Superconducting Correlations Out of Repulsive Interactions on a Fractional Quantum Hall Edge

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Phys. Rev. Lett. **122**, 236802 — Published 13 June 2019

DOI: 10.1103/PhysRevLett.122.236802

Superconducting correlations out of repulsive interactions on a fractional quantum Hall edge

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(Dated: May 6, 2019)

We consider a fractional quantum Hall bilayer system with an interface between quantum Hall states of filling fractions $(\nu_{\text{top}}, \nu_{\text{bottom}}) = (1, 1)$ and $(1/3, 2)$, motivated by a recent approach to engineering artificial edges [1]. We show that random tunneling and strong repulsive interactions within one of the layers will drive the system to a stable fixed point with two counterpropagating charge modes which have attractive interactions. As a result, slowly decaying correlations on the edge become predominantly superconducting. We discuss the resulting observable effects, and derive general requirements for electron attraction in Abelian quantum Hall states. The broader interest in fractional quantum Hall edge with quasi-long range superconducting order lies in the prospects of hosting exotic anyonic boundary excitations, that may serve as a platform for topological quantum computation.

Introduction. Combining superconductivity and fractional quantum Hall edge states opens the possibility to engineer exotic topological phases of matter with anyonic boundary excitations [2–8]. A possible route to this is by using the proximity effect with a bulk superconductor and a quantum well in a hybrid structure [9–11]. Another, less studied possibility is that of intrinsic superconductivity on the edge. Evidently, on a one-dimensional edge there is no true long-range order and correlation functions decay algebraically. One can nevertheless refer to a superconducting phase as the one where the slowest decaying correlation function is of superconducting nature, i.e., a pairing correlator [12]. Such power-law (or quasi-long-range) superconducting order may still be relevant for topological quantum computing applications [13], c.f. [14] in the context of Majorana bound states.

In a recent experimental work [1] it has been demonstrated that in an engineered bilayer system it is possible to structure and control co- and counterpropagating edge modes in both the integer and fractional quantum Hall regimes. The present work takes advantage of this new paradigm and shows that one can design chiral modes with bare repulsive interaction in the presence of disorder to induce *attractive* interaction between the resulting effective modes. This gives rise to a phase with algebraically-decaying superconducting order.

To describe our results qualitatively, let us recall the pioneering work [15] of Kane, Fisher, and Polchinski (KFP) for $\nu = 2/3$ edge hosting counterpropagating $\nu = 1/3$ and $\nu = 1$ modes. Random tunneling and sufficiently strong interaction between the two modes can drive the system to a fixed point with decoupled neutral and charge modes. The charge of the latter, $2e/3$, is determined by the constituent bare modes and charge conservation. The fixed point is approached upon, for example, lowering the temperature, and can be understood as a renormalization of the interaction between the neutral and charge mode (the interaction is an irrelevant perturbation and renormalizes to zero). The novel aspect in our proposal is to consider an additional $\nu = 1$ $(1e)$ charge mode interacting with the KFP modes, see Fig. 1. As in the conventional KFP theory, both charge modes decouple from the neutral mode upon decreasing temperature. However, now there is a set of KFP fixed points, parametrized by the interaction between the $2e/3$ and $1e$ charge modes. Our main finding is that this fixed-point interaction can be *attractive*, even when the bare interactions of the high-temperature limit are repulsive. We further substantiate this claim by studying the renormalization group flow in a fine-tuned strongly-interacting model where the $2e/3$ and the neutral mode are already decoupled on the level of the bare Hamiltonian. We then move on to study the new fixed point. We find that the fixed point has superconducting correlations of the charge modes: their pairing correlation function decays slower than any charge density correlation function. Finally, we outline how our model can be realized in an engineered quantum Hall bilayer system and how one can detect the attractive interactions at the fixed point by using 3 experimental probes: multiterminal shot noise, tunneling spectroscopy, and ground state charge in a quantum dot geometry.

Model. We consider a system with 3 relevant edge modes. We assume a right-moving $\nu = 1/3$ mode and a pair of counterpropagating $\nu = 1$ modes. In terms of a three-component chiral boson field $\boldsymbol{\phi} = (\ \phi_{1/3}\ \ \phi_{-1}\ \ \phi_1\ )^T$, our model is described by the imaginary-time action

$$S = \int d\tau dx \frac{1}{4\pi}\left[\partial_x\boldsymbol{\phi}\mathbf{K}i\partial_\tau\boldsymbol{\phi} + \partial_x\boldsymbol{\phi}\mathbf{V}\partial_x\boldsymbol{\phi}\right] \qquad (1)$$
$$+ \int d\tau dx \left[\xi(x)e^{i\mathbf{c}\cdot\boldsymbol{\phi}} + \xi^*(x)e^{-i\mathbf{c}\cdot\boldsymbol{\phi}}\right] ,$$

where in the first line $\mathbf{K} = \text{diag}(3, -1, 1)$ and the V-

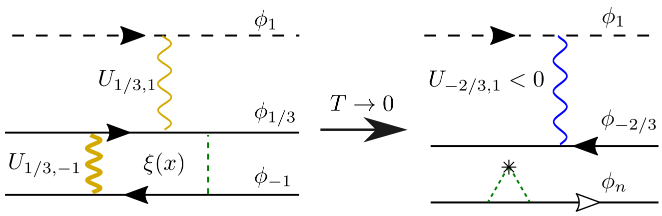


Figure 1. Pictorial description of the main result. On the left, we show the configuration of the bare edge modes. The configuration of edge modes can be experimentally realized in a bilayer structure, see Fig. 3a. There is strong Coulomb interaction, $U_{1/3,-1}$, as well as random tunneling, $\xi(x)$, between the modes $\phi_{1/3}$, ϕ_{-1}. The third mode ϕ_{-1} is weakly coupled with repulsive interaction to the mode $\phi_{1/3}$. As temperature is lowered the random tunneling renormalizes the strongly-coupled modes to a fixed point described by a neutral mode ϕ_n and a charge mode $\phi_{-2/3}$, shown on the right. Remarkably, the interaction between the charge modes $\phi_{-2/3}$ and ϕ_1 can be attractive while the neutral mode is decoupled due to the disorder.

matrix is

$$\mathbf{V} = \begin{pmatrix} 3v_{1/3} & U_{1/3,-1} & U_{1/3,1} \\ U_{1/3,-1} & v_{-1} & U_{-1,1} \\ U_{1/3,1} & U_{-1,1} & v_1 \end{pmatrix} . \tag{2}$$

We assume that the mode ϕ_1 is physically far from the other two, so that $U_{1/3,1}, U_{-1,1} \ll U_{1/3,-1}$. Finally, the second line of Eq. (1) describes disordered tunneling of electrons between the counterpropagating 1/3 and 1 modes; here the tunneling vector is $\mathbf{c} = (3, 1, 0)$ and ξ is a δ-correlated random coefficient, $\langle \xi(x)\xi(x') \rangle = W\delta(x - x')$, with zero average. The random tunneling term is non-linear in the boson fields and leads to non-trivial renormalization of the V-matrix.

Let us first ignore the interactions $U_{1/3,1}$, $U_{-1,1}$ and consider the problem of two modes $\phi_{1/3}$, ϕ_{-1}. This is exactly the model studied by KFP [15] in the context of the edge of the $\nu = 2/3$ quantum Hall state. The amplitude of random tunneling obeys the renormalization group (RG) equation [16] $\frac{dW}{dl} = (3-2\Delta_{3,1,0})W$. Here the scaling dimension is $\Delta_{3,1,0} = (2 - \sqrt{3}c)/\sqrt{1-c^2}$ where $c = (2U_{1/3,-1}/\sqrt{3})/(v_{1/3} + v_{-1})$; the perturbation is relevant, $\Delta_{3,1,0} < 3/2$, when $0.34 \lesssim c \lesssim 0.98$. Thus, for sufficiently large positive (repulsive interaction) $U_{1/3,-1}$, the random tunneling operator $e^{i[3\phi_{1/3}+\phi_{-1}]}$ is a relevant perturbation and its amplitude grows upon lowering the temperature. This tunneling operator is the only relevant one as long as we ignore tunneling to the mode ϕ_1. The latter mode can be ignored due to its larger separation [1], see also Discussion below. Next, we study how the increasing W under RG transformation affects the elements of the V-matrix.

Neutral mode basis and perturbative RG. Following Ref. [15], it is convenient to work in the basis where the random tunneling $e^{i[3\phi_{1/3}+\phi_{-1}]}$ is diagonal. This is the basis of a right-moving neutral mode and a left-moving charge mode,

$$\phi_n = \frac{1}{\sqrt{2}} \left(3\phi_{1/3} + \phi_{-1} \right) , \quad \phi_{-2/3} = \sqrt{\frac{3}{2}} \left(\phi_{1/3} + \phi_{-1} \right) . \tag{3}$$

The random tunneling, which conserves charge, only couples to ϕ_n: in this "neutral mode basis" $\boldsymbol{\phi} = \left(\phi_n \;\; \phi_{-2/3} \;\; \phi_1 \right)^T_{(n)}$ and the tunneling vector becomes $\mathbf{c} = (\sqrt{2}, 0, 0)_{(n)}$. Also, $\mathbf{K} = \mathrm{diag}(1, -1, 1)_{(n)}$ and

$$\mathbf{V} = \begin{pmatrix} v_n & U_{n,-2/3} & U_{n1} \\ U_{n,-2/3} & v_{2/3} & U_{-2/3,1} \\ U_{n1} & U_{-2/3,1} & v_1 \end{pmatrix}_{(n)} , \tag{4}$$

where the matrix elements are simple linear combinations of the elements from Eq. (2). In particular, the interaction between the charge modes $\phi_{-2/3}$, ϕ_1 is $U_{-2/3,1} = \frac{1}{\sqrt{6}}(3U_{-1,1} - U_{1/3,1})$. We see that the interaction is attractive, $U_{-2/3,1} < 0$, when $U_{1/3,1} > 3U_{-1,1}$. This can happen when the mode $\phi_{1/3}$ is the nearest one to ϕ_1, as in Fig. 1. As we show below, the attractive interaction between two charge modes makes the superconducting pair correlations between them the slowest decaying correlation function in the system, which we call superconducting state in 1D.

Evidently, in the bare non-renormalized V-matrix the seemingly attractive interaction is just a result of a basis change from a system with purely repulsive interactions. The off-diagonal elements $U_{n,-2/3}$, U_{n1} that couple the neutral mode to the two charge modes ensure that there are no superconducting correlations. However, we will show next that under renormalization, the elements $U_{n,-2/3}$, U_{n1} will flow to zero due to disorder in the neutral mode, while $U_{-2/3,1}$ remains approximately constant. In the original basis this corresponds to $U_{1/3,1}$, $U_{-1,1}$ flowing to negative values, i.e., attraction, see Fig. 2.

The weak-disorder RG flow of $\mathbf{V}$ was studied by Moore & Wen [17], who found that a relevant disorder operator $e^{i\sqrt{2}\phi_n}$ drives the V-matrix towards a fixed point which is diagonal in the neutral sector. Therefore, $U_{n,-2/3}$ and U_{n1} are both irrelevant and flow to weak coupling [18]. Furthermore, the disorder operator $e^{i\sqrt{2}\phi_n}$ commutes with $\partial_x\phi_{-2/3}\partial_x\phi_1$, so we expect $U_{-2/3,1}$ to be marginal, with weak renormalization stemming from its non-commutation with $U_{n1}\partial_x\phi_n\partial_x\phi_1$. We confirm this intuition by finding the flow equations [19] in the limit of weak disorder and weak couplings in the KFP fine-tuned (yet generic in terms of the resulting physics) point $U_{n,-2/3} = 0$ [corresponding to $U_{1/3,-1} = 3(v_{1/3}+v_{-1})/4$]. Numerical solution of the RG equations produces the flow diagram shown in Fig. 2, presented in terms of the original couplings $U_{-1,1}$, $U_{1/3,1}$.

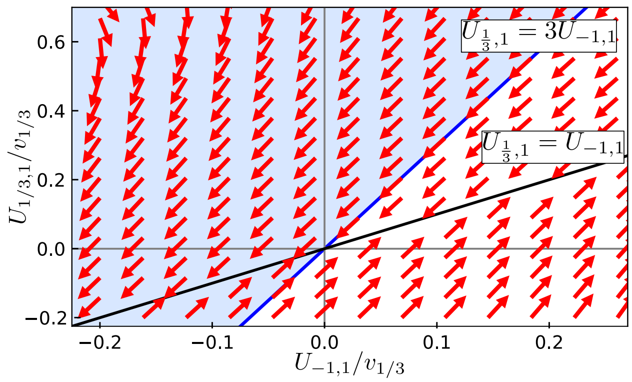


Figure 2. Flow of the interaction parameters $U_{-1,1}\,, U_{1/3,1}$ that couple the mode ϕ_1 to the other modes. The bare interactions in the blue region, $3U_{-1,1} - U_{1/3,1} < 0$, flow to a superconducting phase. There is a line of stable fixed points $U_{-1,1} = U_{1/3,1}$ $[U_{n1} = 0]$ (black). Near this line, the tree-level RG is accurate and flow is along lines of constant $3U_{-1,1} - U_{1/3,1}$ (dark blue line). Far from the line $U_{-1,1} = U_{1/3,1}$ one needs to account for terms beyond tree-level RG. The operator $3U_{-1,1} - U_{1/3,1}$ is marginally irrelevant and no longer remains constant during the flow. The flow diagram is calculated at the fine-tuned point where $U_{n,-2/3} = 0$. We took $v_1 = v_{-1} = 2v_{1/3}$.

Strong-disorder fixed point. Perturbative treatment of random tunneling is only valid at high energies. To describe the non-perturbative low energy regime we follow KFP and postulate a strong-disorder fixed point V-matrix,

$$\mathbf{V}_{\text{f.p.}} = \begin{pmatrix} v_n & 0 & 0 \\ 0 & v_{2/3} & U_{-2/3,1} \\ 0 & U_{-2/3,1} & v_1 \end{pmatrix}_{(n)} . \tag{5}$$

At the fixed point we have a decoupled right-moving neutral mode ϕ_n, a right-moving $\nu = 1$ charge mode ϕ_1, and a left-moving charge mode $\phi_{-2/3}$. The latter two are coupled via an interaction that is attractive, $U_{-2/3,1} < 0$, as long as the bare interactions satisfy $3U_{-1,1} < U_{1/3,1}$. The set of fixed point V-matrices (5) can also be obtained even without random tunneling by fine-tuning the bare interactions in Eq. (2) in such a way that the neutral mode decouples. Such a fine-tuning yields $U_{-2/3,1} = \sqrt{2/3}U_{1/3,1} > 0$, assuming repulsive bare interactions. Thus, renormalization by random tunneling is essential for obtaining an attraction out of repulsion.

The charge sector action can be diagonalized by a hyperbolic rotation

$$\begin{pmatrix} \phi_1 \\ \phi_{\frac{-2}{3}} \end{pmatrix} = \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix} \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}, \; \tanh 2\chi = \frac{-2U_{\frac{-2}{3},1}}{v_{\frac{2}{3}} + v_1}. \tag{6}$$

Using Eq. (6), one finds that the scaling dimension Δ of a generic vertex operator $O = \exp i(c_n\phi_n + c_1\phi_1 + c_{2/3}\phi_{-2/3})$ is

$$\Delta_{\mathbf{c}} = \frac{1}{4}(c_1 + c_{2/3})^2 e^{2\chi} + \frac{1}{4}(c_1 - c_{2/3})^2 e^{-2\chi} + \frac{1}{2}c_n^2 \,. \tag{7}$$

The attractive $U_{-2/3,1}$ in Eq. (5) makes the pairing correlation function the slowest decaying one. The superconducting pairing correlation function in the original basis is $O_{SC} \sim e^{i(\phi_1 - \phi_{-1})}$ [this operator creates two counterpropagating electrons in the $\nu = 1$ modes]. Its dimension is calculated by first expressing ϕ_{-1} in terms of ϕ_n and $\phi_{-2/3}$: $e^{i(\phi_1 - \phi_{-1})} = e^{i(\phi_1 - \frac{1}{\sqrt{2}}[\sqrt{3}\phi_{-2/3} - \phi_n])}$, and then using Eq. (7). We find the scaling dimension [20] $\Delta_{SC} = \frac{1}{4}(1 + \sqrt{\frac{3}{2}})^2 e^{-2\chi} + \frac{1}{4}(1 - \sqrt{\frac{3}{2}})^2 e^{2\chi} + \frac{1}{4}$. For $\chi \gtrsim 0.26$ we have $\Delta_{SC} < 1$, so the pairing correlator decays slower than the neutral mode correlator $e^{i[3\phi_{1/3} + \phi_{-1}]} = e^{i\sqrt{2}\phi_n}$. Likewise, the diagonal density operator $O_{c,\pm 1}(x) \sim \partial_x\phi_{\pm 1}$ has $\Delta = 1$ irrespective of $U_{-2/3,1}$ and so the density perturbation decays faster than pairing. Finally, we consider the off-diagonal density operator [21] $O_{CDW} \sim e^{i(\phi_1 + \phi_{-1})}$. We find $\Delta_{CDW} = \frac{1}{4}(1 + \sqrt{\frac{3}{2}})^2 e^{2\chi} + \frac{1}{4}(1 - \sqrt{\frac{3}{2}})^2 e^{-2\chi} + \frac{1}{4}$. Since $\chi > 0$ for $U_{-2/3,1} < 0$ [Eq. (6)], we always have $\Delta_{SC} < \Delta_{CDW}$. Thus, superconducting pair correlations are the slowest decaying ones in the strong coupling fixed point. Next, we discuss the measurable effects of this attraction.

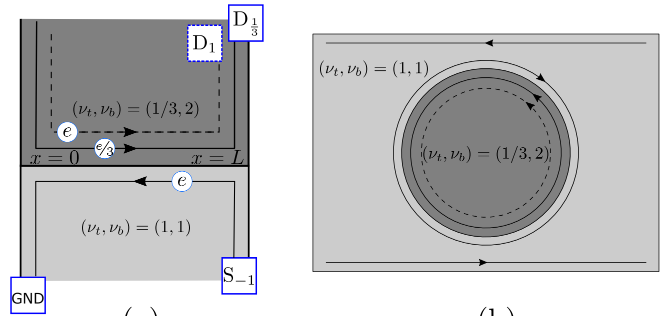
Consequences of attraction. The relatively long-ranged pairing correlations are a direct consequence of the attractive interaction $U_{-2/3,1} < 0$ in Eq. (5). Thus, one way to probe our proposed fixed point is to measure $U_{-2/3,1}$ or its sign. Since the fixed point action is that of a non-chiral spinless Luttinger liquid, one is faced with the known task of measurement of the Luttinger liquid parameter. Next, we outline three possible ways to do this. We focus on the experimentally relevant bilayer quantum Hall system, see Fig. 3.

Signature of attraction in shot noise. It is well-known that the interaction parameter in a non-chiral Luttinger liquid can be measured with a.c. shot noise [22–24]. The attractive interactions in our setup can be measured in a similar experiment, see Fig. 3a. Employing the theory of inhomogeneous Luttinger liquid [25], we solve [19] the problem of a bare incoming mode ϕ_{-1} scattered off an interacting region at the superconducting fixed point. In particular, the charge reflected into the mode ϕ_1 (drain D_1 in the bottom layer) is fractional with a non-universal magnitude. Its sign however is given directly by $U_{-2/3,1}$. *Thus, a smoking gun signature of the emergence of the attraction would be negative current measured at* D_1 ("Andreev reflection" at the edge) [26]. The reflected charge can be measured in a time-domain experiment and requires access to frequencies $\omega \gtrsim v/L$ where $v = \max(v_{2/3}, v_1)$ and L is the length of the scattering region.

Signature of attraction in tunneling conductance. One can also measure $U_{-2/3,1}$ from tunneling conductance [27–32] in the interacting region, for example by using a point-contact to an auxiliary $\nu = 1$ edge. For describing the tunneling Hamiltonian, consider the vertex operator $e^{i(n_1\phi_1 - n_{-1}\phi_{-1} + 3n_{1/3}\phi_{1/3})}$ that creates an excitation of total charge $n_1 + n_{-1} + n_{1/3}$ on the edge; here $n_1, n_{-1}, n_{1/3} \in \mathbb{Z}$. The contribution to the tunneling current from the above operator exhibits a power-law bias voltage dependence [27], [33] $I \propto V^{2\alpha-1}$ where the exponent $\alpha = \Delta_{n_1,n_{-1},n_{1/3}} + \frac{1}{2}n$ is determined by the scaling dimension $\Delta_{n_1,n_{-1},n_{1/3}}$ [obtained from Eq. (7) after transforming the vertex operator into the neutral mode basis by using Eq. (3)] and the number of electrons n removed from the auxiliary edge. The total tunneling current is a sum of elementary tunneling processes, but will be dominated at small voltages by those with a low value of α. For moderate interaction strengths χ [Eq. (6)] the dominant contributions are the 1-electron tunneling operators $e^{i\phi_1}$ and $e^{-i\phi_{-1}}$, as well as the 2-electron tunneling operator $e^{i(\phi_1 - \phi_{-1})}$. Their respective tunneling amplitudes t_1, t_{-1}, and $t_1 t_{-1}$, are in principle controllable by gating, so that different 1-electron contributions can be turned on and off. The signature of attractive interactions ($\chi > 0$) is that $\Delta_{1,-1,0} < \Delta_{1,0,0} + \Delta_{0,-1,0}$, meaning that when tunneling to both $\nu = 1$ edges is present, the current is less suppressed by a small bias than one would expect from uncorrelated tunnelings to each edge separately.

Signature of attraction in a mesoscopic droplet. Finally, one can perform a fully thermodynamic measurement in a Coulomb blockaded quantum Hall droplet, see Fig. 3b. This is akin to ideas of "attraction from repulsion" that have been implemented in other systems [34], compare also proposals to probe neutral modes in the context of quantum Hall edges [35]. The signature of attraction in the Coulomb blockaded droplet is $2e$-periodic charge transitions as a gate charge is varied [19]. This signature can be measured in a thermodynamic capacitive measurement of the charge or in a transport measurement of the Coulomb peak spacings.

Discussion. Our proposal relies on the tunneling between the modes $\phi_{1/3}$ and ϕ_{-1} being the most RG relevant perturbation. Typically, the tunneling $e^{i(\phi_1+\phi_{-1})}$ between the $\nu = 1$ modes is also relevant and leads to a trivial localization of the $\nu = 1$ modes. This effect should however be present only at very low temperatures since we expect the bare amplitude of the $\nu = 1$ tunneling to be very weak due to the large separation of the $\nu = 1$ modes. The $\nu = 1$ tunneling can also be entirely avoided by considering a setup with spin-polarized Landau levels where ϕ_1 and ϕ_{-1} have opposite spins and the tunneling between them is forbidden by spin conservation. This can be achieved with an interface between $(\nu_{\text{top}}, \nu_{\text{bottom}}) = (1/3, 2)$ and $(\nu_{\text{top}}, \nu_{\text{bottom}}) = (1, 1)$, assuming the $\nu = 2$ state consists of opposite-polarized $\nu = 1$ states. This state also satisfies the requirement that $\nu_{\text{bottom}} \geq \nu_{\text{top}}$ holds on both sides [1].



(a) (b)

Figure 3. Experimental probes to measure attractive interactions in (a) shot noise, and (b) Coulomb blockaded Hall droplet. This configuration can be experimentally realized in a bilayer structure at an interface of bulk fillings $(\nu_{\text{top}}, \nu_{\text{bottom}}) = (1, 1)$ and $(1/3, 2)$. We have drawn the bare modes at the interface in the same order as in Fig. 1 left. These modes are renormalized to an effective neutral and charge modes (e and $2e/3$), see Fig. 1 right. The dashed edge mode lives on the bottom layer and solid ones on the top. We have not drawn the $\nu = 1$ mode of the bottom layer that encircles the entire system since it is not relevant to the physics at the interface.

In our model we assumed that $U_{1/3,-1}$ is the largest interaction while the other two were treated perturbatively, which ensures that $e^{i[3\phi_{1/3}+\phi_{-1}]}$ is relevant and KFP fixed point is reached. Thus, we rely on the double-inequality $U_{1/3,-1} > U_{1/3,1} > 3U_{-1,1}$ to approach the fixed point with attractive interactions. We find that Coulomb interaction screened by a nearby gate electrode [36],[19] allows both inequalities to be satisfied.

One may ask how essential the bilayer construction is to manifest our theory. For example, edge reconstruction in a $\nu = 1/n$ Laughlin state can give rise to a $\nu = 1/m$ stripe in the bulk-vacuum interface. Disordered tunneling between the two inner modes gives rise to counterpropagating neutral and a charge modes. The propagation directions of these modes are determined by comparing the two filling fractions. If $n < m$, the charge mode is co-propagating with the outermost $\nu = 1/m$ mode. Therefore, there are no emerging superconducting correlations even if there is attraction between the two charge modes. (Interactions between co-propagating modes do not affect the scaling dimensions of the operators involved, since the V-matrix can be diagonalized with an orthogonal transformation [27].) In the more interesting scenario $n > m$, the charge modes are counterpropagating and superconducting correlations may in principle emerge. In this case the the interaction is attractive when $U_{\frac{1}{n},\frac{1}{m}} > \frac{n}{m} U_{-\frac{1}{m},\frac{1}{m}}$. However, for an interaction falling monotonically with distance, we expect $U_{\frac{1}{n},\frac{1}{m}} < U_{-\frac{1}{m},\frac{1}{m}}$ because the outermost mode $+1/m$ is closer to $-1/m$ rather than the bulk mode $1/n$. This is why we do not expect to find superconducting correlations in such a simple model of edge reconstruction. This problem is circumvented in the bilayer setup, see Fig. 1. Here the two-dimensional electron gas (2DEG) is replaced by a bilayer of 2DEGs whose individual filling fractions can be tuned. The resulting boundary consists of chiral mode structure which can be controlled on-demand by tuning back gate voltages and the magnetic field. Finally, we note that our proposal also works for an interface between $(\nu_{\rm top}, \nu_{\rm bottom}) = (2/3, 1)$ and $(\nu_{\rm top}, \nu_{\rm bottom}) = (0, 2)$, assuming that the $\nu = 2/3$ edge consists of counterpropagating $\nu = 1$ and $\nu = 1/3$ modes [37, 38].

We thank M. Heiblum, R. Lutchyn, and D. Pikulin for discussions. J.I.V. thanks the Aspen Center for Physics which is supported by National Science Foundation grant PHY-1607611. M.G. was supported by the Israel Science Foundation (Grant No. 227/15), the German Israeli Foundation (Grant No. I-1259-303.10), the US-Israel Binational Science Foundation (Grant No. 2016224), and the Israel Ministry of Science and Technology (Contract No. 3-12419). Y.G. was supported by DFG RO 2247/11-1 and CRC 183 (project C01), and the Italia-Israel project QUANTRA.

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[18] The modes ϕ_n and ϕ_1 are mutually chiral and therefore a stable fixed point requires $v_1 > v_n$ [17].
[19] See Supplementary Material, where we present the full RG equations at $U_{n,-2/3} = 0$, discuss in more detail the signatures of attraction, and outline the geometrical requirements to find attraction from repulsion in a bilayer system.
[20] The pairing (charge 2) operator $e^{i(\phi_1 - 2\phi_{-1} - 3\phi_{1/3})} = e^{i(\phi_1 - \phi_{-1})} e^{-i\sqrt{2}\phi_n}$ has the same scaling dimension as O_{SC} while other pairing operators are less relevant [32]. The existence of these equally relevant pairing operators does not change the observable effects outlined below.
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