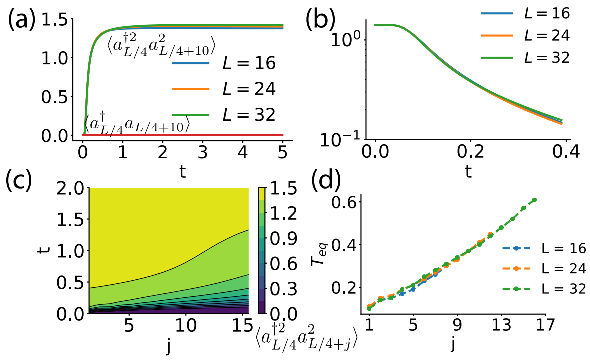

FIG. 2: (a) Time evolution of single-photon and photon pair correlators as a function of time with various system size L. The average photon density $\bar{n} = 1$. (b) Time evolution of $\langle a^{\dagger}_{L/4}{}^2(t{=}\infty)a^2_{L/4+10}(t{=}\infty)\rangle - \langle a^{\dagger}_{L/4}{}^2(t)a^2_{L/4+10}(t)\rangle$. Exponential saturation of the pair correlators (plotted in log scale) shows the dark-state cooling time is independent of system size. (c) Pair correlation function as a function of distance and time for $L = 32$. The unit of time is κ^{-1}. (d) Equilibrium time $T_{\rm eq}$ as function of distance j. The linear dependence of the equilibrium time as function distance is due to the finite propagation speed of entanglement.

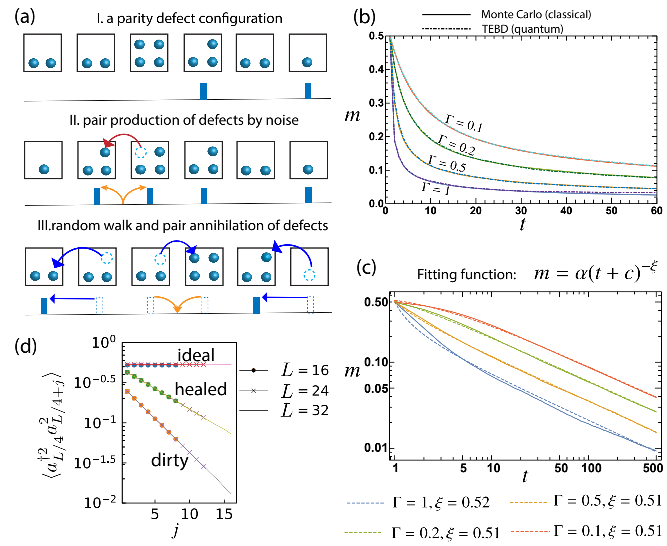

FIG. 3: (a) Illustration: I. particular odd parity defect (blue bars) configuration; II. pair production of defects from single-photon hopping noise; III. random walk of defects due to conditional hopping, and the induced pair annihilation process of defects (healing). (b) Defect density (m) as a function of time (t). Classical Monte Carlo simulation (solid) with $L = 100$ sites and quantum trajectory with matrix product state simulation (dot-dashed) with $L = 30$ sites of the time evolution of the average defect density. (c) Log-log plot of the classical Monte carlo simulation (solid) for $L = 100$, with the fitted curves (dashed) showing the asymptotic power-law scaling $m(t) \sim (\Gamma t)^{-1/2}$. (d) Pair-correlators normalized with the average photon density $\bar{n} = 0.5$ at different system size $L = 16, 24, 32$ in the situations: **i**. ideal (no noise); **ii**. dirty case (with noise but no healing); **iii**. healed case (with noise and healing).

are created in pairs from the pure pair condensate.

The wave function of a particular defect configuration can be given by $|D'_{n_d}\rangle \propto \prod_{i=1}^{2n_d} a'^{\dagger}_{d_i} |D_{2(n-n_d)}\rangle$, where n_d denotes the number of pairs of odd-parity defects and their positions are labeled by d_i. Several solutions of the parity-sector problem are discussed as follows.

To begin with, we note that the different parity sectors are not coupled together via the jump operators. Similar to what has been considered in the numerical simulation in Fig. 2, one can start with an initial product state in the all-even sector (easy to prepare experimentally with pulses in the presence of onsite nonlinearity). In this case, the jump operator will only drive the system to the dark state $|D_{2n}\rangle$ in Eq. (4), in the absence of unwanted noise. We note that noise is always present in experimental systems which either brings the state to different sectors. Therefore, one only expects to prepare the targeting dark state with the jump operators before the unwanted decoherence dominates. If one aims to stabilize the dark state, extra measurement or stabilizing schemes are needed as discussed below.

A more general solution is: by imposing measurement and feedback operation on the parity of each site, i.e., $P_j = (-1)^{n_j}$, it is possible to keep projecting the many-body state to a particular parity sector. A jump operator describing such measurement and feedback operation to stabilize parity-all-even sector is

$$c_j = \Gamma(a^{\dagger}_{j+1}a_j + a^{\dagger}_{j-1}a_j)\left(\frac{1-P_j}{2}\right), \qquad (5)$$

where Γ is the hopping rate. Note that this jump operator applies a hopping term connecting that particular site to its nearest neighbors conditioned by the the parity on the particular site being odd. It causes the odd-parity defect to take a random walk and eventually annihilate with another parity defect, as illustrated in Fig. 3(a). This diffusive defect annihilation process resembles a chemical reaction described by the formula: **df** + **df** → 0, where **df** stands for a single defect. Therefore, no defect will exist in the steady state if the total photon number is even ($2n$), and so the steady state becomes a pure pair condensate. We call such a process "*healing*".

Since the parity measurement at time $t + dt$ will post-select the direction (left or right) to which the defect has hopped at time t, the defect dynamics can be exactly mapped to a classical stochastic dynamics of the diffusive annihialation problem. For a 1D chain, the dynamics of defect density exhibits power law decay: $m(t) \sim =(\Gamma t)^{-1/2}$ [48, 49]

The classical Monte Carlo simulation (with 100 sites) of the time evolution of the average defect density $m(t)$ quantitatively agrees with the quantum TEBD simulation (with 30 sites) for a 1D chain, as shown in Fig. 3(b). The former is plotted in a log-log scale in Fig. 3(c), where the associted fitting curves confirms an asymptotic power-law $m(t) \sim (\Gamma t)^{-1/2}$. We have checked in both types of simulations that the time evolution of $m(t)$ is almost independent of system size.

In the presence of additional noise, such as incoherent single-photon hopping described by the jump operator $l'_j{=}a^{\dagger}_j a_{j+1}$, there exists a finite defect production rate Γh. The defect production can be balanced by the diffusive defect annihilation process with hopping rate Γ, and the defect density

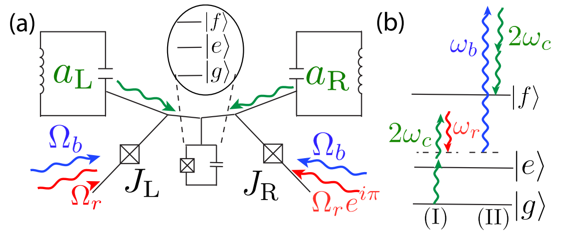

FIG. 4: Proposed circuit-QED setup. (a) Two high-Q cavities at frequency ω_c and an three-level anharmonic oscillator($|g\rangle, |e\rangle$ and $|f\rangle$) coupled to the Josephson junction modes J_L and J_R. Both of the Josephson junction modes are driven by a two-tone drive with frequency ω_r and ω_b. (b) Process (I) shows a pairs of cavity photons combine with a driving ω_r to produce a virtual junction excitation $g \to e$. The effective coupling is given by $(a_L^2 - a_R^2)|e\rangle\langle g|$. The minus sign in the effective coupling comes from the relative phase of the Rabi frequency Ω_r as shown in (a). Process (II) gives the effective coupling $(a_L^{\dagger 2} + a_R^{\dagger 2})|f\rangle\langle e|$. These two virtual processes are detuned from the respective resonance condition by a frequency δ. The combination of these two processes yields the desired interaction.

approaches a residual steady-state density m_s with a characteristic relaxation time τ. We have the following scaling (when $h \ll 1$): $m_s \sim h^{1/\delta}$ and $\tau \sim \Gamma^{-1}h^{-\Delta}$ [48]. Note that even a large hopping rate Γ cannot reduce the steady-state density m_s, but only the healing time τ. A generic scaling law $\Delta\xi = 1/\delta$ should be satisfied [50]. For a 1D chain, we have $\delta = 2$ and $\Delta = 1$ [59].

In Fig. 3(d), we use TEBD to calculate the steady-state pair correlators $\langle a_{L/4}^{\dagger 2} a_{L/4+j}^2 \rangle$ for system sizes $L = 16, 24, 32$, in the following situations: **i**. ideal case (no noise: $h = 0$); **ii**. dirty case (in the presence of single photon hopping noise: $h > 0$, without healing: $\Gamma = 0$); **iii**. healed case (with noise and healing: $h, \Gamma > 0$). We see that in the presence of noise which proliferate the parity defects, the steady state ends up with a mixed state and the correlators decays exponentially, while the healing process significantly slows down the decay.

Due to the complexity of the parity operator, one typically can only realize such a jump operator with an active parity measurement in circuit-QED setup through either continuous [51] or discretized repeated [52, 53] measurement schemes, instead of using continuous autonomous stabilization. Nevertheless, in the situation that we first impose hard-core condition for occupation more than three photons, i.e., $a_j^{\dagger 3} = 0$, the parity condition can be simply converted to occupation condition, and we can effectively re-express the jump operator in Eq. (5) as $c_j' = (a_j^\dagger a_{j+1} + \text{H.c.}) n_j (n_j - 2)$, up to a constant factor of 2. This jump operator can potentially be implemented continuously and hence autonomously stabilize the targeting pure photon pair condensate. Similarly, one can either actively or autonomously monitor and stabilize the total photon number in the system.

Finally, we can also passively stabilize the parity sector via energetic constraint in the case of hard-core condition $a_j^{\dagger 3} = 0$. This is achieved by assign the following energy penalty term $\delta H = -V_0 n_j(n_j - 2)$ in the Hamiltonian H [54]. Therefore, the configuration with single-photon occupation (odd parity) on any site is projected out of the low-energy sector.

Experimental realization with Circuit QED.—We illustrate the experimental scheme with a two-site jump operator $l = (a_L^{\dagger 2} + a_R^{\dagger 2})(a_L^2 - a_R^2)$. The generalization to a 1D chain is straightforward. Consider a system consisting of the two high Q cavities a_L and a_R, and an anharmonic oscillator. The anharmonic oscillator is modeled by a three-level system ($|g\rangle, |e\rangle$ and $|f\rangle$) and is coupled to a cavity a_λ and a Josephson junction mode J_λ, where $\lambda = L, R$, at both sides respectively. Both junction modes are driven by a two-tone drive $\Omega_\lambda(t) = \Omega_{\lambda,r} e^{i\omega_r t} + \Omega_{\lambda,b} e^{i\omega_b t}$ as shown in Fig. 4.

We engineer a two-photon jump operator via four-wave mixing induced from the junction modes J_L and J_R. The drive ω_r (ω_b) is used to introduce an exchange of two photons of cavity mode a_λ^2 ($a_\lambda^{\dagger 2}$) with the excitation $g \to e$ ($e \to f$). The four-wave mixing interaction of the pump ω_r (ω_b) is proportional to $\sum_\lambda \Omega_{\lambda,r} a_\lambda^2 |e\rangle\langle g|$ ($\sum_\lambda \Omega_{\lambda,b} a_\lambda^{\dagger 2} |f\rangle\langle e|$). The minus sign in the jump operator can be engineered by introducing a π phase shift between $\Omega_{L,r}$ and $\Omega_{R,r}$. The effective Hamiltonian is of the form

$$H' = -\frac{\chi}{2} \sum_{\lambda=L,R} a_\lambda^{\dagger 2} a_\lambda^2 + g_1 T_- |e\rangle\langle g| + g_2 T_+^\dagger |f\rangle\langle e| + h.c.$$

where $T_\pm = a_L^2 \pm a_R^2$, χ is the Kerr nonlinearity induced from the junction modes and g_1 (g_2) is proportional to the Rabi frequency Ω_r (Ω_b) as shown in Fig. 4a.

To obtain the jump operator in Eq. (2), we combine the two-photon loss process (T_-) and two-photon creation process ($T_+^\dagger$) in equation above. We can achieve this by detuning the two four-wave mixing processes by δ as shown in Fig. 4b so that only a cascade of two such processes is possible [39]. The detunning $\delta \gg g_1, g_2$ allows a two-photon exchange processes via a Raman transition. The effective Hamiltonian becomes

$$H_{\text{eff}} = -\frac{\chi}{2} \sum_{\lambda=L,R} a_\lambda^{\dagger 2} a_\lambda^2 + \frac{1}{\delta} \begin{bmatrix} |g_2|^2 T_+^\dagger T_+ & g_1 g_2 T_+^\dagger T_- \\ g_1^* g_2^* T_-^\dagger T_+ & |g_1|^2 T_-^\dagger T_- \end{bmatrix} \quad (6)$$

where the 2×2 matrix acts on the anharmonic oscillator basis $\{|f\rangle, |g\rangle\}$. In Eq. (6), the term $T_+^\dagger T_- |f\rangle\langle g|$ gives the desired two-photon process coupled to $g \leftrightarrow f$ transition. Assuming the decay rate(κ) of process $f \to g$ is much greater than all of the other coupling constant in Eq. (6). The system can be described $H_s = -\frac{\chi}{2} \sum_{\lambda=L,R} a_\lambda^{\dagger 2} a_\lambda^2 + \frac{|g_1|^2}{\delta} (a_L^{\dagger 2} - a_R^{\dagger 2})(a_L^2 - a_R^2)$ and the jump operator $l_s = \sqrt{\frac{2}{\kappa}} \frac{g_1 g_2}{\delta} (a_L^{\dagger 2} + a_R^{\dagger 2})(a_L^2 - a_R^2)$. The jump operator gives rise to Eq. (1) for the array case. The self-Kerr and the cross-Kerr nonlinearity terms in Hamiltonian H_s can be eliminated by adding an extra pair of Josephson junction and a two-level system [60].

Conclusion and outlook.— We have discovered a photon pair jump operator which can dissipatively prepare and stabilize an exotic two-photon pair-condensate with phase-nematic order, with a circuit-QED implementation. We have further proposed a conditional hopping operator to stabilize the dark state in a particular parity sector. Such a scheme can also be realized with Rydberg polaritons or ion-trap systems. An interesting future direction would be using such higher-order dissipators for autonomous quantum error correction using bosonic codes.

Acknowledgments

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[58] See Supplemental Material for more details on the derivation dark state exact solution.

[59] See Supplemental Material which includes Ref. [48–50, 55–57]

[60] See Supplemental Material for more details.