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# Bedforms Produced on a Particle Bed by Vertical Oscillations of a Plate

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We describe a new mechanism that produces bedforms and characterize the conditions under which it operates. The mechanism is associated with pressure gradients generated in fluid saturated particle bed by a plate oscillating in the water above it. These vertical pressure gradients cause oscillatory bed failure. This facilitates particle displacement in its interior and transport at and near its surface that contribute to the formation of a heap under the plate. Flows over erodible beds generally cause shear stresses on the bed and these induce bed failure. Failure driven by pressure gradients is different from this. We report on bedforms in a bed of glass beads associated with such fluctuating pressure gradients. We measure the development of the profiles of heaps as a function of time and determine the tangential and normal motion of areas on the beds surface, and estimate the depth of penetration of the tangential transport. The measurements compare favorably with a simple model that describes the onset of failure due to oscillations in pressure.

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The wide variety of observed bedforms, including ripples and dunes, reflects the sensitivity of their development to the conditions under which they form. Ripples have even been observed under turbulence where there was no mean flow and no mean shear [1, 2]. There is, as yet, no agreement over the mechanisms responsible for such features.

A dominant aspect of the conditions at play during deposition is the flow of air or water over the sediments, which continues to a certain extent in the interstitial fluid of the bed. The flow can be continuous or oscillating, driven by currents, winds, tides, or waves. Wherever bedforms develop in nature, the flow tends to be turbulent. The flow and the turbulence impose shear stresses on the bed, and the action of these stresses has rightly attracted extensive scrutiny (e.g., [3–8]). Bed failure under a mean shear is responsible for many interesting bedforms. Shear stresses, however, are only part of the picture. Bed failure has also been predicted to occur because of pressure gradients in the bed [9–12], which can be associated with surface waves [13, 14] and internal waves [15].

Not only shear stresses, but also pressure gradients are intrinsic to fluid motion. Vertical pressure gradients within a horizontal bed produce a vertical force on the particles in the bed. When this vertical force is large enough, it can lift the particles near the surface of the bed and so cause bed failure. In this Letter, we show that this type of failure can lead to the development of bedforms.

Oscillatory forcing in the form of vibrations is known to have a fluidizing influence in systems of dry grains [16] and of particle suspensions [17]. Oscillatory motion of the grains is analogous to thermal motions in Brownian or molecular systems, and it is possible to relate the macroscopic rheological properties and the diffusional properties of the medium at the grain scale [18]. In our system the grains, driven by oscillations in the fluid, experience small local rearrangements that permit the slow global deformation of the bed and the growth of the heap. This behavior is similar to the creep deformation seen in aggregates of dense, dry, frictional grains [19, 20], except it is driven by pressure, rather than shear forces. The link between microscopic dynamical mechanisms, at the grain scale, to macroscopic rheological behaviors has not yet been made.

We employed a simple system that explicitly incorporates vertical pressure gradients in a flow that, at lowest order, has a stagnation point at the bed. This is a flow that involves deviatoric straining with no rotation. Here, we consider an oscillatory version of such a flow with a horizontal bed surface and vertical symmetry axes.

*Heap formation.—* The oscillatory stagnation-point flow is a simplification of the laboratory and natural flows that involve only turbulent fluctuations. It can be thought of as a turbulent fluctuation of a single frequency acting on the bed. The pressure associated with the fluctuation drives a fluid flow in the porous bed that exerts forces on the spheres within it. Consequently, there are oscillatory forces applied both on the surface and within the bed. As the horizontal distance from the axis increases, the magnitude of the shear stresses on the surface of the bed increases from zero and that of the vertical pressure gradients within it decreases from a maximum. The experiments indicate that both superficial stresses

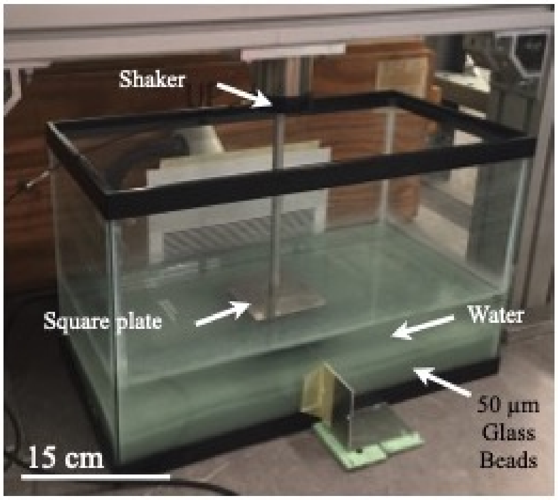


FIG. 1: A photograph of the apparatus. The tank, 50 cm long and 30 cm wide, was first filled to a depth of 8 cm with glass beads with a mean diameter, $d$, of 0.5 mm and then filled with water to submerge a 7.5 cm square plate. A cross beam held an electro-magnetic shaker over the tank that oscillated the plate up and down through the water. The plate was centered in the tank.

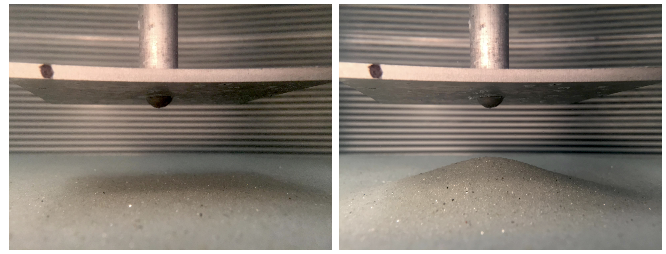

FIG. 2: *Left:* The bed viewed from the side at the beginning of an experiment. Visible above the bed is the oscillating plate and behind it scale bars spaced by 4 mm. The plate was 2.7 cm over the bed and 8.6 cm below the surface of the water. *Right:* An image of the same bed after $2 \times 10^5$ cycles (1.5 hours), during which time the plate oscillated at 15 Hz with a peak-to-peak displacement of 1.5 mm, and a heap formed.

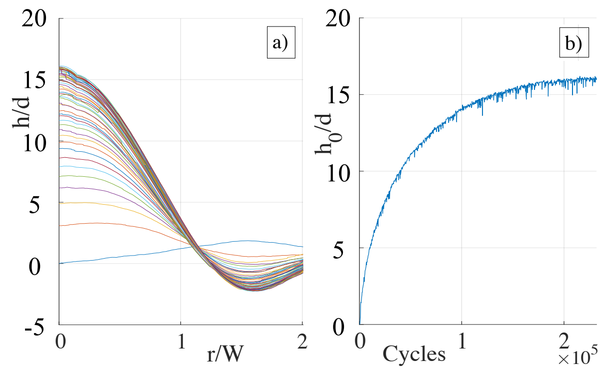


FIG. 3: a) The evolution of the surface of the bed, $h$, as a function of radius, $r$, at intervals of 1800 cycles (120 s). The image was obtained through analysis of videos of the heap. The heap grew upward in time from its initial shape indicated by the nearly horizontal blue line. b) The evolution of the center height, $h_0$, under the same conditions as in FIG. 2.

and pressure gradients are involved in the formation of the heap.

A steady stagnation point flow would be expected to produce a depression if the flow were into the bed and an elevation if there were flow away from the bed. However, when the flow is oscillatory, there is no mean shear stress on the bed and no mean force within it. In this case, the development of a heap raises the question of what broke the symmetry. We address this in the present Letter.

In FIG. 1, we show the experimental apparatus. The particles were glass spheres with a mass density, $\rho_s$, of $2.7\,\mathrm{g/cm^3}$, an average diameter, $d$, of 0.50 mm and an approximately uniform distribution between 0.43 and 0.60 mm (Potters Industries, P-0230); the plate was square with side lengths $2W = 7.5$ cm. Heaps formed above the bed when the plate was submerged at depths greater than 3 cm beneath the surface of water and oscillated at frequencies between 10 and 40 Hz with peak-to-peak amplitudes, $A$, between 0.02 to 0.20 cm at heights, $H$, above the bed between 1.6 and 3.3 cm. The heaps developed in about 1.5 hours and required many oscillations of the plate to do so – on the order of a hundred of thousand. In other words, the heap was a cumulative effect of many small motions in the bed. This phenomenon is new and startling.

In FIG. 2, we show photographs of the bed at the beginning and end of the heap development. The evolution of the surface of the bed and the evolution of the center height, $h_0$, of the heap are plotted in FIGS. 3a) and 3b), respectively. A laser sheet was used to illuminate the surface, a video then captured its evolution, and image analysis was employed to determine its development [21].

Vibrations of the tank influenced the growth of the heap and steps were taken to minimize them. When oscillations of the plate did not disturb the free surface, the proximity of the plate to the free surface had little influence on the growth of a heap. Dye showed that the flow above the bed was oscillatory, not turbulent, and indicated a vortex the size of the gap, with axis parallel to the edge, forming at each edge of the plate and rotating clockwise on the left edge and counter-clockwise on the right. Increasing the amplitude, at fixed frequency and geometry, increased the radius of the heap. The heap would cease to form at fixed frequency and geometry if the amplitude was too small. Decreasing the frequency at fixed amplitude and geometry increased the height of the heap, but the radius remained the same. Increasing the height of the plate above the bed decreased the radius of the heap, and its height is decreased if the plate is too near the bed; but not influenced for heights sufficiently distant. These features of the heap formation will

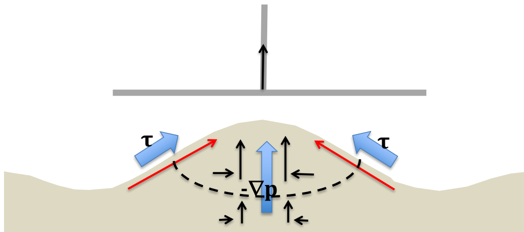


FIG. 4: The directions of the stresses $\nabla p$ and $\tau$ indicated in the sketch correspond to those during upward motions of the plate; they point in the opposite directions during downward motions. On the other hand, the motions of grains break this symmetry and tend to move in the direction of the stresses only during upward motions of the plate – they are locked in place when the plate moves downward. The dashed line gives an indication of the failed region.

be detailed in a subsequent publication [22]; in this Letter, we focus on the existence of the heaps and provide an indication of the mechanisms that are responsible for their formation. We consider the tallest heap, produced when the plate oscillated at a frequency of 15 Hz with a peak-to-peak displacement of 1.5 mm.

Our hypothesis is that failure of the bed resulted from vertical pore-pressure gradients larger than the buoyant specific weight of the bed material. This failure occurred periodically when the plate moved upward. It is this failure of the bed that broke the symmetry of the oscillatory forcing. The bed failed on the upward motion, but was impeded from returning to its original state by small rearrangements of the particles that took place during the deformation that accompanies failure. Associated with these rearrangements were irreversible vertical and radial strains that contributed to the growth of the heap. These strains occurred every cycle and, as a consequence, the heap gradually grew irreversibly.

The cyclical straining was accompanied by tangential transport at and below the surface of the bed. This transport was associated with the oscillatory shear stresses in the viscous boundary layer in the fluid near the surface of the bed. The magnitude of these shear stresses increases from zero with the horizontal distance from the axis [23, 24]. The shear stresses were directed inward when the plate was moving upward and the bed was expanded, and outward when the plate was moving downward and the bed was compressed. Tangential motions of particles at and near the surface of the bed were possible when the bed was expanded, but not when it was contracted. The resulting transport contributed to the growth of the heap away from its center. FIG. 4 provides a sketch of these mechanisms.

*Theoretical analysis.*—We employed a simple theoretical model to quantify bed failure. We approximated the sediment bed as a continuous porous medium so that flow extends into the bed in a calculable way. The model comprises an array of long plates of width $2W$, periodic in the $x$-direction, that oscillate out of phase with a circular frequency $f$ and peak-to-peak amplitude $A$. The plates oscillate about a distance $H$ above the bed in the $y$-direction in an incompressible fluid with mass density $\rho$. The oscillating, spatially-periodic, stagnation-point flow of each plate was matched with that of its neighbors. We focus on the plate with its center at $x = 0$. The pressure, $p'$, in excess of the hydrostatic, exerted on the surface of the bed by the resulting unsteady, inviscid flow under this plate is, for $0 < x < W$, $p' = \rho(A/H)\pi^2 f^2 \left(-x^2 + W^2\right) \sin(2\pi f t)$ and, for $W < x < 2W$, $p' = \rho(A/H)\pi^2 f^2 \left(x^2 + 3W^2 - 4Wx\right) \sin(2\pi f t)$. This pressure drives an oscillatory flow within a bed of particles of material mass density $\rho_s$ and concentration $c$ in the region $y < 0$. We ignored the compressibility and inertia of the interstitial fluid and followed Darcy [25] in assuming that the average fluid velocity in the bed was proportional to the gradient of the pressure, which, by conservation of mass, was a solution to Laplace's equation in $x$ and $y$, with $x$ and $y$ horizontal coordinates, with their origin on the bed under the center of the plate. The lowest spatially periodic component of the solution is

$$\begin{aligned} p' \;&\doteq\; (32/\pi)\, \rho A f^2 \left(W^2/H\right) \exp\left[\pi y/(2W)\right] \\ &\times\; \cos\left[\pi x/(2W)\right] \sin(2\pi f t)\,. \end{aligned} \qquad (1)$$

Bed failure occurs at $x = 0$ when, during the upward motion of the plate, the upward derivative of the pressure first equals the buoyant weight of the bed, $(\rho_s - \rho)\, cg$. Then, from Eq. (1),

$$\left(A f^2\right)_0 = (\rho_s - \rho)\,(H/W)\, cg/\,(16\rho)\,. \qquad (2)$$

If the product $Af^2$ exceeds this critical value, failure extends into the bed and spreads laterally. Although the model is an oversimplification, it does provide the scalings of Eq. (2) for the initiation of the bed failure.

The phase diagram shown in FIG. 5 defines the conditions under which bedforms arose. By varying the frequency, $f$, and the peak-to-peak amplitude, $A$, of the plate oscillation, we varied the strength of the interactions between the flow and the sediment bed. The model predicts bed failure caused by pressure gradients to occur for values of $Af^2$ larger than a constant; a curve of constant $Af^2$ is shown in FIG. 5. The particular form of the curve is distinctive of bed failure due to pressure gradients. The dependence of bed failure on the acceleration of the vertical oscillation is in contrast to a dependence on the velocity of horizontal vibrations, $Af$, seen in transition from stick-slip to continuous sliding in motions of a bumpy rigid plate over a particle bed [26]. When heaps do not form, the particles of the bed are seen to vibrate and there is a surficial, oscillatory, tangential flow.

To measure the tangential motion, we deposited colored glass beads of the same size and density as those of the bed in a filled circle and in a thin ring about three sphere diameters in extent, centered on the axis. We then

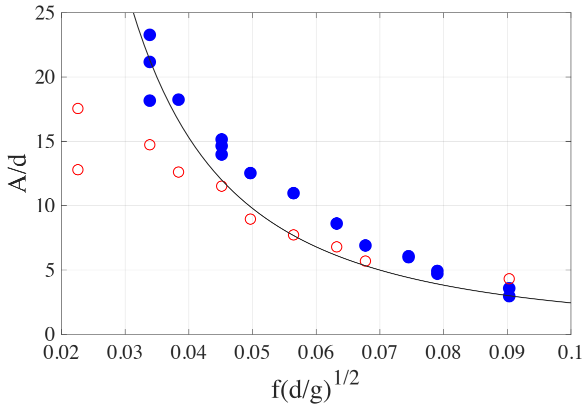


FIG. 5: A phase diagram that illustrates the conditions under which heaps were observed to develop. The filled and open circles indicate points at which failure of the bed did and did not occur, respectively. The solid line is a curve of constant $Af^2$, drawn through the lowest point of failure, at which the corresponding value of $(Af^2)_0$ was 0.24 m/s$^2$. This is a small fraction of the gravitational acceleration because of the factor of 16 in Eq. (2).

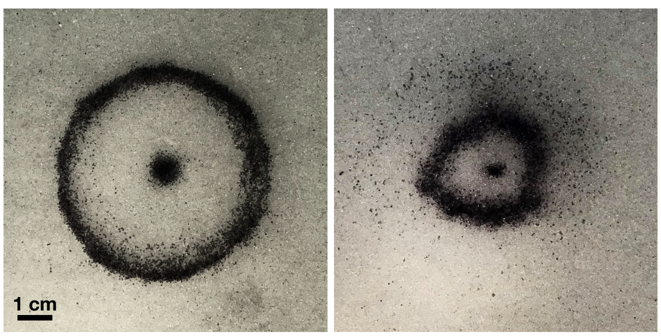


FIG. 6: The motions of a filled circle and of a ring on the surface. *Left:* The circle and ring of colored beads initially. *Right:* The circle and ring after $2 \times 10^5$ cycles.

imaged the colored sand and measured the evolution of the location of the peak distribution of color. From the images, we determined the motion of the sediment at the surface of the bed, and estimated the diffusivity of sediments near the surface.

The peak of the color distribution of the ring of spheres, with an extent on the surface of about 20 diameters, and a mean radius roughly equal to 75 diameters moved inward to roughly 40 diameters from the axis and eventually diffused. A circle filled with colored spheres centered on the axis with a radius of 15 diameters shrank slightly, stayed on the surface, and was otherwise unaffected as the heap grew. In FIG. 6 we show images of the initial and final states of the circle and ring [27].

To provide information on the flow beneath the surface, we buried a ring of colored glass beads about three diameters beneath the surface at a radius of 55 sphere diameters from the center. After about $10^5$ cycles, these spheres appeared on the surface at roughly 40 sphere diameters from the center. These experiments indicate that there was motion on and below the surface of the bed that extended from the edge of the plate to about 2 cm from its center.

To determine the depth beneath the surface to which the tangential motion persisted, we employed a flexible circular cylinder with a radius equal to the distance from the axis to the point of inflection of the profile of an undisturbed heap. The cylinder was buried in the bed and extended from the base of the bed to a depth below its surface that was different in different experiments. Cylinders with tops beneath the surface of less than 10 bead diameters were seen to influence the formation of the heap; those with tops below 10 diameters did not.

*Conclusions.*— Our novel experimental configuration makes it possible to test simple models of the role of pressure gradient fluctuations on granular beds. The data are consistent with a proposed model, and indicate that both irreversible straining of the bed and tangential transport contributed to the growth of the heap. Irreversible vertical and tangential straining of the bed dominated growth near the center of the plate. Tangential transport on and just below the surface was important near the edges of the plate and diminished towards the center. Both the straining of the bed and transport on and near its surface depended upon the flow of fluid and enabled particle motion while the plate was moving upward. Upward and inward displacements of particles in the failed region of the bed could not recover completely because of particle rearrangements during expansion; shear stresses on the bed surface, directed inward during expansion, displaced particles, while those directed outward during contraction did not.

This mechanism is different from the creep [28] or fluidization [29] in dense, dry and fluid saturated [30] granular aggregates, in which fluctuations are driven by a steady shear stress; however, the ratcheting associated with the difference in particle motions on expansion and compression is similar to that proposed for the upward motion of the larger particles in the Brazil-nut effect [31]. Finally, it has been pointed out [32] that if compressibility of the fluid is important, there is a possibility that the oscillations of the plate could resonate with a standing wave in the fluid of the bed [33]. This would reinforce the oscillations and contribute to bed expansion, when the upward and downward particle motions differ.

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[21] See Supplemental Material at [URL will be inserted by publisher]. The video shows the formation and the evolution of the heap under the following conditions: frequency of 15 Hz, amplitude of 1.4 mm and height above the bed of about 27 mm . There are 966 frames and each frame is taken 16 seconds apart. A figure with a sketch of the laser sheet illuminating a developing heap is also provided.

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