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Optical Bound States in the Continuum with Nanowire Geometric Superlattices

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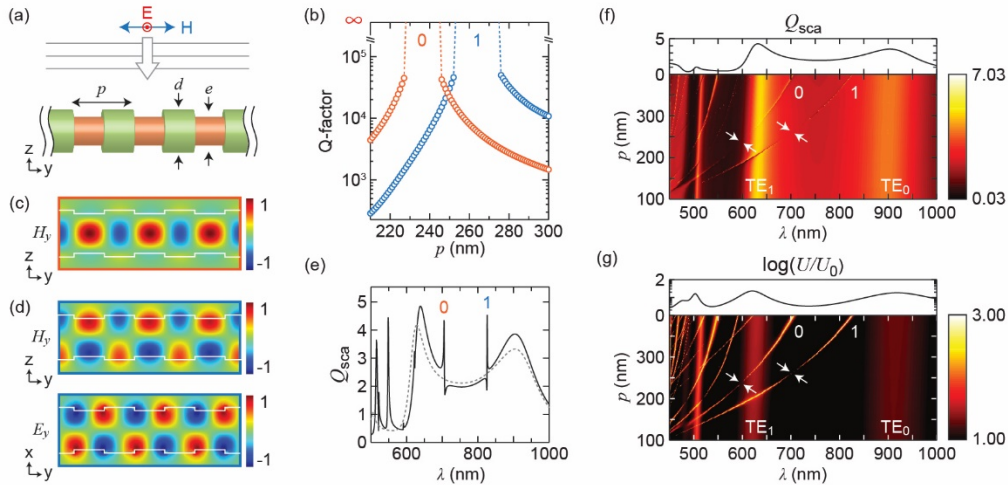
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Perfect trapping of light in a subwavelength cavity is a key goal in nanophotonics. Perfect trapping has been realized with optical bound states in the continuum (BIC) in waveguide arrays and photonic crystals, yet the formal requirement of infinite periodicity has limited the experimental realization to structures with macroscopic planar dimensions. Here, we characterize BICs in a silicon nanowire (NW) geometric superlattice (GSL) that exhibits one-dimensional periodicity in a compact cylindrical geometry with subwavelength diameter. We analyze the scattering behavior of NW GSLs by formulating temporal coupled mode theory to include Lorenz-Mie scattering, and we show that GSL-based BICs can trap electromagnetic energy for an infinite lifetime and exist over a broad range of geometric parameters. Using synthesized NW GSLs tens of microns in length and with variable pitch, we demonstrate the progressive spectral shift and disappearance of Fano resonances in experimental single-NW extinction spectra as a manifestation of BIC GSL modes.

20 Trapping light in subwavelength structures is of utmost importance in wave physics [1-3]
21 and central to a wide range of photonic and optoelectronic applications [4-7]. Localized optical
22 modes with infinite lifetimes, namely optical bound states in the continuum (BICs), can exist in
23 the radiation continuum, and they have been described in one-dimensional (1D) arrays of
24 coupled waveguides [8-10] and two-dimensional (2D) photonic crystals (PCs) [3,11,12]. Despite
25 the subwavelength size of each optical resonator unit cell, however, BIC structures require
26 infinite periodicity to formally satisfy the BIC condition [3], necessitating macroscopic quasi-
27 infinite planar structures for experimental realization of both the 1D array and 2D PC examples.
28 To reduce the physical dimensions of BIC cavities, recent theoretical studies have investigated
29 the presence of BICs in 1D structures with lateral 2D confinement, such as 1D arrays of
30 dielectric spheres [13,14] or disks [15,16], as well as supercavity modes in individual dielectric
31 nanorods [17,18]. Optical BICs in these theoretical examples are reported to exist because of
32 symmetry mismatch, accidental decoupling [13,15], or topological protection [14]. Although a
33 detuned quasi-BIC has been observed in the microwave regime from a chain of millimeter-sized
34 ceramics [19], experimental demonstration of BICs in the optical regime with laterally-confined
35 1D nanostructures has to our knowledge not been reported.

36 In this letter, we describe the perfect trapping of light in single Si nanowire (NW)
37 geometric superlattices (GSLs) through BICs above the light cone. A NW GSL has a
38 subwavelength diameter that is periodically modulated along the NW axis [20-22], as shown in
39 Fig. 1(a). The optical confinement defined by the NW diameter gives rise to well-defined, strong
40 Mie resonances [23-25], allowing NWs to strongly interact with external plane waves. Moreover,
41 a NW GSL exhibits an additional set of unique photonic modes that are dependent on the pitch
42 (p), outer diameter (d), and inner diameter (e) of the GSL. As shown herein, in a NW GSL under

43 transverse-electric (TE) polarized plane wave illumination, GSL guided resonances [26] with
 44 different orbital angular momenta [13,15] can be excited and couple to Mie resonances to
 45 produce sharp Fano resonances. For a certain set of geometric parameters, these GSL modes
 46 undergo complete destructive interference, resulting in disappearance of the Fano features and
 47 formation of optical BICs. Full wave simulations and theoretical modeling using temporal
 48 coupled-mode theory (TCMT) formulated to include Lorenz-Mie scattering theory describe the
 49 origin of Fano resonances in different angular channels and the appearance of optical BICs. We
 50 discuss the geometric parameters for which a GSL satisfies the BIC condition and verify
 51 theoretical predictions with experimental measurements on single Si NW GSLs. This work
 52 realizes 1D BICs for the first time in the optical regime with true nanoscale lateral footprints, and
 53 we expect this result to motivate further research into the design of optical nanocavities.



54
 55 **FIG. 1.** Optical BICs in a NW GSL. (a) Geometry of a NW GSL under TE-polarized plane wave
 56 illumination, where the length of each segment is $p/2$. (b) Q-factors of two GSL eigenmodes with
 57 varying p in a NW GSL with $d=200$ nm, $e=170$ nm. Modes are labeled with angular numbers
 58 $m=0$ or $m=1$. (c) H_y pattern of $m=0$ GSL eigenmode. (d) H_y (upper) and E_y (lower) patterns of
 59 $m=1$ GSL eigenmode. (e) Q_{sca} spectrum of a NW GSL with $d=200$ nm, $e=170$ nm, and $p=400$

60 nm (solid black curve) and of a NW with $d=185$ nm (gray dashed curve). (f-g) Heatmaps of Q_{sca}
61 (f) and $\log(U/U_0)$ (g) for a NW GSL with varying p for fixed $d=200$ nm and $e=170$ nm. Single
62 spectra for a uniform NW with $d=185$ nm are presented on top of each heatmap.

63 As shown in Fig. 1(b), eigenmode analysis of NW GSL structures reveals GSL modes
64 with quality factors (Q-factors) that diverge to infinity within a range of p , indicating that these
65 GSL modes are optical bound states with infinite lifetimes. The y -component of the
66 electromagnetic (EM) fields, H_y and E_y , of the two GSL eigenmodes are given in Fig. 1(c) and (d)
67 [see *Supplemental Material* for full EM profiles]. Each GSL mode is assigned with angular
68 numbers of $m=0$ or 1 based on the azimuthal order of field maxima. In Fig. 1(c), the $m=0$ GSL
69 mode has a definitive TE polarization (*i.e.* $E_y=0$; not shown), and H_y exhibits an
70 antiferromagnetic ordering of magnetic dipoles. In contrast, the $m=1$ GSL mode in Fig. 1(d) is
71 hybrid-polarized, so both E_y and H_y are nonzero and must be considered.

72 Bulgakov *et al.* [13,15] have categorized BICs arising in confined 1D geometries based
73 on symmetry and propagation constant, and static BICs may have either even or odd symmetry
74 under inversion. Odd modes are always symmetry protected from the free-space radiation
75 whereas even modes become decoupled from the radiation continuum only with certain
76 geometric parameters. The $m=0$ mode in Fig. 1(c) exhibits even symmetry and belongs to the
77 latter case, and the BIC condition is achieved by tuning p as shown in the orange trace in Fig.
78 1(b). The $m=1$ mode in Fig. 1(d), however, is odd in E_y but even in H_y . Thus, it is symmetry-
79 protected against the decay into the transverse-magnetic (TM) diffraction channel but reaches the
80 bound state only when it also decouples from the TE continuum through the proper choice of p
81 (Fig. 1(b), blue trace). Previously, we reported a coupled-excitation of guided modes in a NW
82 GSL under excitation with a transverse-magnetic (TM) polarized plane wave [22], and although

83 those modes have a similar symmetry to the $m=1$ GSL mode in Fig. 1(d), Fig. 1(c) shows that the
 84 GSL modes are not limited to the guided modes and can possess different symmetry types.

85 A scattering efficiency (Q_{sca}) spectrum of a NW GSL with $p=400$ nm is shown in Fig. 1(e)
 86 along with a reference Q_{sca} spectrum for a uniform NW. Because the value of p places the
 87 structure outside the range needed for a BIC, the Q_{sca} spectrum of the GSL exhibits two Fano
 88 resonances resulting from coupling between the GSL mode and the background Mie resonance
 89 in the same angular channel. In the Q_{sca} heatmap in Fig. 1(f), two sharp branches of GSL modes
 90 denoted with angular numbers $m=0$ and 1 red-shift with increasing p while the background Mie
 91 resonances, denoted TE_0 and TE_1 , do not shift because of the fixed diameters. The $m=0$ and 1
 92 GSL branches show vanishing points at p values of 237 and 262 nm, respectively, where the
 93 modes become completely bound. These features are more clearly observed in the heatmap of
 94 confined energy (U/U_0) in Fig. 1(g). While the p values producing a BIC, marked with arrows in
 95 Fig. 1(f) and (g), fall in the ranges of infinity Q-factor for each mode in Fig. 1(b), the range of p
 96 satisfying the BIC condition are much narrower than the BIC ranges predicted by eigenmode
 97 calculations because of the directional illumination in plane wave simulations.

98 TCMT can be used to predict the optical coupling behavior in a NW GSL and has been
 99 used to interpret similar effects in photonic crystal slabs [26-28] and in spherical nanoparticles
 100 [29-32]. Here, we employ TCMT in the context of NWs by relating resonance parameters to the
 101 exact solutions of Mie coefficients [23]. For the scattering of a uniform, cylindrical NW, H_y
 102 under a TE plane wave ($H_x; H_z=0$) is given by

$$103 \quad H_y = \sum_{m=-\infty}^{\infty} \left[h_m^+ H_m^{(2)}(k\rho) + h_m^- H_m^{(1)}(k\rho) \right] e^{im\phi} \quad ,$$

104 (1) where h_m^+ and h_m^- are amplitudes of the incoming and outgoing waves, $H_m^{(1)}$ and $H_m^{(2)}$ are the
 105 m th-order Hankel functions, k is a wavevector, and ρ and ϕ are the polar coordinates [31]. We

106 define a reflection coefficient by $R_m \equiv h_m^-/h_m^+$, and a single-mode TCMT expression is given by

$$107 \quad \frac{d}{dt} A_m^{Mie} = (-i\omega_m^{Mie} - \gamma_m^{Mie}) A_m^{Mie} + \kappa_m^{Mie} h_m^+ \\ 108 \quad (2)$$

109 with $h_m^- = h_m^+ + d_m^{Mie} A_m^{Mie}$, where A_m^{Mie} , ω_m^{Mie} and γ_m^{Mie} are the amplitude, eigenfrequency, and
 110 radiative decay rate of an m th-order Mie resonance, respectively, and κ_m^{Mie} and d_m^{Mie} are coupling
 111 coefficients to the incoming and outgoing plane waves, respectively. Absorptive loss is neglected
 112 for simplicity, and $\kappa_m^{Mie} = d_m^{Mie} = i\sqrt{2\gamma_m^{Mie}}$ by time-reversal symmetry [28]. The total Q_{sca} of a
 113 NW [31] is

$$114 \quad Q_{sca} = \frac{2}{kr} \sum_{m=-\infty}^{\infty} \left| \frac{1-R_m}{2} \right|^2, \\ 115 \quad (3)$$

116 where r is the NW radius. Noting the similarity of Eq. (3) to the Mie scattering formula, we can
 117 relate the scattering coefficient, $\left| \frac{1-R_m}{2} \right|$ in Eq. (3), with an exact Mie scattering coefficient [23] to
 118 yield

$$119 \quad \left| \frac{1-R_m}{2} \right| = \left| \frac{\gamma_m^{Mie}}{i(\omega - \omega_m^{Mie}) + \gamma_m^{Mie}} \right| = |a_m|, \\ 120 \quad (4)$$

121 where a_m is an m th-order electric Mie coefficient responsible for scattering of a NW under TE
 122 polarization (analogously we can use the magnetic Mie coefficient, b_m , for TM polarization).

123 Rearranging Eq. (4), as shown in the *Supplemental Material*, we get

$$124 \quad \omega_m^{Mie} = \frac{2i \cdot a_m \cdot Q_m^{Mie} \cdot \omega}{(2i \cdot Q_m^{Mie} + 1) a_m \pm 1}, \\ 125 \quad (5)$$

126 where $Q_m^{Mie} = \omega_m^{Mie}/2\gamma_m^{Mie}$ is the Q-factor of a Mie resonance on the order of 5-10 that can
 127 easily be estimated from numerical spectra. With ω_m^{Mie} and γ_m^{Mie} as functions of a_m , the
 128 modified TCMT can correctly produce the asymmetric line shapes of NW Mie resonances.

129 For a NW GSL, the full TCMT equation becomes

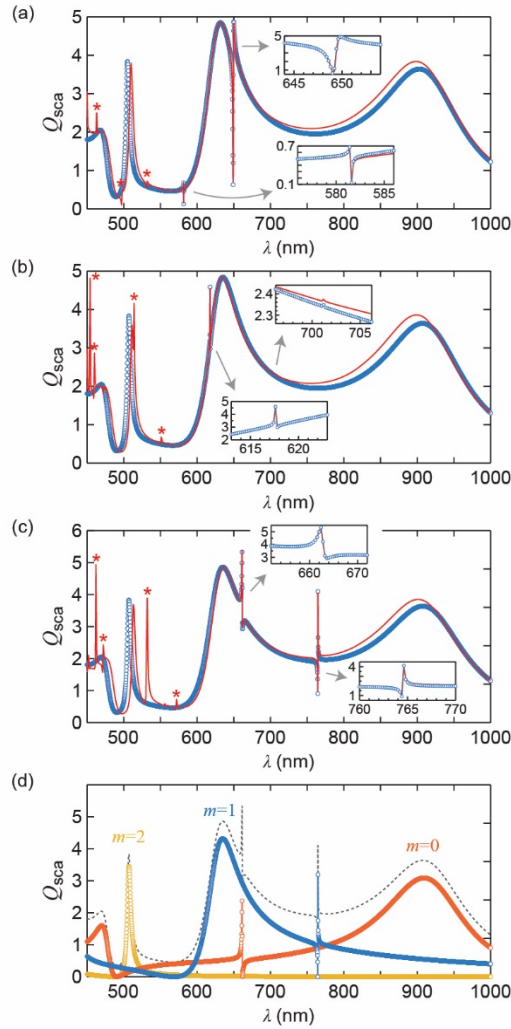
$$130 \quad \frac{d}{dt} \begin{pmatrix} A_m^{Mie} \\ A_m^{GSL} \end{pmatrix} = \left[-i \begin{pmatrix} \omega_m^{Mie} & \omega_m^c \\ \omega_m^c & \omega_m^{GSL} \end{pmatrix} - \begin{pmatrix} \gamma_m^{Mie} & 0 \\ 0 & \gamma_m^{GSL} \end{pmatrix} \right] \begin{pmatrix} A_m^{Mie} \\ A_m^{GSL} \end{pmatrix} + \begin{pmatrix} \kappa_m^{Mie} \\ \kappa_m^{GSL} \end{pmatrix} h_m^+ ,$$

131 (6)

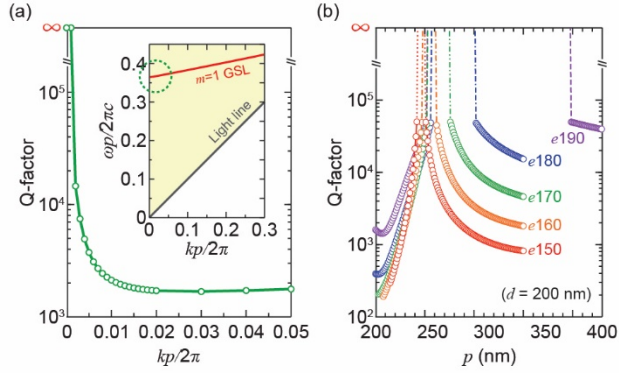
132 and $h_m^- = h_m^+ + d_m^{Mie} A_m^{Mie} + d_m^{GSL} A_m^{GSL}$, where A_m^{GSL} , ω_m^{GSL} and γ_m^{GSL} are the amplitude,
 133 eigenfrequency, and radiative decay rate of an m th-order GSL mode, ω_m^c is the coupling strength
 134 between the Mie and GSL modes, and κ_m^{GSL} and d_m^{GSL} are coupling coefficients of GSL modes to
 135 the incoming and outgoing plane waves, respectively.

136 We only consider the coupling of modes within the same angular channel [33], and using
 137 Eq. (6) we can fit the numerical Q_{sca} spectra to reproduce all scattering features. As an example,
 138 Fig. 2(a)-(c) display Q_{sca} for $p=220, 260,$ and 320 nm at a fixed $d=200$ nm and $e=170$ nm, where
 139 total Q_{sca} obtained from TCMT (circles) are overlaid with numerical simulations (red curves).
 140 The case of $p=260$ nm satisfies the BIC condition, but shorter and longer p do not. Fano
 141 resonances appear for the shorter and longer p cases because the GSL modes couple with the Mie
 142 resonance ($\omega_m^c \neq 0$). At $p=260$ nm, however, the $m=1$ Fano peak almost completely vanishes at
 143 701 nm (Fig. 2(b)). The analytical Q_{sca} in Fig. 2(b) is obtained with both ω_1^c and $\gamma_1^{GSL} \approx 0$,
 144 implying the emergence of a perfectly bound optical state. Because the $m=0$ GSL mode becomes
 145 bound at a slightly different p than $m=1$ (*c.f.* Fig. 1(b)), it is still observed at ~ 618 nm but with a
 146 vanishing linewidth of ~ 0.2 nm. Fig. 2(d) shows the total Q_{sca} (dashed curve) and separate Q_{sca}
 147 spectra from each angular channel calculated using Eq. (6) for the $p=320$ nm NW GSL (circles).
 148 The two Fano resonances at ~ 661 and ~ 765 nm are separately observed in the $m=0$ and 1 angular

149 channels, respectively, and the long tails of the asymmetric Mie resonances permit Fano
 150 resonances to appear far away from the Mie maxima of the same channel.



151
 152 FIG. 2. Q_{sca} from TCMT and from full wave simulations. (a-c) Total Q_{sca} calculated by modified
 153 TCMT (blue circles) overlaid with numerical calculations (red curve) for a NW GSL with $d=200$
 154 nm, $e=150$ nm, and $p=220$ nm (a), $p=260$ nm (b), or $p=320$ nm (c). Insets show magnified views
 155 near Fano resonances. Peaks marked with asterisks result from higher order GSL modes not
 156 included in the TCMT. (d) Total Q_{sca} (dashed curve) from panel (c) decomposed into each
 157 angular channel (circles).



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159 FIG. 3. Geometric dependence of Q-factor. (a) Plot of Q-factor as a function of $kp/2\pi$. Inset:
 160 band structure of the $m=1$ mode. (b) Q-factors of an $m=1$ mode as a function of p with $d=200$ nm
 161 and various e .

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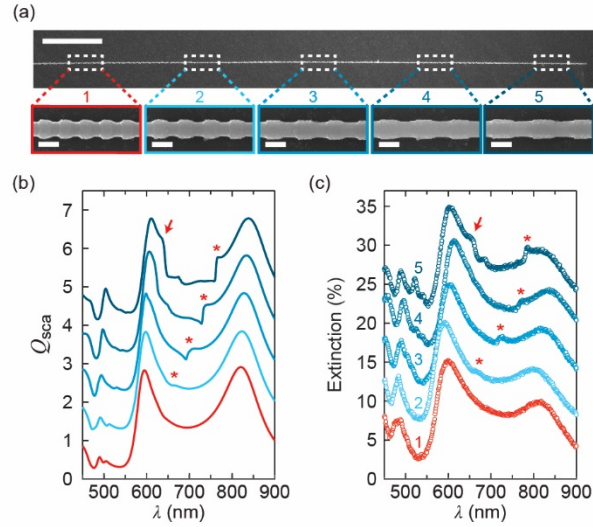
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The appearance of a BIC depends sensitively on illumination and structural geometry. For instance, BICs only appear at a Γ point because the symmetry of the BIC is distorted with a nonzero axial wavevector. As an example, Fig. 3(a) shows the Q-factor of the $m=1$ BIC mode of a NW GSL, calculated from the circled area in the inset band diagram, as a function of $kp/2\pi$ starting at Γ . The Q-factor decreases from infinity as the wavevector deviates from Γ because the loss of illumination symmetry allows the mode to couple to the Mie resonance. Moreover, even at Γ , the range of p that produces a BIC (or infinite Q-factor) changes with different values of e at a fixed d , and there is a substantial widening of the p range producing a BIC as e approaches d , as shown in the eigenmode calculations in Fig. 3(b). When d and e are similar, the magnitude and mode volume of dipoles within each diameter segment are similar in magnitude [Supplemental Material Fig. S1], so a broad set of p can produce the total destructive interference needed to form a BIC. However, as e deviates from d , the p range supporting a BIC narrows and eventually disappears (frequencies of BICs formed at different p and e are summarized in Fig. S2 in Supplemental Material).



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177 FIG. 4. Experimental extinction measurement of NW GSLs. (a) SEM image (upper panel) of a
 178 NW containing five GSL sections; scale bar, 10 μm . Magnified views (lower panels) of each
 179 GSL segment corresponding to the boxed regions in the upper panel; scale bars, 200 nm.
 180 Geometric parameters are $d=189\pm 2$ nm, $e=141\pm 2$ nm, $p=201\pm 4$ nm (GSL 1), $d=185\pm 1$ nm,
 181 $e=135\pm 2$ nm, $p=250\pm 4$ nm (GSL 2), $d=186\pm 1$ nm, $e=147\pm 2$ nm, $p=300\pm 3$ nm (GSL 3), $d=185\pm 1$
 182 nm, $e=153\pm 2$ nm, $p=348\pm 4$ nm (GSL 4) and $d=183\pm 1$ nm, $e=148\pm 2$ nm, $p=400\pm 6$ nm (GSL 5).
 183 (b-c) Simulated Q_{sca} (b, spectra offset by 1) and measured extinction (c, spectra offset by 5%) of
 184 GSLs. Red traces in both graphs represent spectra of a GSL at the BIC condition. Arrows and
 185 asterisks indicate the $m=0$ and 1 Fano resonances, respectively.

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We experimentally verified the scattering characteristics of NW GSLs fabricated by the ENGRAVE (Encoded Nanowire Growth and Apppearance through Vapor-liquid-solid growth and Etching) technique [20,21]. A d close to 200 nm was chosen to allow direct comparison with the scattering heatmap in Fig. 1(f), and p was varied from 200 nm to 400 nm with 50 nm increments to investigate the spectral shift and disappearance of the Fano resonances. To minimize variation in d and e , five 10 μm -long GSL sections with different p were encoded

192 simultaneously in a single NW with 10 μm uniform segments separating each GSL, as shown by
193 the scanning electron microscope (SEM) images in Fig. 4(a). Polarization-resolved transmissive
194 single-NW extinction was measured in the visible range using a home-built laser microscope
195 [22]. Simulated Q_{sca} corresponding to measured geometries and measured extinction spectra are
196 shown in Fig. 4(b) and (c), respectively. Q_{sca} was simulated with a Gaussian beam (full-width-at-
197 half-maximum of 1.5 μm) in the presence of material absorption to properly reflect the
198 experiment.

199 For simulated spectra of a GSL with $p=400$ nm (uppermost in Fig. 4(b)), two Fano
200 resonances for $m=0$ and $m=1$ GSL modes are observed at ~ 645 and ~ 762 nm as marked by the
201 arrow and asterisk, respectively. Compared to the sub-nm linewidth shown in Fig. 1(e), a
202 substantial broadening of the Fano lineshape is observed because of absorptive loss and the finite
203 beam [34]. An additional small peak at ~ 676 nm comes from the use of a finite beam [22]. As p
204 decreases, the $m=0$ peak gradually merges into the broad Mie resonance peak centered at ~ 600
205 nm, and the $m=1$ peak (red asterisks) blueshifts and progressively decreases in magnitude. At
206 $p=230$ nm (red curve), the Fano peak vanishes because the mode becomes decoupled from the
207 TE and TM radiation continua. The same pattern is observed in the experimentally measured
208 extinction in Fig. 4(c). The extinction of GSL 5 (uppermost) shows the $m=1$ Fano resonance at
209 ~ 785 nm (red asterisk). The $m=1$ Fano resonance blueshifts with decreasing p , and it eventually
210 vanishes for GSL 1 (red circles), corresponding to the formation of a BIC. As a result, the
211 extinction of GSL 1 looks identical to the typical extinction spectrum of a uniform NW,
212 demonstrating the inaccessibility of the trapped modes by far-field illumination. Inclusion of
213 absorption in eigenmode calculations shows a significant reduction of the Q-factors to ~ 150 , and

214 experimental Q-factors, obtained from fitting the spectra, yield values of 95-180 that
215 qualitatively agree with calculations (see *Supplemental Material* Fig. S3).

216 In conclusion, we have demonstrated that NW GSLs support unique photonic modes that
217 can be completely bound under a certain set of geometric parameters, and this report represents
218 the first experimental demonstration of a BIC in a laterally-confined 1D geometry in the optical
219 regime. The bottom-up growth of Si NW GSLs through the ENGRAVE process offers several
220 technological advantages such as mechanical robustness from single-crystalline materials, ability
221 to electro-generate photons inside the cavity through doping [35], and ease of device integration
222 through templated growth [36]. Because the subwavelength lateral dimensions provide a true
223 nanoscale footprint, these findings could enable the design of compact high-Q photonic devices
224 such as single-NW photodetectors, lasers, sensors, and photonic circuits.

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