Topologically Protected Long Edge Coherence Times in Symmetry-Broken Phases

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Phys. Rev. Lett. **122**, 240605 — Published 21 June 2019

DOI: 10.1103/PhysRevLett.122.240605

Topologically-Protected Long Edge Coherence Times in Symmetry-Broken Phases

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(Dated: May 31, 2019)

We argue that symmetry-broken phases proximate in phase space to symmetry-protected topological phases can exhibit dynamical signatures of topological physics. This dynamical, symmetry-protected "topological" regime is characterized by anomalously long edge coherence times due to the topological decoration of quasiparticle excitations, even if the underlying zero-temperature ground state is in a non-topological, symmetry-broken state. The dramatic enhancement of coherence can even persist at infinite temperature due to prethermalization. We find exponentially long edge coherence times that are stable to symmetry-preserving perturbations, and not the result of integrability.

Practical quantum computation requires systems with long coherence times. This has driven recent theoretical interest in the limits and causes of decoherence in quantum many-body systems where, typically, local quantum information is rapidly scrambled. One tactic to store and process quantum information is to use topological edge modes. Combining these with many-body localization [1–9], information can be protected for infinite times, even at effectively infinite temperature [10–14]. Another avenue is to take advantage of prethermalization, wherein some observables retain memory of the initial state on a "prethermal plateau" before finally reaching their equilibrium values, leading to exponentially long coherence times [15–20].

In this Letter we demonstrate an anomalous dynamical regime—characterized by long edge coherence times—that appears only in symmetry-broken phases proximate in phase space to symmetry-protected topological phases (SPTs) [21–31]. The essential observation is that the presence of a nearby SPT phase can modify the nature of quasiparticle excitations even when the symmetry protecting the topological order is spontaneously broken at zero temperature. The topologically "decorated" [32] quasiparticles inherited from the SPT cannot be created or annihilated at the edges of the system, leading to exponential increases in coherence times (see Fig. 1). Neither fine-tuning nor integrability are required. Even more remarkably, this protection of edge coherence remains at finite temperature and can persist all the way to infinite temperature thanks to prethermalization. Aspects of SPT physics, therefore, are retained in the dynamics even if the underlying zero-temperature ground state is symmetry-broken.

Though we will focus on SPTs, a motivation for this work comes from the ongoing experimental search for quantum spin liquids [33–35], which are another form of topological paramagnets. Given the fact that many spin liquid candidate materials exhibit magnetically ordered ground states, the question arose as to whether remnants of a nearby topological paramagnetic phase could be detected in their dynamical properties. Indeed, such a "proximate spin liquid" regime was recently reported in α-$RuCl_3$ [36, 37]. In this Letter we answer this question in the affirmative, by providing an example of a proximate SPT regime whose anomalous dynamical properties are sharply defined.

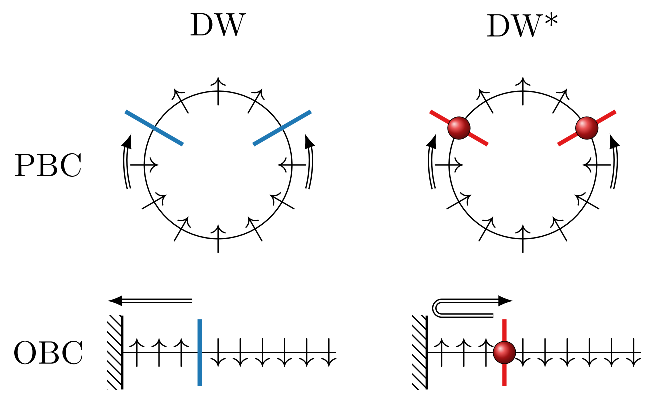

FIG. 1. Sketch of the dominant processes that tunnel between the two ferromagnetic ground states. Domain walls (DW) are represented by blue bars, and their decorated counterparts (DW*) are red and carry a $\mathbb{Z}_2$ charge. Under periodic boundary conditions (PBC), the two types of domain walls are equivalent. With open boundary conditions (OBC), however, the decorated domain walls cannot be annihilated at the edges without breaking the symmetry, so will "bounce off" instead. Decorated domain walls are therefore unable to flip the edge spin without breaking the symmetry.

Below we define a simple model of a proximate SPT regime that demonstrates exponential enhancement in edge coherence times. To understand its dynamics, we consider the regular and decorated quasiparticles inherent to the model. This quasiparticle picture is confirmed at zero temperature, where we accurately predict the coherence times via perturbation theory. We then proceed to show that the regime is robust to symmetry-preserving

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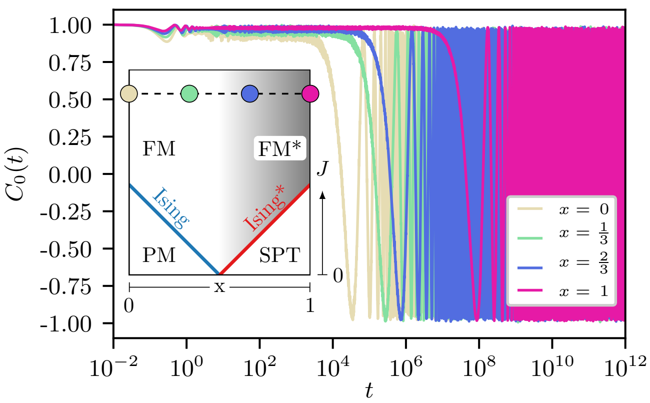

FIG. 2. Autocorrelation of the edge spin at zero temperature computed with exact diagonalization (ED) for 14 spins and OBC. The parameters are $J = 5.2$, $B = 1.424$, and $V(g_1, \ldots, g_5)$ is chosen to break integrability completely [40]. *Inset:* Sketch of the phase diagram for Eq. (1) as a function of x and J. Phases are described in the text. The location of the dots corresponds to the data by color.

perturbations, independent of integrability, and holds at all temperatures.

Model and phase diagram. We rely on the simplest model of an SPT phase in one dimension, a variant of the Haldane chain [38] protected by a global $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry [30, 32, 39]. Consider a spin-$\frac{1}{2}$ chain with two alternating species, σ and τ, with a global $\mathbb{Z}_2^\sigma \times \mathbb{Z}_2^\tau$ symmetry generated by $\prod_i \sigma_i^x$ and $\prod_i \tau_i^x$. (We use the convention $\sigma_0, \tau_0, \sigma_1, \tau_1, \ldots, \tau_{(L/2)-1}$ to label the L spins.) We adopt a Hamiltonian

$$\hat{H}(x) = J\hat{H}_{\mathrm{FM},\sigma} + (1-x)\hat{H}_{\mathrm{PM}} + x\hat{H}_{\mathrm{SPT}} + V, \qquad (1)$$

where $0 \leq x \leq 1$, $\hat{H}_{\mathrm{FM},\sigma} = -\sum_i \sigma_i^z \sigma_{i+1}^z$, $\hat{H}_{\mathrm{PM}} = -\sum_i \sigma_i^x + B\tau_i^x$, and $\hat{H}_{\mathrm{SPT}} = -\sum_i \tau_{i-1}^z \sigma_i^x \tau_i^z + B\sigma_i^z \tau_i^x \sigma_{i+1}^z$. Finally, V includes generic symmetry-preserving perturbations to break integrability, as described in the Supplemental Material [40]. As shown in the inset of Fig. 2, this model interpolates between three different phases: a ferromagnet for the σ spins at large J, a trivial paramagnet at small J and x near 0, and an SPT ("topological paramagnet") at small J and x near 1. Starting from either paramagnetic phase, J drives an Ising transition to a ferromagnet for the σ spins, and B controls the energy scale for the τ spins, which remain paramagnetic across the whole phase diagram [41].

A standard result is that the two paramagnetic phases have the same bulk properties, but are different at the boundary: the SPT has a free spin-$\frac{1}{2}$ at each edge, which is protected as long as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry survives [30, 32]. A lesser-known result is that these edge modes actually survive at the phase transition, leading to a "topological" variant of the Ising transition on the topological side (the red Ising* line), by forcing an anomalous conformal boundary condition [42–44]. In the ferromagnetic phase, however, one would naively expect the topological physics to be lost since the protecting symmetry is spontaneously broken.

Decorated quasiparticle picture. We show instead that the dichotomy between $x = 0$ and $x = 1$ extends beyond the Ising transition to the ferromagnetic phase, a distinction rooted in the changing nature of the quasiparticles. As usual for a ferromagnet, quasiparticle excitations are domain walls, separating domains of opposite magnetization (for the σ spins). What is unusual, however, is that there are two kinds of domain walls in this model: the regular domain walls (RDW), generated by H_{PM}, and the "decorated" domain walls (DDW), generated by H_{SPT} [40]. The latter kind is decorated in the sense that it carries a charge for the $\mathbb{Z}_2^\tau$ symmetry [32, 40, 42].

This decoration is inconsequential in the bulk, where domain walls are always created or annihilated in pairs—but it has a drastic effect at the edge of the system. Flipping an edge spin changes the number of domain walls by ± 1, which leads to a change in the total $\mathbb{Z}_2^\tau$ charge sector whenever the domain wall is decorated. Such a process necessarily breaks the $\mathbb{Z}_2^\tau$ symmetry and is therefore disallowed. This means that DDWs cannot flip an edge spin without breaking the symmetry, while RDWs can. Note that the PM (resp. SPT) phase corresponds to the condensation of regular (resp. decorated) domain walls.

These considerations are, of course, irrelevant for static properties of the FM ground states, which contain no domain walls. On the other hand, dynamical properties are dominated by the dynamics of domain walls, and it hence makes a difference whether they are decorated or not. SPT proximity effects are thus invisible in static bulk properties, but are revealed in dynamical properties of the edge. The remainder of the text will therefore be devoted to the dynamical properties of the model.

Let us consider the autocorrelation of the edge spin at temperature T, $C_T(t) = \mathrm{Re}\,\langle \sigma_0^z(t)\sigma_0^z(0)\rangle_T$. Fig. 2 and Fig. 4 (a) show $C_T(t)$ for various cases, and Fig. 3 shows the coherence time as a function of x, defined as the typical decay time of $C_T(t)$ [45]. As seen in Fig. 3, for OBC, the edge coherence time is larger by several orders of magnitude at $x = 1$ than at $x = 0$, while no such increase is observed in the case of periodic boundary conditions. This dramatic increase in edge coherence is due to the dominance of DDWs in the region close to $x = 1$ (dubbed FM*).

$T = 0$ dynamics. To confirm the quasiparticle picture we have outlined, we first work at zero temperature. Although the dynamics of a $T = 0$ ferromagnet become trivial in the strict thermodynamic limit, we work at finite system sizes, which will provide a useful diagnostic of the "hidden" topological effects in the FM* region. In this case, the notion of "coherence time" is nothing but the period of the Rabi oscillations between the two ground states, as seen in Fig. 2. Deep in the ferromagnetic phase, there are indeed two nearly-degenerate ground states, $(|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ (resp. $|\downarrow\rangle$) is a

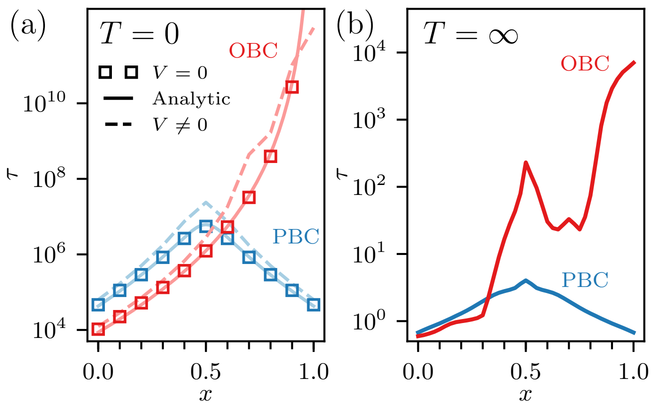

FIG. 3. **($\mathbf{T = 0}$)** Comparison of the coherence time (data) with its analytical prediction (lines) Data is computed on 14 spins via ED with parameters $(J, B) = (5.2, 1.27)$. The symbols were obtained with $V = 0$, while the dashed lines were obtained with $V \neq 0$. **($\mathbf{T = \infty}$)** Comparison of coherence times for OBC and PBC at infinite temperature on 14 spins with $(J, B) = (1.57, 9.03)$ and V chosen so that the model is not integrable — see Figs. 4 (c) & (d). Numerical details are given in the Supplemental Material [40].

state with $\sigma_i^z = +1$ (resp. -1) and $\tau_i^x = +1$. The Rabi period is simply the inverse of the ground state energy splitting ΔE. While the coherence time τ is infinite in the thermodynamic limit for all x, one can see in Fig 2 and 3 that its finite-size value has a systematic x dependence — it grows exponentially with x — thereby revealing a fundamental difference between the dynamics of the two sides.

We first study the special case $V = 0$, which qualitatively captures the $V \neq 0$ behavior as long as $T = 0$. Within degenerate perturbation theory, the splitting ΔE is proportional to the tunneling rate from $|\uparrow\rangle$ to $|\downarrow\rangle$. With PBC, the lowest order tunneling process occurs at order $L/2$ and corresponds to two domain walls being nucleated, propagating around the system, and annihilating each other. (See Fig. 1.) Such a process can occur for a pair of either RDWs or DDWs, leading to

$$\Delta E_{\mathrm{PBC}}(x) \propto \Delta E_{\mathrm{DW}} + \Delta E_{\mathrm{DW}^*}, \tag{2}$$

where $\Delta E_{\mathrm{DW}} = \left(\frac{1-x}{4(J+xB)}\right)^{L/2}$ is the contribution for RDWs and $\Delta E_{\mathrm{DW}^*}(x) = \left(\frac{x}{4(J+(1-x)B)}\right)^{L/2}$ is the contribution for DDWs. Note that (2) is symmetric under $x \leftrightarrow 1-x$, reflecting the equivalence of RDWs and DDWs under PBC.

Open boundary conditions change the situation significantly. Given the facts that (i) going from one ground state to another involves flipping all the σ spins, including at the edges, and (ii) DDWs cannot flip an edge spin, it is clear that only RDWs contribute to the splitting. (See Fig. 1 for illustration.) Hence

$$\Delta E_{\mathrm{OBC}}(x) \propto \Delta \widetilde{E}_{\mathrm{DW}}, \tag{3}$$

where the tilde signifies that the RDW contribution is slightly modified compared to PBC: $\Delta \widetilde{E}_{\mathrm{DW}} = \frac{1}{1-x}\left(\frac{1-x}{2(J+xB)}\right)^{L/2}$. This is manifestly *asymmetric* under $x \leftrightarrow 1-x$ and indeed vanishes in the limit $x \to 1$, leading to a diverging coherence time on the topological side. Fig. 3 (a) shows that Eqs. (2) & (3) accurately predict the coherence times in this simple limit. Turning V back on makes the $T = 0$ coherence time finite at $x = 1$ for finite L, but still larger than the $x = 0$ coherence by a factor that is exponential in L (as shown in Fig. 3(a)).

$T > 0$ *Dynamics.* At non-zero temperatures, there is a finite density $\rho \sim e^{-\Delta/T}$ of domain wall quasiparticles, where Δ is the energy gap of the excitation [46, 47]. For x close to 1, decorated domain walls have a lower gap than regular ones, and therefore are expected to dominate the dynamics at low T. For higher T, on the other hand, there is a finite density of both kinds of domain walls, so the naive expectation is that topological effects will disappear.

Surprisingly, we find instead that the enhancement of coherence from $x = 0$ to $x = 1$ with open boundary conditions persists even at $T = \infty$ (Fig. 3 and Fig. 4). (Results at intermediate temperatures $0 < T < \infty$ are similar [40].) We have checked that this behavior does not rely on integrability. The level spacings, shown in Fig. 4.(c) have good level repulsion with a shape characteristic of GOE statistics [3]. The many-body density of states in panel (d) is normally distributed, as is required to be representative of the thermodynamic limit [48] (see [40] for more details). While the coherence time initially increases exponentially with L, it eventually saturates to a L-independent value, as expected for a thermalizing system. This behavior can be seen in Fig. 4 (a) and (e).

To understand the survival of coherence at infinite temperature, we appeal to the physics of prethermalization. As shown in Fig. 4.(e), the dominant parameter that controls the coherence time is B, which sets the energy scale for the τ spins. It is therefore instructive to consider the case of $B \gg 1$ and to rewrite the Hamiltonian as

$$\hat{H} = -B\left[x\hat{N}^* + (1-x)\hat{N}\right] + \hat{V}_p, \tag{4}$$

where $\hat{N}^* = \sum_i \sigma_i^z \tau_i^x \sigma_{i+1}^z$, $\hat{N} = \sum_i \tau_i^x$ and $\hat{V}_p$ contains all the $\mathcal{O}(1)$ terms that are independent of B. The operator $\hat{N}^*$ counts the number of "mismatched decorations": domain walls without a $\mathbb{Z}_2^\tau$ charge attached, or $\mathbb{Z}_2^\tau$ charges without a domain wall.

While there are symmetry-respecting processes which can flip the edge spin, one can show that they necessarily have to change the $\hat{N}^*$ sector. (For instance, σ_0^x anticommutes with $\hat{N}^*$.) Such processes are exponentially suppressed with B due to the so-called ADHH theorem [49]. The theorem states, roughly, that if $e^{2\pi i \hat{N}^*} = 1$ and $\hat{N}^*$ is a sum of commuting projectors — which is indeed the case here — then $\hat{N}^*$ is approximately conserved until at

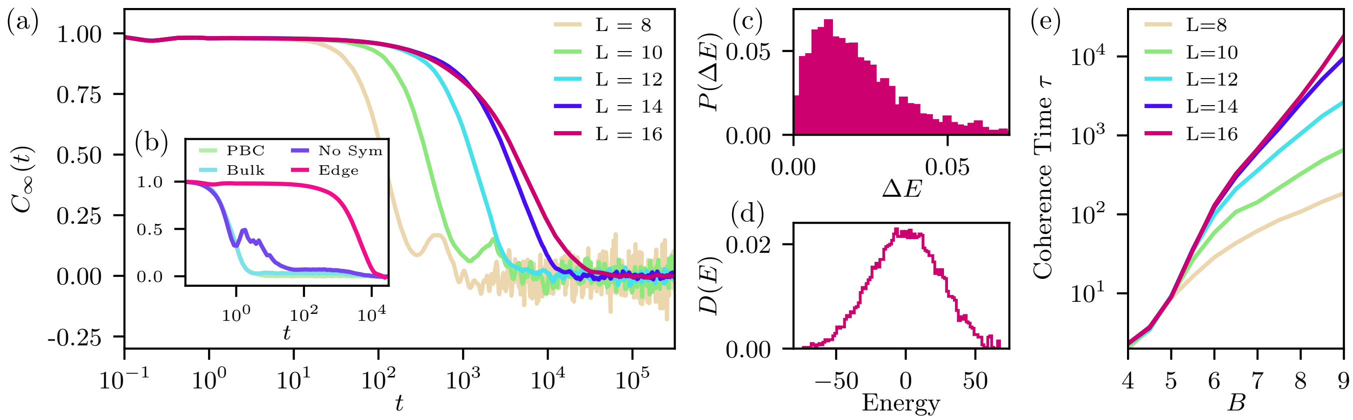

FIG. 4. **(a)** Autocorrelation $C_\infty(t)$ at $x = 1$ and $T = \infty$ under OBC and varying system size. $C_\infty(t)$ remains close to one for a time τ until it drops to its thermal value of 0, and τ increases exponentially with system size until its saturation. **(b)** The same autocorrelation $C_\infty(t)$ under various conditions on 14 sites. 'Edge' is same as in the main panel, 'bulk' corresponds to $\sigma^z_{L/4}$, 'PBC' corresponds to periodic boundary conditions, and 'No Sym' corresponds to a system where the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry was broken explicitly with edge perturbations $\sigma^x_0\tau^z_1$ and $\sigma^y_0\tau^z_1$. **(c)** Histogram of the differences in adjacent energy levels showing the non-integrability of the model and **(d)** normalized density of states in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ even/even sector on 16 spins **(e)** Coherence time for $x = 1$. Here $J = 1.57$ and $B = 8.42$ in (a) – (d). Numerical details are given in the Supplemental Material [40].

least a (quasi)-exponentially long time $\tau \sim e^{Bx/h}$, where h is the norm of the second-largest term after $\hat{N}^*$. (See [49] for the precise statement.) For x close to 1 [50], the second largest term is in $\hat{V}_p$, so h is $\mathcal{O}(1)$ and we expect $\tau \sim e^{Bx}$. We find indeed in Fig. 4 (e) that the large-L saturation value of τ increases exponentially with B for $x = 1$. For x away from 1, the second largest term is $\hat{N}$, leading to $\tau \sim e^{x/(1-x)}$ (excluding special values of x at which the sum of N and N^* have integer spectrum, leading to extra peaks in the coherence, see Fig 3 (b)).

This enhancement of the coherence is "topological", since only the coherence of the edge is exponentially enhanced and, unlike previous applications of the ADHH theorem [19, 20], it is also symmetry-protected. Explicitly, this means that adding terms which break the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry can immediately destroy the anomalously long edge coherence times. The term $\sigma^x_0\tau^z_1$, for instance, commutes with $\hat{N}^*$ but breaks the $\mathbb{Z}^\tau_2$ symmetry and is able to flip the edge spin and suppress the coherence, as shown in Fig. 4 (b). This provides a clear example of (prethermal) SPT physics even at infinite temperature, in a regime where the protecting symmetry is spontaneously broken at zero temperature.

Discussion. We have demonstrated the existence of a proximate SPT regime, characterized by anomalously long edge coherence times. The key to the model's dynamics is the behavior of its two species of quasiparticles: regular and decorated domain walls. The DDWs, which are inherited from the SPT phase, cannot be created or annihilated near the edges of the system without breaking the symmetry, giving rise to a dramatic increase in edge coherence. At $T = 0$, we have confirmed the quasiparticle picture within perturbation theory. The enhancement of edge coherence was shown to be stable to perturbations, and to survive to all temperatures thanks to prethermalization.

The existence of a proximate SPT regime has several broader implications. Regarding the low temperature physics, we have shown how the dynamics of low-lying quasiparticles in a "trivial" ordered phase can be infected by a topological phase nearby in phase space, leading to anomalous edge behavior. We expect this proximity effect to extend much beyond the DDW picture we used here; anomalous surface properties are expected in any D-dimensional ordered phase in proximity to a topological paramagnet. The anomalous behavior on the symmetry-broken side can also be understood as a consequence of the anomalous character of the nearby quantum phase transition to the topological paramagnet[42–44]. Such signatures could also be helpful when "prospecting" for a spin liquid in the phase diagram of a candidate spin liquid material which is magnetic at low T.

In our 1D example, the "anomalous surface behavior" described above actually led to a useful resource: an edge spin with extremely long coherence. Unlike the low-T results, which we expect to be general properties of proximate-SPT phases, the high-T protection is arguably much more model-dependent. It indeed relies on a combination of the prethermal physics described in Refs.[19, 20] with the concept of symmetry-protection which underlies SPTs: no *symmetry-respecting* operator can flip the edge spin without changing of $U(1)$ sector, whose value is protected for an exponentially long time. We surmise that combining 1D SPT parent Hamiltonians with prethermalization should provide a systematic way to find new models with long edge coherence at all temperatures.

Acknowledgments. We thank Maksym Serbyn, Ehud Altman and Joel Moore for insightful discussions. We acknowledge support from NSF Graduate Research Fellowship Program NSF DGE 1752814 (D.P.), the Emergent Phenomena in Quantum Systems initiative of the Gordon and Betty Moore Foundation (T.S.), and University of Massachusetts start-up funds (R.V.).

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