Microwave Spectroscopy of a Weakly Pinned Charge Density Wave in a Superinductor

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Phys. Rev. Lett. **122**, 237701 — Published 13 June 2019

DOI: 10.1103/PhysRevLett.122.237701

Microwave spectroscopy of a weakly-pinned charge density wave in a superinductor

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A chain of small Josephson junctions (aka superinductor) emerged recently as a high-inductance, low-loss element of superconducting quantum devices. We notice that the intrinsic parameters of a typical superinductor in fact place it into the Bose glass universality class for which the propagation of waves in a sufficiently long chain is hindered by pinning. Its weakness provides for a broad crossover from the spectrum of well-resolved plasmon standing waves at high frequencies to the low-frequency excitation spectrum of a pinned charge density wave. We relate the scattering amplitude of microwave photons reflected off a superinductor to the dynamics of a Bose glass. The dynamics at long and short scales compared to the Larkin pinning length determines the low- and high-frequency asymptotes of the reflection amplitude.

Interaction between particles gives rise to collective excitations in a many-body system. In the case of Coulomb interaction, these are the well-known plasma oscillation modes. The long-range interaction between particles confined to one or two dimensions (1D or 2D) may be cut-off by the polarizability of the surrounding medium. Translational invariance in 1D or 2D then results in the sound-like plasmon spectrum at low frequency. An ubiquitously present disorder breaks translational invariance and possibly affects the spectrum of low-frequency excitations. The competition between interaction and disorder sets the stage to a possible localization transition in a many-body system. This competition in 1D was addressed by means of perturbative renormalization group (RG) theory by Giamarchi and Schulz [1]. Interaction in 1D can be characterized by a dimensionless parameter K related to the magnitude of zero-point fluctuations of the particles' density (the classical limit is $K \to 0$). It turns out that the disorder potential is irrelevant at $K > 3/2$ and the sound-like mode does exist down to the smallest wave vectors ($q \to 0$). However, at $K < 3/2$ even an infinitesimally weak disorder results in localization, severely affecting the properties of 1D systems at long spatial scales. Attempts to understand the localized phase have led to the notion of Bose glass phase [2] and to establishing its links to the pinned vortices in superconductors [3], domain walls in magnets [4], and "classical" charge-density waves in normal metals [5].

In the localized phase, the static spatial order is destroyed on the scale exceeding the Larkin length, $R_\star$, which was first introduced [3] for the collective pinning of vortices (these were modeled as classical particles). For weak pinning, $R_\star$ exceeds significantly the interparticle distance. Elastic properties of the pinned system on length scales shorter than $R_\star$ are hardly affected by pinning. Respectively, in a non-dissipative system excitations with frequencies $\omega \gtrsim \omega_\star \equiv v/R_\star$ may still be approximated by propagating waves (here v is the propagation velocity) [6]. Pinning drastically changes the excitation spectrum at frequencies $\omega \ll \omega_\star$. The corresponding density of modes arises from the statistics of specific configurations of disorder supporting localized-in-space low-frequency excitations [5–12]. The found [9–12] limiting low-frequency behavior of the density of modes (per unit volume) is $\nu(\omega) \propto \omega^4$.

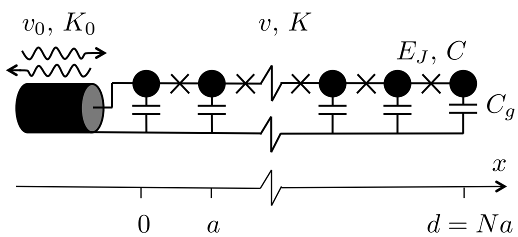

FIG. 1. Microwave photons incident from a waveguide are reflected off a superinductor formed of N Josephson junctions in series.

Conductivity $\sigma(\omega)$, being sensitive to the frequency dependence of the transition matrix elements along with that for the density of modes, carries some information regarding the dynamics of charge density waves. It was predicted to have a maximum at $\omega \sim \omega_\star$, see, *e.g.*, [13–16]. Experiments performed with a 2D electron gas in GaAs heterostructures qualitatively confirmed the predictions, but disagreed with them quantitatively, see [17] for a review. In 1D, the physics of charge density waves was addressed in an experiment [18] where nonlinear current-voltage characteristics of Josephson-junction chains were studied. The tell-tale signature of the pinning was the appearance of dissipative current above a threshold voltage and a specific, systematic dependence of the threshold voltage on the parameters of a chain. The threshold voltage is related to the pinning energy at scale $R_\star$ [19, 20], which is the basic property of the static pinning configuration.

In this work, we elucidate a way to study the dynamics of charge-density waves with a special type of

Josephson-junction arrays, known as superinductors. Developed in the context of superconducting quantum devices [21], superinductors are linear elements combining high inductance with a small stray capacitance. An equivalent circuit of a superinductor is shown in Fig. 1. Large inductance and linearity are achieved by making the number of junctions large, $N \gg 1$, and quantum fluctuations of the phase across a single junction small, $E_J/E_C \gg 1$, while having small stray capacitance calls for a very small ratio E_C/E_g [here E_J is the Josephson energy of a single junction, $E_g = 4e^2/(2C_g)$ and $E_C = 4e^2/(2C)$ are the charging energies for an extra Cooper pair associated, respectively, with the superconducting island's stray capacitance C_g and the junction capacitance C; the parameters were $N \sim 10^2$, $E_J/E_C \sim 20$, and $E_C/E_g \sim 10^{-4}$ in the experiment [21]]. The product of E_J/E_C and E_C/E_g turns out to be small, resulting in $K < 1$, so nominally superinductors are insulators. However, the amplitude of quantum phase slips and therefore the pinning potential are exponentially small, $\exp(-\sqrt{32E_J/E_C}) \ll 1$. Unless $N \gg 1$ compensates for that smallness, phase slips are rare and the superinductor faithfully performs its inductance function in a circuit [21–24]. Recently, even longer chains with $N \sim 10^4$ were developed [25] for which the statistics of quantum phase fluctuations allows a finite density of quantum phase slips to appear. This, in turn, enables weak pinning of charge density waves. Realization of the pinning potential depends on the random background charges in the environment of the chain. These are slowly fluctuating in time [26], providing a tool for the ensemble averaging of observables. We evaluate the most accessible one, which is the ensemble-averaged reflection amplitude off a chain, $\langle r(\omega)\rangle$, find its relation to the local density of states of excitations, and predict the low- and high-frequency asymptotes of $\langle r(\omega)\rangle$.

We model the superinductor with the Hamiltonian

$$H_{\rm chain} = \frac{1}{2}\sum_{nm} Q_n C^{-1}_{nm} Q_m - E_J \sum_n \cos(\varphi_n - \varphi_{n+1}), \quad (1)$$

where φ_n and Q_n are canonically conjugated phase and charge of each superconducting island along the chain, $[\varphi_n, Q_m] = 2ei\delta_{n,m}$. The first term in Eq. (1) describes the electrostatic coupling with elements $C_{nn} = (2C + C_g)$ and $C_{nn\pm1} = -C$ of the capacitance matrix, the second term describes the Josephson coupling between successive islands. Hamiltonian (1) can be used if the temperature, charging, and Josephson energy are smaller than the superconducting gap. The small stray capacitance corresponds to a large charge screening length, $\ell_{\rm sc} = a\sqrt{C/C_g} \gg a$, where a is the unit cell length.

In harmonic approximation, Hamiltonian (1) yields the dispersion relation $\omega(q) = v|q|/\sqrt{1 + (vq/\Omega)^2}$, where $v = a\sqrt{2E_JE_g}/\hbar$ is the plasmon velocity and $\Omega = \sqrt{2E_JE_c}/\hbar$ is the single-junction plasma frequency. Modes with $\omega(q) \approx v|q|$ are adequately described by a harmonic string Hamiltonian,

$$H_0 = \int dx \left[\frac{\pi v K}{2\hbar}\Pi^2 + \frac{\hbar v}{2\pi K}(\partial_x\theta)^2\right], \quad (2)$$

acting on states with energies within the bandwidth $\sim \hbar\Omega$. Here θ and $\Pi = -(\hbar/\pi)\partial_x\varphi$ are two canonically conjugated fields, $[\theta(x), \Pi(x')] = i\hbar\delta(x - x')$, where φ and $\rho = -(1/\pi)\partial_x\theta$ are the coarse-grained phase and Cooper-pair density along the chain, respectively. The parameter $K = \pi\sqrt{E_J/(2E_g)}$ is related to the low-frequency impedance Z of the chain, $K = \pi\hbar/(4e^2Z)$.

Quantum phase slips allow for jumps of the phase differences $\varphi_n - \varphi_{n+1}$ between the minima of the Josephson energy in Eq. (1). Rare phase slips can be accounted for by adding [27] a perturbative term to the Hamiltonian (2),

$$H = H_0 - \frac{\lambda}{a}\int dx \cos(2\theta + \chi). \quad (3)$$

Operators $e^{\pm 2i\theta(x)}$ appearing in Eq. (3) create $\pm 2\pi$-kinks at position x in the field $\varphi(x)$. The classical field $\chi(x)$ leads to the Aharonov-Casher effect in the probability amplitudes of phase slips [28, 29].

Field $\chi(x) = 2\pi\int^x dx' \rho_{\rm b}(x')$ is random, $\rho_{\rm b}(x)$ is the density of coarse-grained offset charges. A maximal disorder corresponds to offset charges fluctuating independently and randomly in each superconducting island. It yields a Gaussian correlator

$$\langle \cos\chi(x)\cos\chi(x')\rangle = \frac{1}{2}a\delta(x - x'), \quad (4)$$

where $\langle \cdots \rangle$ stands for disorder averaging, for the field $\cos\chi$ (a similar relation holds for the field $\sin\chi$) on spatial scales larger than $\ell_{\rm sc}$. Furthermore, a recent experiment [26] reported timescale $t_c \sim 1$min for the offset charge fluctuations of a single island. Extrapolating their results to a chain yields a timescale t_c/N for scrambling the Aharonov-Casher phase in a chain of N junctions.

Due to the separation of scales in the plasmon spectrum at $C_g/C \ll 1$ and the known full solution [30, 31] of the phase-slip problem at $C_g/C = 0$, it is possible, in contrast to the phenomenological treatments [27, 32], to derive Hamiltonian (3) and evaluate λ in terms of microscopic parameters,

$$\lambda = \frac{8}{\sqrt{\pi}}(2E_J^3E_c)^{1/4}e^{-\sqrt{32E_J/E_c}} \quad \text{at} \quad E_J \gg E_c. \quad (5)$$

Once λ is known, the $\mathcal{O}(1)$ uncertainty in the bandwidth $\sim \hbar\Omega$ translates, at $K \ll 1$, into a negligible $\mathcal{O}(K)$ uncertainty in the low-frequency ($\omega \ll \Omega$) observables which we aim to evaluate.

In the thermodynamic limit (infinitely long chain), the perturbative RG flow associated with the Hamiltonian

(3) is given by the Giamarchi-Schulz scaling [1]

$$\frac{dD(\Lambda)}{dl} \simeq (3-2K)D(\Lambda)\,. \tag{6}$$

The unitless function $D(\Lambda)$ here describes the evolution of the phase slip probability in the process of coarse-graining, Λ is the running momentum cutoff, and $dl = -d\Lambda/\Lambda$. The initial condition for Eq. (6) is $D(\Lambda_0) = v\lambda^2/(a\Omega^3)$ with λ of Eq. (5) and $\Lambda_0 \sim \Omega/v$. Equation (6) signals a transition between a superfluid phase ($K > K_c$) and a Bose glass ($K < K_c$) at $K_c = 3/2$. At small fugacity $\exp(-\sqrt{32E_J/E_c})$, see Eq. (5), one may disregard the renormalization [1] of K in finding the transition condition, $E_g = (2\pi^2/9)E_J$.

In the classical limit ($K \to 0$ and $\hbar \to 0$ at fixed finite $K/\hbar$), one may neglect the kinetic term in Eq. (3) and estimate the Larkin length for the static pinning of the charge density from energy arguments: On one hand, a deformation of a static field $\bar\theta(x)$ by 2π over the length R costs an elastic energy $\sim \hbar v/(KR)$. On the other hand, for a constant field $\bar\theta$, the disorder-averaged pinning energy vanishes, while its typical value for a given disorder configuration is estimated as $\sim \lambda\sqrt{R/a}$ using the correlator (4). The pinning energy dominates the elastic one if $R > R_\star$, where $R_\star = a\{\hbar v/[aK\lambda(R_\star)]\}^{2/3}$. Using here the renormalized phase slip amplitude $\lambda(R) = \lambda(R/\ell_{\rm sc})^{-K}$ instead of its bare value (5) allows us to account for quantum fluctuations on short length scales $R \lesssim R_\star$. Solving then for $R_\star$, and using $v/a = \Omega\sqrt{C/C_g}$, we find the generalized Larkin length,

$$R_\star = a\left(\frac{\hbar\Omega}{K\lambda}\right)^{2/(3-2K)}\left(\frac{C}{C_g}\right)^{(1-K)/(3-2K)}. \tag{7}$$

Remarkably, $1/R_\star$ coincides with the momentum scale at which the perturbative RG, cf. Eq. (6), breaks down, $D(1/R_\star) \sim 1$. (Note that $R_\star$ diverges at $K = 3/2$).

We are now ready to formulate the problem of the elastic scattering of microwave photons off a Josephson-junction chain of a finite length d. For this, we consider the same Hamiltonian (3) but with spatially nonuniform parameters such that it describes a waveguide with plasmon velocity v_0 and impedance $Z_0 = \pi\hbar/(4e^2K_0)$ (and without phase slips, $\lambda = 0$) at $x < 0$, and the superinductor at $0 < x < d$, see Fig. 1. The equations of motion derived from Eq. (3) yield $\ddot\theta = -v^2\partial_x^2\theta + (2\lambda/a)\sin(2\theta+\chi)$ together with the boundary conditions expressing the continuity of current, $\partial_t\theta(0^+,t) = \partial_t\theta(0^-,t)$, and voltage, $(v/K)\partial_x\theta(0^+,t) = (v_0/K_0)\partial_x\theta(0^-,t)$, at the interface between the waveguide and the chain, as well as the absence of current, $\partial_t\theta(d,t) = 0$ at the other end of the chain. Taking the classical limit, we can now define the reflection amplitude $r(\omega)$ at frequency ω, such that the solution of these equations is expressed as $\theta(x,t) = \bar\theta(x) + \psi(x)e^{-i\omega t}$, where $\bar\theta(x)$ is a static charge density that minimizes the (classical) energy, and $\psi(x)$ describes small oscillations around it. The linearized equation of motion takes a form similar to the Schrödinger equation,

$$\omega^2\psi = -v^2\partial_x^2\psi + V(x)\psi \quad \text{at} \quad 0 < x < d\,, \tag{8}$$

with the normalization condition

$$\psi(x) = e^{i\omega x/v_0} + r(\omega)e^{-i\omega x/v_0} \qquad \text{at} \qquad x < 0\,, \tag{9}$$

and potential $V(x) = (4\pi K\lambda v/\hbar a)\cos(2\bar\theta(x) + \chi(x))$ which is determined both by the offset charge disorder and the static charge density $\bar\theta(x)$.

The impedance of typical waveguides is of the order of the vacuum impedance, $Z_{\rm vac} \approx 377\,\Omega$; thus, $K \ll K_0$. Using this, we can find an expression for $r(\omega)$ in terms of the properties of the chain valid at arbitrary disorder configuration. Namely, we observe that, at $K/K_0 \to 0$, the chain is disconnected from the waveguide. Thus, it admits a set of discrete bound states with eigenfrequencies ω_n and eigenfunctions $\psi_n(x)$, which satisfy the Schrödinger equation (8) with non-radiative boundary conditions, $\partial_x\psi_n(0^+) = 0$ and $\psi_n(d) = 0$, corresponding to the absence of accumulated charge at $x = 0^+$ and to zero current at $x = d$, respectively. We also impose the normalization condition $1/d\int_0^d dx\,\psi_n^2(x) = 1$. At small but finite K/K_0, these solutions become quasi-bound states: they emit plasmons in the waveguide, such that $\psi(x<0) = \psi(0^-)e^{-i\omega_n x/v_0}$ with $\psi(0^-) = \psi_n(0^+)$ according to the boundary condition for current continuity at $x = 0$. The energy stored in the bound state is $E_n = \omega_n^2 d/(\pi v K)$, while the energy emitted in the waveguide is characterized by the Poynting vector $P = \omega_n^2\psi_n^2(0^+)/(\pi K_0)$. The rate of energy loss, $\Gamma_n \equiv P/E_n = \psi_n^2(0^+)(K/K_0)v/d$, gives the level width of the quasi-bound state. We can now use the Breit-Wigner formula to account for the contribution of all quasi-bound states, $r(\omega) = -1 + i\sum_n \Gamma_n/(\omega - \omega_n + i\Gamma_n/2)$. Let us now introduce a Green function of the chain, which solves

$$\left[\omega^2 + v^2\partial_x^2 - V(x)\right]G(x,x';\omega) = \pi v\delta(x-x') \tag{10}$$

with boundary conditions $\partial_x G(0,x';\omega) = G(x,d;\omega) = 0$. Introducing the local plasmon density of states

$$\nu(x,\omega) = -\frac{2\omega}{\pi^2 v}\operatorname{Im}G(x,x;\omega)\,, \tag{11}$$

defining $\nu_0 = 1/(\pi v)$, and using the complete basis of normalized eigenstates $\psi_n(x)$ to express the Green function allows us to find a relation between the real part of the reflection amplitude and the density of states at the edge of the chain,

$$r'(\omega) = -1 + \frac{K}{K_0}\frac{\nu(x=0,\omega)}{\nu_0}\,. \tag{12}$$

The random charge realization changes on time scale t_c/N far exceeding the typical plasmon propagation time d/v. For a static disorder, $r(\omega)$ is the sum of narrow peaks corresponding to plasmon resonances in the finite-size chain. We now evaluate its disorder average, assuming that the measurement time exceeds t_c/N, thus facilitating the averaging.

At large frequencies, $\omega \gg \omega_\star$, the plasmon wavelength v/ω is much smaller than $R_\star$. This justifies neglecting the spatial variations of the static field $\bar{\theta}$ appearing in the disorder potential in Eq. (8), which then becomes Gaussian. The potential induces both forward- and back-scattering characterized by a frequency-dependent mean-free path evaluated, within Born approximation, and using Eqs. (4) and (7), as

$$\ell(\omega) = v\tau(\omega) = R_\star(\omega/\omega_\star)^2\,. \tag{13}$$

Ignoring the back-scattering for a while, we readily find that the local density of states is expressed as $\nu(0,\omega) = 1/d\sum_n \delta(\omega-\omega_n)$, where $\omega_n = (n+1/2)\Delta + \gamma_n$ corresponds to a spectrum of plasmon resonances nominally spaced by $\Delta = \pi v/d$ and randomly shifted by $\gamma_n \approx 1/(2d\omega_n)\int_0^d dx V_0(x)$, where V_0 is the "smooth" component of V. Gaussian average over V_0 yields an "inhomogeneous broadening" of the peaks in $\langle\nu(0,\omega)\rangle$, which acquire a Gaussian lineshape.

Accounting for both forward- and back-scattering in the evaluation of $\langle\nu(0,\omega)\rangle$ at large frequency $\omega \gg \omega_\star$ can be performed with the Fokker-Planck method, see Sec. I of the Supplemental Material (SM) [33]; it only modifies the width of the Gaussian lineshapes, compared with the forward-scattering case. The result is

$$\langle\nu(0,\omega)\rangle = 2\nu_0\sum_m \frac{\Delta}{\sqrt{2\pi}\gamma(\omega)}e^{-[\omega-(m+1/2)\Delta]^2/[2\gamma^2(\omega)]}\,, \tag{14}$$

with the frequency-dependent width of the resonances,

$$\gamma(\omega) = \Delta\cdot\frac{\omega_{\rm cr}}{\omega} \quad \text{and} \quad \omega_{\rm cr} = \sqrt{3}\omega_\star\left(\frac{d}{R_\star}\right)^{1/2}\,. \tag{15}$$

The Gaussian-shaped resonances are well-separated as long as $\gamma(\omega) \ll \Delta$, corresponding to the frequency range $\omega \gg \omega_{\rm cr}$ or, equivalently, for a chain's length $d \ll \ell(\omega)$.

At frequencies below the crossover, $\omega \lesssim \omega_{\rm cr}$, the resonances overlap, gradually suppressing the amplitude of oscillations of $r(\omega)$. Using the Poisson summation formula to transform Eq. (14), we find

$$\langle\nu(0,\omega)\rangle = 2\nu_0\left[1-2\cos\left(2\pi\omega/\Delta\right)e^{-2(\pi\omega_{\rm cr}/\omega)^2}\right] \tag{16}$$

in the frequency range $\omega_\star \ll \omega \ll \omega_{\rm cr}$.

The leading term in Eq. (16) is independent of frequency, as at $\omega \gg \omega_\star$ the adjustment of the static configuration $\bar{\theta}(x)$ to the external charge disorder realization is not important. In contrast, at $\omega \lesssim \omega_\star$ the plasmon wavelength becomes of the order of the correlation length of $\bar{\theta}(x)$. In this limit, the oscillatory part of $\langle r(\omega)\rangle$ remains exponentially small, but – in addition – the leading term in Eq. (16) becomes a function of $\omega/\omega_\star$. The $\omega/\omega_\star \to 0$ asymptote of that function is a power-law with a universal exponent,

$$\langle\nu(x=0,\omega)\rangle = C\nu_0(\omega/\omega_\star)^4 \quad \text{at} \quad \omega/\omega_\star \ll 1\,. \tag{17}$$

To see the universality of the exponent, we consider a sequence of models bridging the limits of weak and strong pinning, and then apply the ideas [6, 9–12] developed for the density of states in the *bulk* to derive the *edge* property $\langle\nu(x=0,\omega)\rangle$. Then, we find the constant $C \approx 0.032$ by a numerical simulation.

To motivate Eq. (17), we may assume the chain to be half-infinite, as the boundary condition at $x = d$ should not matter due to the wave localization. The low-frequency plasmon spectrum will be contributed by the soft oscillation modes in special, barely stable configurations of disorder built in the vicinity of the $x = 0$ edge of the chain. Next, we generalize the considered-so-far Gaussian model of disorder in continuum by substituting Hamiltonian (3) and correlator (4) with

$$H = H_0 - \frac{\lambda}{\sqrt{ac}}\sum_j \cos(2\theta(x_j)+\chi_j)\,, \tag{18}$$

where χ_j and x_j are, respectively, the random phases associated with and random locations of discrete impurities. By varying their density c from a large value down to $c \ll 1/a$ one crosses over between the limits of weak collective and strong individual-impurities pinning. The latter limit is amenable to the analytical treatment [6, 9].

Infinitely-strong pinning on separate impurities reduces the system to a sequence of independent segments [6]. Finite-strength impurities allow for special configurations carrying soft excitations with arbitrarily-low frequency, as was noticed in [9]. Closely following that work, we consider the needed three-impurity configurations in the vicinity of $x = 0$. The only difference from [9] is that we impose the boundary condition $\partial_x\bar{\theta}(0) = 0$ on the static charge distribution. Following [9], we find impurity configurations resulting in the low-frequency local modes of the pinned elastic string. It is tedious but straightforward (see Sec. II of SM [33]) to show that the boundary condition at $x = 0$ does not affect the $\omega \to 0$ asymptote of the density of states, $\langle\nu(\omega)\rangle \propto \omega^4$. As argued in [10–12], the functional form of the asymptote is universal and remains the same in the limit of weak pinning, which is of direct interest in the context of this work.

The considerations that led to Eq. (17) are substantiated – and the proportionality coefficient in it is found – in a numerical simulation presented in Sec. III of SM [33], and whose result is illustrated in Fig. 2.

Using Eq. (12) and the results (14), (16), and (17), we can now predict the overall evolution of the photon reflection amplitude with the increase of frequency. The microwave photons are fully reflected with $\langle r'\rangle \approx -1$ at

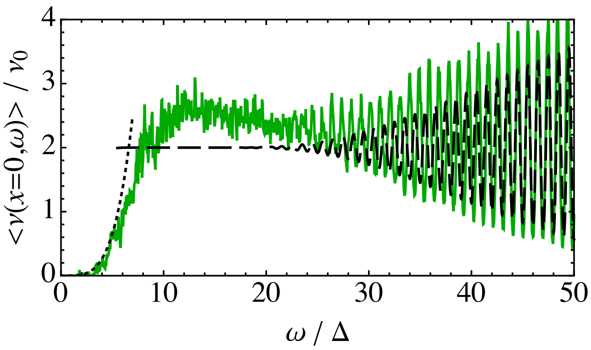

FIG. 2. Frequency dependence of the edge density of states, numerically evaluated and averaged over 5000 disorder configurations in a chain of length $d = 7.4R_\star$ (straight line). For comparison, analytically obtained asymptotes, Eq. (14) (dashed line) and Eq. (17) (dotted line), are also plotted.

$\omega \ll \omega_\star$; according to Eq. (17), the average reflection increases approaching $\langle r' \rangle \approx -1+2K/K_0$ at $\omega \sim \omega_\star$. Upon further increase of ω, the prominence of the spatial resonances in $\langle r'(\omega) \rangle$ raises, and they become well-resolved at $\omega \gg \omega_{\rm cr}$, see Eqs. (16) and (14). In the latter regime, the width of resonances translates into the quality factor $Q = \omega/[(4\ln 2)\gamma(\omega)] \propto \omega^2$. We emphasize that the Q-factor of the resolved resonances, according to our theory, comes from the ensemble averaging applied to the elastic plasmon propagation. We note in passing, that the inelastic scattering, considered in [34, 35] in the context of plasmon decay at $d \to \infty$, results in a minor contribution to $1/Q$ of well-resolved spatial resonances at finite d, see Sec. IV of SM [33].

In conclusion, microwave photon scattering off a superinductor may open a new way to study pinning in a one-dimensional quantum system. This work was devoted to the theory of reflection amplitude in the limit of small quantum fluctuations ($K \ll 1$). The reflection amplitude was recently measured [25] for a variety of superinductors; in a qualitative agreement with our predictions, the increase of the Q-factor with the microwave frequency was seen for samples with the highest probability of quantum phase slips.

We acknowledge stimulating discussions with V. Manucharyan. This work is supported by the DOE contract DE-FG02-08ER46482 (LG), and by the ARO grant W911NF-18-1-0212, a Google gift to Yale University, the European Union's FP7 programme through the Marie-Skłodowska-Curie Grant Agreement 600382, and ANR through grant No. ANR-16-CE30-0019 (MH).

[1] T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988).

[2] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B **40**, 546 (1989).

[3] A. I. Larkin, Sov. Phys. JETP **31**, 784 (1970).

[4] Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[5] H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978).

[6] L. P. Gorkov, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 384 (1977) [JETP Lett. **25**, 358 (1977)].

[7] M. V. Feigel'man, Sov. Phys. JETP **52**, 555 (1980).

[8] M. V. Feigel'man and V. M. Vinokur, Phys. Lett. **87**A, 53 (1981).

[9] I. L. Aleiner and I. M. Ruzin, Phys. Rev. Lett. **72**, 1056 (1994).

[10] M. M. Fogler, Phys. Rev. Lett. **88**, 186402 (2002).

[11] V. Gurarie and J. T. Chalker, Phys. Rev. Lett. **89**, 136801 (2002).

[12] V. Gurarie and J. T. Chalker, Phys. Rev. B **68**, 134207 (2003).

[13] H. Fukuyama and P. A. Lee, Phys. Rev. B **18**, 6245 (1978).

[14] R. Chitra, T. Giamarchi, and P. Le Doussal, Phys. Rev. Lett. **80**, 3827 (1998).

[15] H. A. Fertig, Phys. Rev. B **59**, 2120 (1999).

[16] M. M. Fogler and D. A. Huse, Phys. Rev. B **62**, 7553 (2000).

[17] T. Giamarchi, in I. V. Lerner *et al.* (eds.), *Strongly Correlated Fermions and Bosons in Low-Dimensional Disordered Systems* (Kluwer Academic Publishers, 2002).

[18] K. Cedergren, R. Ackroyd, S. Kafanov, N. Vogt, A. Shnirman, and T. Duty, Phys. Rev. Lett. **119**, 167701 (2017).

[19] N. Vogt, R. Schäfer, H. Rotzinger, W. Cui, A. Fiebig, A. Shnirman, and A. V. Ustinov, Phys. Rev. B **92**, 045435 (2015).

[20] N. Vogt, J. H. Cole, and A. Shnirman, New. J. Phys. **18**, 053026 (2016).

[21] V. E. Manucharyan, J. Koch, L. I. Glazman, and M. H. Devoret, Science **326**, 113 (2009).

[22] N. A. Masluk, I. M. Pop, A. Kamal, Z. K. Minev, and M. H. Devoret, Phys. Rev. Lett. **109**, 137002 (2012).

[23] M. T. Bell, I. A. Sadovskyy, L. B. Ioffe, A. Yu. Kitaev, and M. E. Gershenson, Phys. Rev. Lett. **109**, 137003 (2012).

[24] T. Weißl, B. Küng, E. Dumur, A. K. Feofanov, I. Matei, C. Naud, O. Buisson, F. W. J. Hekking, and W. Guichard, Phys. Rev. B **92**, 104508 (2015).

[25] R. Kuzmin, R. Mencia, N. Grabon, N. Mehta, Y.-H. Lin, and V. E. Manucharyan, arXiv:1805.07379.

[26] D. Ristè, C. C. Bultink, M. J. Tiggelman, R. N. Schouten, K. W. Lehnert, and L. DiCarlo, Nature communications **4**, 1913 (2013).

[27] V. Gurarie and A. M. Tsvelik, J. Low Temp. Phys. **135**, 245 (2004).

[28] D. A. Ivanov, L. B. Ioffe, V. B. Geshkenbein, and G. Blatter, Phys. Rev. B **65**, 024509 (2001).

[29] J. R. Friedman and D. V. Averin, Phys. Rev. Lett. **88**, 050403 (2002).

[30] S. E. Korshunov, Sov. Phys. JETP **68**, 609 (1989).

[31] K. A. Matveev, A. I. Larkin, and L. I. Glazman, Phys. Rev. Lett. **89**, 096802 (2002).

[32] M. Bard, I. V. Protopopov, I. V. Gornyi, A. Shnirman, and A. D. Mirlin, Phys. Rev. B **96**, 064514 (2017).

[33] Details on the derivation of Eq. (14) including backscattering, the prefactor of Eq. (17) in the strong-pinning regime, the numerics, and the smallness of inelastic scattering rate can be found in the SM, which includes

Refs. [9, 12, 34, 36–39].
[34] M. Bard, I. V. Protopopov, A. D. Mirlin, Phys. Rev. B **98**, 224513 (2018).
[35] H.-K. Wu and J. D. Sau, arXiv:1811.07941.
[36] I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* (John Wiley, New York, 1988).
[37] Jie Lin, K. A. Matveev, and M. Pustilnik, Phys. Rev. Lett. **110**, 016401 (2013).
[38] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Annals of Physics **321**, 1126 (2006).
[39] B. Rosenow, A. Glatz, and T. Nattermann, Phys. Rev. B **76**, 155108 (2007).