Many-Body Localization in the Presence of a Central Qudit

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Many body localization in the presence of a central qudit

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We consider a many-body localized system coupled globally to a central d-level system. Under an appropriate scaling of d and L, we find evidence that the localized phase survives. We argue for two possible thermalizing phases, depending on whether the qudit becomes fully ergodic. This system provides one of the first examples of many-body localization in the presence of long-range (non-confining) interactions.

A fundamental shift in our understanding of non-equilibrium quantum systems has occurred via the discovery of many-body localization (MBL), where sufficiently strong disorder induces stable localization [1–4]. MBL generalizes the notion of Anderson localization to the presence of interactions and is widely believed to be the only generic method for breaking the eigenstate thermalization hypothesis (ETH [5–7]) in isolated quantum systems. Since its inception, MBL has been shown numerically for a variety of models [2, 3], mathematically proven to exist under minimal assumptions [8], and been generalized to situations such as time periodic (Floquet) drive [9, 10], where MBL is particularly important to avoid heating to a featureless infinite temperature state.

MBL is commonly considered for the case of local interactions, with the exception of [11], where long-range confining interactions behave short-ranged with regards to the relevant degrees of freedom. Absent confinement, long-range interactions generically entangle spatially separated degrees of freedom, destroying the MBL phase. Perhaps the simplest example of this is the central spin-1/2 model, where it was found that a single globally coupled impurity immediately destroys localization in an infinite spin chain for arbitrarily weak couplings [12, 13][14]. One may suspect that this delocalization is generic for non-confining interactions, as a single spin-1/2 represents in some sense the minimal quantum bath providing thermalization.

In this paper, we show that this intuition is incorrect. Specifically, inspired by quantizing the drive degrees of freedom in Floquet MBL, we show that an appropriate limit of a d-level system ("qudit") coupled to a disordered spin chain may display an MBL-ETH transition at finite coupling. We argue that this phase transition survives the thermodynamic limit under the condition that $d \gtrsim \sqrt{L}$ asymptotically, where L is the length of the spin chain. The resulting phase diagram has many surprising features, such as decreased thermalization for larger d and the potential for an inverted mobility edge.

Model— As a starting point, we consider a model of MBL in the presence of global periodic drive, adapted from Zhang et al. [15]:

$$H = \frac{H_z + H_x}{2} + \cos(\Omega t)\frac{H_z - H_x}{2}$$
$$H_z = \sum_i (h + g\sqrt{1-\Gamma^2}G_i)\tau_i^z + \tau_i^z\tau_{i+1}^z$$
$$H_x = g\Gamma \sum_i \tau_i^x$$

where τ are Pauli matrices and G_i are random Gaussian variables of zero mean and unit variance describing on-site disorder. When the drive frequency is high, the system is effectively described by the average Hamiltonian $\frac{1}{2}(H_z+H_x)$, which exhibits an MBL-ETH transition.

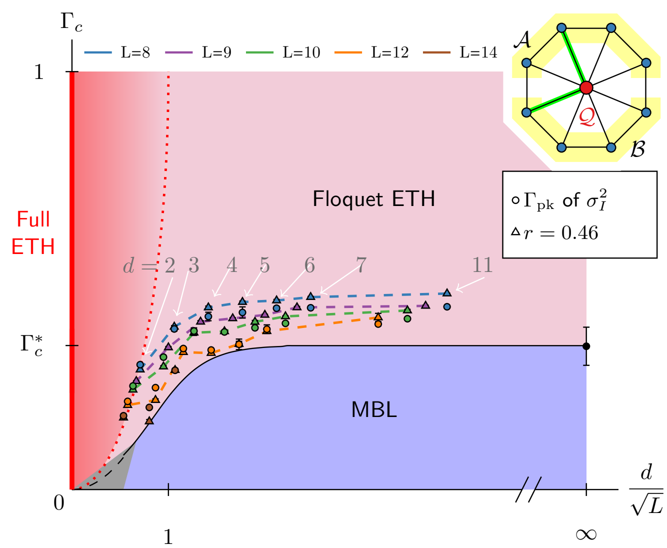

FIG. 1. Proposed infinite temperature phase diagram for the central qudit model upon taking $L \to \infty$ with $d/\sqrt{L}$ fixed. In addition to MBL and ETH phases of the spin chain, the dotted line indicates the crossover from fully thermal qudit to athermal qudit ("Floquet ETH"). The behavior of the phase boundary near $d/\sqrt{L} = 0$ is unclear; a possible $\Gamma_c \sim L^{-1}$ scaling [12] is indicated by the dashed line. Two finite size estimators of the critical Γ – defined near Eq. 2 and in Fig. 2 – are plotted. The value $r = 0.46$ is taken to be halfway between the thermal ($r = 0.53$) and non-thermal ($r = 0.39$) values [2].

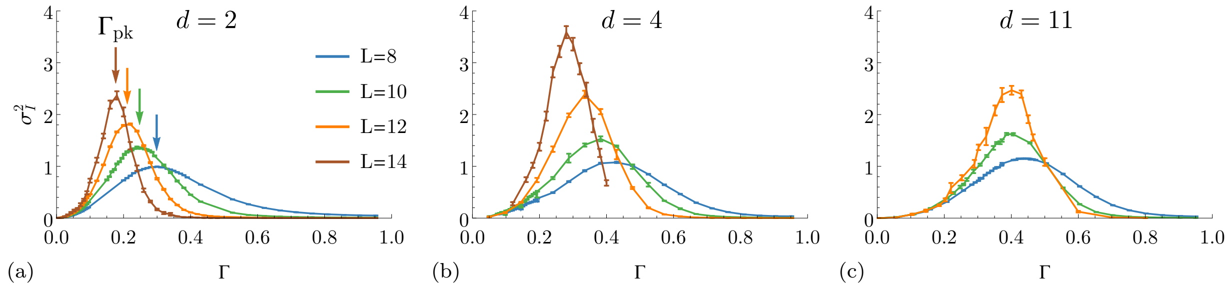

FIG. 2. Half chain mutual information variance σ_I^2 for various qudit sizes. The leftward drift of the peaks with system size L is seen to slow down with increasing d.

The coupling Γ controls the strength of disorder as well as the degree of noncommutativity between the zeroth and first harmonics of H. We take $h = 0.809$, $g = 0.9045$, and $\Omega = 3.927$, for which we numerically verify that an MBL-ETH transition is present at $\Gamma_c \approx 0.33$.

Interesting insight may be obtained by examining this model in the Floquet extended zone picture [16]. Writing the wave function in Fourier harmonics, $|\psi(t)\rangle = \sum_{n=-\infty}^{\infty} |\psi^{(n)}(t)\rangle e^{in\Omega t}$, $|\psi^{(n)}\rangle$ may be considered as the wave function dressed by n photons. This wave function evolves under the extended zone Hamiltonian:

$$H_{\rm EZ} = \sum_n \left(\frac{1}{2}H_+ + \Omega n\right) \otimes |n\rangle\langle n| + \frac{1}{4}H_- \otimes \left(\sum_n |n+1\rangle\langle n| + \text{h.c.}\right), \quad (1)$$

where $H_\pm = H_z \pm H_x$ are the zeroth $(+)$ and first $(-)$ Fourier modes of H. We introduce an extended Hilbert space $|n\rangle$ corresponding to photon occupation numbers.

In numerically solving such an extended zone Hamiltonian, one often truncates the photon Hilbert space, for instance restricting $n = -N_c, -N_c + 1, \ldots, N_c$. In order to obtain the proper Floquet result, one must extrapolate $N_c \to \infty$. If instead we maintain a finite truncation, the photon degrees of freedom form a d-level system – a "qudit" – with $d = 2N_c + 1$. In the $d \to \infty$ limit, we recover Floquet physics, for which an MBL-ETH transition is expected in this model. Keeping d finite, as in the case of a qubit ($d = 2$), Ponte et al. have argued in a similar model that ETH is expected for all finite couplings in the thermodynamic limit [12]. The remainder of this paper will be devoted to understanding the crossover between these limits, thereby uncovering the physics of MBL in the presence of a central qudit. Note that alternative choices of truncation would allow one to instead think of the degree of freedom as a central spin-S or photon with finite occupation, a picture relevant to cavity QED. These other truncations are discussed in the Supplement Material [17].

Numerical results— We investigate the behavior of this model up to $L = 14$ spins and $d = 11$ using the shift-invert method [18]. By targeting the ten states with energy closest to 0, we effectively work in the infinite temperature limit. We see that these ten states describe the same energy density by observing that there are no small scale structures in the disorder-averaged many-body density of states near zero energy [19, 20]. We compare these results to the full Floquet dynamics ($d = \infty$) by approximating the exact dynamics over one period with ≥ 16 time steps.

In our model, thermalization of the localized spins can occur through direct spin-spin interactions, qudit-mediated interactions, or some combination thereof. To distinguish entanglement between the spins from entanglement with the qudit, we consider the mutual information (MI) between two halves of the spin chain (see Fig. 1 for definition of $\mathcal{A}$ and $\mathcal{B}$):

$$I(L/2) \equiv I(\mathcal{A}, \mathcal{B}) = S(\rho_{\mathcal{A}}) + S(\rho_{\mathcal{B}}) - S(\rho_{\mathcal{AB}}). \quad (2)$$

By subtracting entanglement with the qudit, $S(\rho_{\mathcal{AB}}) = S_{\rm qudit}$, we find that I captures the bipartite correlations between $\mathcal{A}$ and $\mathcal{B}$ more faithfully than $S(\rho_{\mathcal{A}})$.

We calculate mutual information and qudit entanglement entropy for each of the eigenstates and $200 - 6000$ realizations of disorder, as well as the level statistics ratio r [2]. Let us begin by discussing $I(L/2)$. For large $d = 11$ approaching the Floquet limit, it increases from a nearly system size independent area law in the MBL phase at small Γ to a thermal volume law, approaching the Page value $S_{\rm Page} = \frac{L}{2} - \frac{1}{2\log 2}$ [21], for large Γ (see Supplement [17]). It has been found elsewhere that shot-to-shot fluctuations of the entanglement entropy are a useful detector of the MBL-ETH phase transition, peaking sharply near the transition [22, 23]. Here we obtain the variance of the MI, σ_I^2, due to intersample variations between disorder realizations and intrasample variations between eigenstates (Fig. 2). Treating the peak values $\Gamma_{pk}(L)$ as a finite size approximation of the critical point, we see that for large d, the peak shifts only weakly with L. This is consistent with the Floquet MBL-ETH phase transition at finite Γ in taking first $d \to \infty$, then $L \to \infty$. By contrast, at the smallest value of $d = 2$,

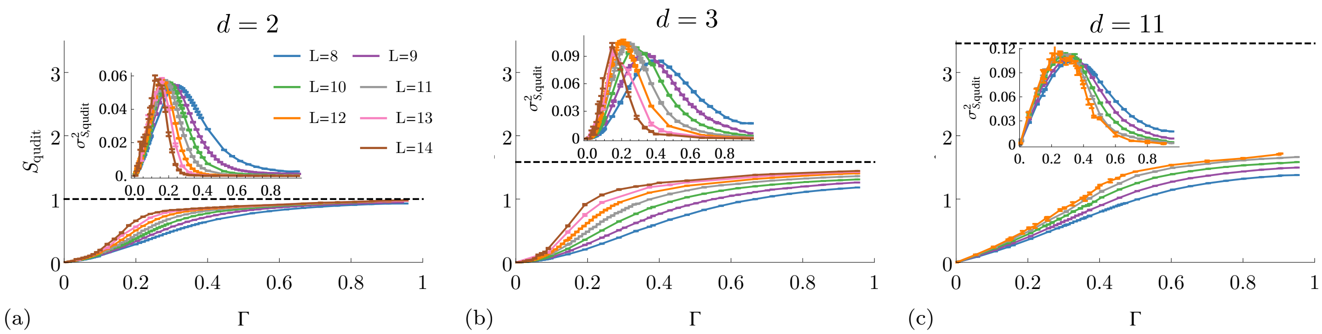

FIG. 3. Entanglement entropy $S_{\mathrm{qudit}} \equiv S(\rho_{\mathcal{Q}}) = S(\rho_{\mathcal{AB}})$ between the qudit and the spin chain. The dashed line corresponds to the Page value, $\log_2 d - d\left(2^{L+1}\log 2\right)^{-1} \approx \log_2 d$. The insets show the variance of S_{qudit}.

the peak shifts sharply with L, consistent with the expected absence of an MBL-ETH phase transition in the thermodynamic limit. The behavior for $d \sim 5$ is intermediate to these two limits, and its crossover behavior will be addressed in more detail later.

The qudit entanglement entropy S_{qudit} and its variance, $\sigma^2_{\mathrm{S,qudit}}$, are shown in Fig. 3, while r is shown in the Supplement [17]. One striking difference between S_{qudit} and $I(L/2)$ is immediately apparent – for large d, the qudit entropy does not reach its maximal value, and thus the qudit does not thermalize. Despite the lack of thermalization in the qudit, the level statistics ratio still saturates the Gaussian orthogonal ensemble value of $r \approx 0.53$ for $\Gamma > \Gamma_c$ in the large d limit. On the other hand, for $d = 2$, the qudit entropy and its fluctuations closely track $I(L/2)$, suggesting that thermalization of the spin chain is mediated by the central qudit. These numerics together suggest that thermalization of the qudit and the spin chain do not always go hand in hand, confirming the expectation that the limits $d \to \infty$ and $L \to \infty$ do not commute. We now address how these limits may be taken to obtain the phase diagram shown in Fig. 1.

The appropriate scaling of d vs. L can be argued by first decoupling them, i.e., taking $\Gamma = 0$. Then eigenstates of the full problem become direct products of eigenstates of H_z with those of the qudit. The qudit states behave like non-interacting charged particles in an external electric field with nearest neighbor hopping proportional to the many-body energy of the H_z eigenstate. In the Floquet limit, $d \to \infty$, the qudit will be Wannier-Stark localized with a characteristic spread given by the ratio of the hopping strength $\langle H_z \rangle$ to the potential tilt Ω, for which the variance of the qudit occupation is given by $\Delta_Q^2 \equiv \langle n^2 \rangle - \langle n \rangle^2 = \frac{1}{2}\langle H_z \rangle^2/\Omega^2$ [17]. The many body spectrum has characteristic width $\sigma_{\langle H_z \rangle} \sim \sqrt{L}$, hence averaging over eigenstates gives $\Delta_Q^2 \sim L$.

This scaling of Δ_Q^2 is further argued to be robust for small Γ in the Supplement [17]. However, numerically we find that this result holds nonperturbatively as well, giving $\Delta_Q^2 \sim L$ for Γ throughout the phase diagram (Fig. 4). Therefore, we argue that the relevant ratio controlling thermalization is $d/\sqrt{L}$, as in Fig. 1. For $d \gg \sqrt{L}$, the spin chain is insufficient to act as a bath for the qudit, and thus no thermalization of the qudit occurs. For $d \ll \sqrt{L}$, the spin chain can thermalize the qudit and vice versa. Taking the limit $L \to \infty$ with $d/\sqrt{L}$ small but finite, our data is unable to confirm whether the qudit fully thermalizes, or rather whether the qudit entropy gradually crosses from athermal to thermal as we take $d/\sqrt{L} \to 0$; we leave this topic for future study.

Having identified $d/\sqrt{L}$ as the relevant scale for understanding the qudit's role in thermalization, we may now plot the finite size approximants to Γ_c (Fig. 1). We see that once L is reasonably "large" ($L \gtrsim 10$) the finite size Γ_c from level statistics and MI variance seem to approach a single curve, which we postulate will become a sharp MBL-ETH phase transition in the thermodynamic limit. For $d/\sqrt{L} \lesssim 1$, the MBL-ETH transition indicated by these two measures is consistent with that obtained from the qudit entanglement entropy, while going to $d/\sqrt{L} \gtrsim 1$, this is no longer true, consistent with a crossover from qudit-mediated thermalization [17]. Finally, we note that the prediction of $\Gamma_c \sim 1/L$ at arbitrary finite d [12, 13] maps in our phase diagram to $\Gamma_c \sim d^2/L$ for $d/\sqrt{L} \ll 1$. We are unable to obtain data for transitions in this limit, so leave clarification of the bottom left corner of the phase diagram for future work.

Discussion— Our data suggest that three distinct phases exist for the disordered spin chain coupled to a central qudit: (1) Both spin chain and qudit are athermal (MBL), (2) both the spin chain and the qudit are thermal (full ETH), and (3) the spin chain is thermal but the qudit is athermal. We refer to this last phase as Floquet ETH because it is necessarily obtained in the Floquet limit, $d/\sqrt{L} \to \infty$. By contrast, for full ETH to occur, the spin chain must act as a bath for the qudit states and vice versa. In the thermodynamic limit, this should manifest as observables for both the spins and the qudit exhibiting criticality at the same value of

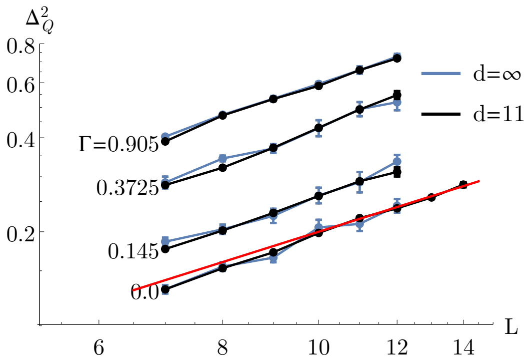

FIG. 4. Variance of the qudit wavefunction versus size of the spin chain, for Γ in both MBL and ETH phases and near criticality. The red line is given by $0.02L$.

Γ. We cannot currently probe this effect, given the small region of L and d space accessible. However, drifts in $\Gamma_{\rm pk}$ obtained from $S_{\rm qudit}$ (Fig. 3, inset) and $I(L/2)$ (Fig. 2) appear to be consistent with a $\Gamma_c = 0$ transition as found in earlier works [12, 13]. The full ETH phase is certainly obtained for $d/\sqrt{L} = 0$, e.g., by taking $L \to \infty$ while keeping d finite. While we cannot rule out the possibility that this phase extends to nonzero $d/\sqrt{L}$, implying a phase transition between the thermal full ETH and Floquet ETH phases, we expect that Floquet ETH will be immediately obtained as $d/\sqrt{L}$ is increased from zero.

Most surprising is the persistence of MBL at finite $d/\sqrt{L}$. Integrating out the central qudit, we may think of this as MBL in the presence of infinite range interactions. Similar MBL phases have been proposed in the presence of long-range confining interactions by Nandkishore and Sondhi [11], but this work represents the first numerical example of such long-range-interacting MBL to our knowledge. A natural expectation is that thermalization would be easier for larger central qudit size, as larger central qudits have more pathways for the qudit to flip and thus mediate long-range interactions. However, our data suggests the opposite – larger d leads more readily to MBL. In the Supplement, we show how the qudit may be "integrated out" in the high frequency limit and recover the $d/\sqrt{L}$ scaling using this method. Intuitively, the picture that emerges is that, at large d, the spin chain Hamiltonian becomes independent of the qudit occupation due to translation invariance in qudit occupation space, and thus the qudit [24] is no longer able to mediate long-range interactions. Finally, we note that applying the same procedure to models with Floquet-induced localization [25, 26] would lead to localization that is encouraged rather than discouraged by the presence of the central qudit.

We note one further non-trivial corollary to this phase diagram. If we treat d as a proxy for the photon number in a photonic regularization of the Floquet problem, then smaller d would correspond to smaller photon number and, thus, lower many-body energies. Moving to the left in Fig. 1 is then loosely equivalent to decreasing energy. If we take some value of Γ below the Floquet critical point, e.g., $\Gamma = 0.2$, this implies that the system goes from many-body localized at infinite temperature to ergodic at lower temperature: an inverted many-body mobility edge. This analogy is inexact, but numerically we may target lower energy densities at fixed d to determine whether indeed this unexpected inversion holds.

Experimentally, central qudit systems are realized in a variety of settings, such as quantum dots [27, 28] and defect centers [29]. Localization of the spin bath there is less obvious, as the spin-spin interactions are commonly dipolar. Other promising avenues for realizing localization in the presence of a central mode include superconducting qubits coupled in geometry similar to Fig. 1 [30] or spin chains consisting of ultracold atoms globally coupled to a cavity [31–34]. In the latter architecture, the cavity photon number plays the role of the qudit size, as discussed more extensively in the Supplement [17].

In conclusion, we have mapped out the phase diagram of a disordered spin chain interacting with a central qudit. We found that the size of the central qudit plays an important role, with the ratio $d/\sqrt{L}$ appearing to control the crossover from Floquet-like physics at $d/\sqrt{L} \gg 1$ to central qudit-like physics at $d/\sqrt{L} \ll 1$. We expect similar behavior to hold for other models of Floquet MBL, as well as other methods for quantizing the Floquet drive.

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