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## Density oscillations induced by individual ultracold two-body collisions

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Access to single particle momenta provides new means of studying the dynamics of few interacting particles. In a joint theoretical and experimental effort, we observe and analyze the effects of a finite number of ultracold two-body collisions on the relative and single-particle densities by quenching two ultracold atoms with initial narrow wave packet into a wide trap with inverted aspect ratio. The experimentally observed spatial oscillations of the relative density are reproduced by a parameterfree zero-range theory and interpreted in terms of cross-dimensional flux. We theoretically study the long time dynamics and find that the system does not approach its thermodynamic limit. The set-up can be viewed as an advanced particle-collider that allows one to watch the collision process itself.

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The one-dimensional harmonic oscillator is discussed in many text books, from introductory classical and quantum mechanics to quantum optics and field theory [\[1\]](#page-5-1). The physics of the one-dimensional harmonic oscillator is simple: Its classical orbits are sinusoidal and periodic and the quantum propagator has a compact analytical expression. Moreover, the harmonic oscillator allows one to gain intuition for the dynamics of multi-dimensional systems.

This work studies, both experimentally and theoretically, the quench dynamics of an anisotropic threedimensional harmonic oscillator in which the three degrees of freedom are coupled by a point scatterer of varying strength that is located at the origin. Since the point scatterer has a measure of zero, the classical trajectories are not influenced by the point scatterer [\[2\]](#page-5-2). However, the situation changes drastically when one enters the quantum regime since the point scatterer can simultaneously partially reflect and partially transmit the wave packet, or even reflect the wave packet in its entirety [\[2–](#page-5-2)[15\]](#page-5-3).

The quench dynamics of one-dimensional quantum systems has been investigated extensively at the microscopic level [\[9–](#page-5-4)[12,](#page-5-5) [14–](#page-5-6)[24\]](#page-6-0). Examples include the realization of quantum Newton's cradle [\[16,](#page-5-7) [20,](#page-6-1) [21\]](#page-6-2) and the observation of quantum revivals in a system containing around 1000 atoms, addressing questions related to equilibration, thermalization, and their connections to integrability of one-dimensional systems [\[22\]](#page-6-3). The quench dynamics of three-dimensional systems is expected to differ from that of one-dimensional systems in important ways. This letter explores these differences by studying the quench dynamics of an anisotropic harmonic oscillator, including the weakly-attractive and repulsive regimes, where the system behavior is quite intuitive, and the stronglyinteracting regime, where the s-wave scattering length is the largest length scale in the problem and intuition tends to fail.

We realize the three-dimensional anisotropic harmonic

oscillator with point scatterer experimentally by optically trapping two ultracold atoms [Fig.  $1(a)$ ], which interact via a short-range van der Waals potential with tunable scattering length. The dynamics are initiated by a quench of the trap geometry. The system provides a versatile platform for studying few-body dynamics in a regime where a small and predictable number of collisions occur. Since the optical trap is nearly perfectly harmonic, the center-of-mass motion, which is not affected by the interactions, decouples from the relative motion. Thus, we focus on (i) the dynamics in the relative degrees of freedom and (ii) the impact of this motion on the single-particle density. Excellent agreement with our parameter-free theory predictions is found. In the strongly-interacting regime, the resulting density profiles in the relative, low-energy z-coordinate display timedependent oscillatory or fringe pattern, which we interpret as signatures of cross-dimensional dynamics. The single-particle density profiles, in contrast, are smooth except for very short time periods during which the two particles are close to each other. This illustrates that the scattering events impact the single- and higher-order correlation functions differently. While the relative density varies appreciably with time, our calculations reveal an extremely slow approach to equilibrium, manifest in a failure to thermalize over thousands of cycles.

Experimentally, we prepare two  ${}^{6}$ Li atoms in two distinct hyperfine states denoted by  $|1\rangle$  and  $|3\rangle$  [\[25\]](#page-6-4) in the motional ground state of a tightly focussed optical tweezer trap elongated along the x-direction [Fig. [1\(](#page-2-0)ai)] [\[26\]](#page-6-5). At time  $t = 0$ , the system is quenched by instantaneously changing the trap geometry and aspect ratio [Fig. [1\(](#page-2-0)aii)]. We release the atoms into a much weaker dipole trap with inverted geometry, whose weakest frequency is along the z-axis. Since the trap potentials are harmonic to a good approximation, the system before and after the quench is described in terms of the low-energy two-particle Hamiltonian  $H_{\alpha} = H_{rel,\alpha} +$ 



<span id="page-2-0"></span>FIG. 1: (color online) Schematic of system and characterization of initial state. (ai) We prepare the wavepacket of two interacting <sup>6</sup>Li atoms (blue cloud) in the ground state of a tight optical tweezer trap (dark red). We additionaly ramp on a weak crossed beam optical dipole trap (light red, not to scale). (aii) At  $t = 0$  we switch off the tweezer trap to quench the trap geometry so that the cloud quickly oscillates in the  $x$ - and  $y$ -directions while slowly expanding in the  $z$ -direction. (aiii) After a variable expansion time we record the z-positions of both atoms via fluorescence imaging. (b) Schematic energy spectrum of the Hamiltonian  $H_{rel,f}$  after the quench, showing a single "molecular branch" and a nearly harmonic spectrum of "scattering" states". Different magnetic fields lead to very different projections of the initial state onto the eigenstates of  $H_{rel,f}$ . In the weakly-interacting regime at large magnetic fields, many scattering states are populated with roughly equal probability. At lower magnetic fields, the projection onto the single molecular state increases, eventually dominating the two-body dynamics. For the experimental parameters, hundreds of levels contribute to the dynamics.

 $H_{\text{cm},\alpha}$ , where  $\alpha = i, f$  denotes the geometry before (i) and after  $(f)$  the quench,

$$
H_{\text{rel},\alpha} = \frac{-\hbar^2}{2\mu} \vec{\nabla}_{\vec{r}}^2 + \frac{\mu}{2} (\omega_{x,\alpha}^2 x^2 + \omega_{y,\alpha}^2 y^2 + \omega_{z,\alpha}^2 z^2) + \frac{2\pi\hbar^2 a_s}{\mu} \delta(\vec{r}) \frac{\partial}{\partial r} r
$$
 (1)

and

$$
H_{\mathrm{cm},\alpha}=\frac{-\hbar^2}{2M}\vec{\nabla}_{\vec{R}}^2+\frac{M}{2}(\omega_{x,\alpha}^2X^2+\omega_{y,\alpha}^2Y^2+\omega_{z,\alpha}^2Z^2). \eqno{(2)}
$$

Here,  $\vec{r} = (x, y, z)^T$  and  $\vec{R} = (X, Y, Z)^T$  are the relative and center-of-mass position vectors, respectively,  $\mu$  and  $M$  the associated masses, and  $a_s$  the three-dimensional s-wave scattering length characterizing the interaction strength.

The experimentally measured trapping frequencies are  $\omega_{x,i} : \omega_{y,i} : \omega_{z,i} = 2\pi \times (6.4 : 31 : 30)$  kHz before and  $\omega_{x,f} : \omega_{y,f} : \omega_{z,f} = 2\pi \times (640 : 600 : 61.7)$  Hz (aspect ratio of  $\omega_{x/y,f}/\omega_{z,f} \approx 10$ ) after the quench. For all theoretical studies presented, to simplify the calculations, we assume that the initial and final traps are axially symmetric (but about different axis). Specifically, our calculations use  $\omega_{y,i} = \omega_{z,i}, \, \omega_{x,i}/\omega_{z,i} = 0.2098, \, \omega_{z,i}/\omega_{z,f} = 494.3, \, \omega_{x,f} =$  $\omega_{y,f}$ , and  $\omega_{x,f}/\omega_{z,f} = 10$ .

We record the spatial correlations along the z-axis [Fig. [1\(](#page-2-0)aiii)] that develop during the wave packet dynamics using a single-atom and state resolved imaging scheme [\[27\]](#page-6-6). Furthermore, we control the interaction strength by adiabatically adjusting a magnetic offset field in the vicinity of a broad Feshbach resonance located at around 690 G [\[25\]](#page-6-4). This allows us to reach three distinct regimes via the quench, which are set by the role of a bound state in the system  $[Fig. 1(b)]$  $[Fig. 1(b)]$  $[Fig. 1(b)]$ : In the case of a small negative  $a_s$  [in units of the harmonic oscillator length  $a_{\text{ho},z} = \sqrt{\hbar/(\mu \omega_{z,f})}$ , the system is in the

weakly-attractive regime where the quench projects onto a large number of nearly free particle eigenstates. For  $a_s/a_{\text{ho},z}$  = -0.0203, the occupation  $|c_0|^2$  of the lowest eigenstate of  $H_{rel,f}$  immediately after the quench is about 2% (Table S1 [\[28\]](#page-6-7)). For small positive  $a_s$  (e.g.  $a_s/a_{\text{ho},z} = 0.0474$ , in contrast, the particles are deeply bound into a single molecular state both before and after the quench. In this work, we are particularly interested in the paradigmatic "unitary" regime [\[38,](#page-6-8) [39\]](#page-6-9), where the three-dimensional scattering length is the largest length scale in the system or even diverges. In this regime  $(a_s/a_{\text{ho},z} = -4.64), |c_0|^2$  is of order 0.3 (Table S1 [\[28\]](#page-6-7)).

Since the quench does not couple the relative and center-of-mass motions, the center-of-mass wave packet for  $t > 0$  simply performs breathing oscillations at the characteristic time scales  $T_{x/y/z}/2 = \pi/\omega_{x/y/z,f}$ . The relative motion, in contrast, is non-trivial. Since the energy  $\langle E_{\text{rel}} \rangle$  of the  $t > 0$  wave packet in the relative degrees of freedom is much larger than the energy scales set by the trapping frequencies of  $H_{rel,f}$  (Table S1 [\[28\]](#page-6-7)), the dynamics in the relative degrees of freedom involves many eigenstates of  $H_{rel,f}$ . To illustrate this, the circles in Fig. [1\(](#page-2-0)b) schematically show the occupation probabilities  $|c_j|^2$ , which are obtained by expanding the relative portion of the  $t < 0$  wave packet in terms of the eigenstates of  $H_{\text{rel},f}$  [\[40,](#page-6-10) [41\]](#page-6-11), for three different s-wave scattering lengths.

Figure [2](#page-3-0) summarizes the dynamics for  $a_s = -4.64a_{\text{ho},z}$ by displaying  $\langle \rho^2 \rangle$  and  $\langle z^2 \rangle$ , where  $\rho^2 = x^2 + y^2$ . Both observables oscillate smoothly with time but at different frequencies. The times marked by a circle, a square, and arrows are discussed in more detail in Figs. [3,](#page-3-1) [4,](#page-4-0) and [5,](#page-4-1) respectively.

Figure [3](#page-3-1) shows the relative density along the  $z$ coordinate for  $t = 3.5$  ms =  $2.16T_x$ , i.e., after four collisions (Fig. [2\)](#page-3-0), for six different s-wave scattering lengths.



<span id="page-3-0"></span>FIG. 2: (color online) Expectation values of  $\rho^2$  (solid line) and  $z^2$  (dashed line) for  $a_s = -4.64a_{\text{ho},z}$  as a function of time. The circle, square, and arrows mark the times that are considered in Figs. [3,](#page-3-1) [4,](#page-4-0) and [5,](#page-4-1) respectively.

The agreement between the experimental results (circles) and the parameter-free theory results (solid lines) is, except for Fig. [3\(](#page-3-1)f), very good. The theoretical results shown in Fig. [3](#page-3-1) are convolved with a Gaussian to account for the experimental resolution of 4  $\mu$ m = 0.542a<sub>ho.z</sub>. Interestingly, the relative densities shown in Figs. [3\(](#page-3-1)a)- [3\(](#page-3-1)e) contain oscillatory structure or fringes, which change notably with the s-wave scattering length  $a_s$ , on top of a broad background. The fringe pattern changes with time and we have found no unique way to assign  $t$ - and z-independent peak spacings for fixed scattering length. For the smallest positive s-wave scattering length considered [Fig. [3\(](#page-3-1)f)], the initial state is small compared to the harmonic oscillator lengths of the final and initial traps and the coefficient  $|c_0|^2$  is large (Table S1 [\[28\]](#page-6-7)). In this case, finite-range effects might need to be accounted for to obtain quantitative agreement between theory and experiment.

The relative densities, shown in Figs.  $3(a)-3(f)$  $3(a)-3(f)$  $3(a)-3(f)$ , reflect the evolution from a comparatively weakly-interacting regime, in which the molecular state does not play a special role (small  $|c_0|^2$ ), to the strongly-interacting regime, where  $|c_0|^2$  is appreciable but not dominant, to the small molecular bound-state regime, where  $|c_0|^2$  dominates. For large negative  $a_s$  we observe a clear fringe pattern. Since the wave packet in the relative coordinate would, in the absence of the scatterer, simply repeatedly expand and contract, the fringes have to be caused by scattering events. Figure [4](#page-4-0) shows the theoretically determined unconvolved relative density along z for  $a_s = -0.651a_{\text{ho},z}$  during the first scattering event, i.e., for t close to  $t = T_x/2$ . At this time,  $\langle \rho^2 \rangle$  is quite small but  $\langle z^2 \rangle$  is comparatively large. This implies that the majority of the wave packet is located away from the point scatterer. The snapshots in Fig. [4](#page-4-0) illustrate that the fringes emerge as a consequence of the scattering. A portion of the small- $\rho$  wave packet does not get reflected along the  $\rho$ -direction but instead gets "redirected" to leave the small- $\rho$  region along the *z*-direction [schematic in Fig.  $4(g)$ ]. One can think of the scattering



<span id="page-3-1"></span>FIG. 3: (color online) Experiment-theory comparison after four two-body collisions. The relative densities  $n_{rel}(z, t)$  are shown for  $t = 3.5$  ms and six different scattering lengths. The values of  $a_s$  (in units of  $a_{\text{ho},z}$ ) are reported in the upper left corner of each panel. Circles show experimental data and solid lines convolved theory results. Typical error bars are shown for a subset of the experimental data. Note the different yscales in the panels.

event as a cross-dimensional redistribution of flux from the  $\rho$ - to the *z*-direction, creating a newly emitted wave packet portion along the z-direction that subsequently interferes with the "background" wave packet portion. This process is repeated during subsequent scattering events  $(t \approx nT_x/2; n = 2, 3, 4, ...)$ , leading to an increasingly complex fringe pattern in the relative density along  $z$  (see also Fig. S3 [\[28\]](#page-6-7)).

Does the single-particle density, an observable recorded frequently in cold atom experiments, develop a fringe pattern? The answer is yes but only for very short time periods over a length scale that is too small to be observed with the current experimental set-up. Figure [5\(](#page-4-1)a) compares the experimental (diamonds) and convolved theoretical (solid line) single-particle densities along z for  $t = 2$ ms and  $a_s = -4.64a_{\text{ho},z}$ . At this time, which corresponds to two oscillations of  $\langle \rho^2 \rangle$  (Fig. [2\)](#page-3-0), the convolved single-particle density is smooth. It continues to be smooth for times  $t < 0.5T_z$  [red dashed and green dot-dashed lines in Fig. [5\(](#page-4-1)a)]. Since the size of the wave packet is much larger than the Gaussian convolution width  $\sigma$ ,  $\sigma = 0.542a_{\text{ho},z}$ , the convolved and unconvolved single-particle density are indistinguishable on the scale shown in Fig.  $5(a)$ . The behavior of the singleparticle density changes drastically when the wave packet is characterized by a small  $\langle z^2 \rangle$  and a small  $\langle \rho^2 \rangle$ . For  $t \approx 0.5T_z$ , the unconvolved single-particle density [solid



<span id="page-4-0"></span>FIG. 4: (color online) Development of fringe pattern in the relative density during the first collision event  $(t \approx T_x/2)$  for  $a_s = -0.651a_{\text{ho},z}$ . (a)-(d) The lines show the theoretically determined unconvolved relative density  $n_{rel}(z, t)$  for the times t reported in the upper left corner of each panel. (e)-(g) The red solid lines schematically show the wave packet at (e)  $t = 0^+$ , (f)  $t \approx T_x/4$ , and (g)  $t \approx T_x/2$ . The black dashed lines schematically show the equipotential lines of the final trap. The arrows schematically indicate the flux.

line in Fig. [5\(](#page-4-1)b)] exhibits a fringe pattern. The fringe pattern exists only for a short time period. For  $t = 0.501T_z$ (not shown), e.g., the oscillations are no longer visible. Additionally, the limited spatial resolution smoothes out the fringe pattern of the single-particle density such that it cannot be observed in the experiment [dotted line in Fig. [5\(](#page-4-1)b)]. The fringe pattern in the single-particle density keeps "appearing" and "disappearing" at larger times. Figure [6\(](#page-5-8)b) shows that the single-particle density displays intricate fine structure for  $t = 4T_z$  (corresponding to  $\langle z^2 \rangle \approx 0$ ). Figure [6\(](#page-5-8)d) shows that no fringe pattern exists in the single-particle density for  $t = 4.25T_z$  (corresponding to  $\langle z^2 \rangle \approx 70 a_{\text{ho},z}^2$ . Remarkably, the relative density along z is characterized by notable fine structure for both times [Figs.  $6(a)$  and  $6(c)$ ].

The discussion surrounding Figs. [3-](#page-3-1)[6\(](#page-5-8)d) illustrates that collisions impact the single-particle and relative densities differently. In particular, the relative density displays an increasingly large number of oscillations with increasing time while the single-particle density is smooth for all times, except for  $t \approx nT_z/2$ . Given the strong time dependence of the relative density, we ask whether the system, in the large time limit, approaches thermal equilibrium. The answer is, as is expected from Ref. [\[5\]](#page-5-9), that it does not. To gain insight into the long-time dynamics, we analyze cycle-averaged observables, i.e., observables averaged over a period of length  $T_z$  [from  $t = nT_z$  to  $t = (n + 1)T_z$ . Lines in Figs. [6\(](#page-5-8)e) and 6(f) show the unconvolved cycle-averaged relative and single-particle densities  $\langle n_{rel}(z,t) \rangle_{T_z}$  and  $\langle n(z_1,t) \rangle_{T_z}$ , respectively, for



<span id="page-4-1"></span>FIG. 5: (color online) Single-particle densities for  $a_s$  =  $-4.64a_{\text{ho},z}$ . (a) The black solid line and purple diamonds with error bars show the convolved theoretical data and experimental results for  $t = 1.234T_x = 2$ ms; the red dashed and green dot-dashed lines show the convolved theoretical data for  $t = 3T_x$  and  $t = 4.5T_x$ , respectively. (b) The blue solid and red dotted lines show, respectively, the unconvolved and convolved theoretical data for  $t = 5T_x = 0.5T_z$ .

 $a_s = -4.64a_{\text{ho},z}$  and  $n = 1, 10^2$ , and  $10^4$ . In both panels, the three curves are indistinguishable on the scale shown. Thus, despite the intricate dynamics within each cycle, the cycle-averaged observables display essentially no dynamics. The reason for this is that the normalized nearest neighbor energy spacings are rather sharply peaked around 1 (Fig. S1 [\[28\]](#page-6-7)). We emphasize that this behavior is also observed for other scattering lengths. The close to frozen cycle-averaged relative density for large  $n$  indicates a lack of thermalization. Indeed, the thermal relative density [green solid line in Fig. [6\(](#page-5-8)e)] differs visibly from the calculated cycle-averaged relative densities.

In summary, we have presented a joint theoreticalexperimental study that investigated the wave packet dynamics of two ultracold atoms following a "violent" trap quench, which leads to the occupation of many eigenstates of the post-quench Hamiltonian. Following the quench, two-body collisions, through their effect on the structural observables, were observed. The excellent agreement between the experimental and theoretical data together with the time-resolved single-atom detection with high spatial resolution makes the system a promising candidate for future dynamical studies, which are aimed at addressing questions related to thermalization, state engineering, chaos, and integrability. The set-up also promises to be a fertile playground for testing hydro-dynamical formulations [\[42–](#page-6-12)[44\]](#page-6-13), which can potentially be used to simulate the dynamics of few- and many-body systems. Quantitative tests of the hydrodynamics theory with ultracold atoms may yield insights into why the dynamics of quark gluon plasmas seems, somewhat surprisingly, to be governed by hydrodynamic equations [\[45,](#page-6-14) [46\]](#page-6-15).



<span id="page-5-8"></span>FIG. 6: (color online) Unconvolved theory results for the longtime regime for  $a_s = -4.64a_{\text{ho},z}$ . The left and right columns show the relative and single-particle densities, respectively. Panels (a) and (b) show snapshots for  $t = 4T_z$  while panels (c) and (d) show snapshots for  $t = 4.25T_z$ . Lines in panels (e) and (f) show cycle-averaged observables for  $n = 1, 10^2,$  and  $10^4$ . On the scale shown, the lines are essentially indistinguishable. For comparison, the green solid line in panel (e) shows the relative thermal density.

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