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Andrea Addazi, Antonino Marcianò, and Nicolás Yunes

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Can we probe Planckian corrections at the horizon scale with gravitational waves?

Andrea Addazi^{1*} and Antonino Marciano^{1†}

¹ *Center for Field Theory and Particle Physics & Department of Physics, Fudan University, 200433 Shanghai, China*

Nicolás Yunes^{2‡}

*eXtreme Gravity Institute, Department of Physics,
Montana State University, Bozeman, MT 59717, USA*

Future detectors could be used as a *gravitational microscope* to probe the horizon structure of merging black holes with gravitational waves. But can this microscope probe the quantum regime? We study this interesting question and find that (i) the error in the distance resolution is exponentially sensitive to errors in the Love number, (ii) the uncertainty principle of quantum gravity forces a fundamental resolution limit, and (iii) conclusions about the structure of spacetime at small distances rely on assumptions about the properties of the (unknown) compact objects considered.

Introduction. The recent discovery of gravitational waves (GWs) [1–5] has raised some new and interesting ideas in fundamental black hole (BH) physics. From the possibility to observe parity violation in gravity inspired by quantum gravity [6–8], to measuring corrections to the dispersion relation [9], GWs are becoming an important probe of fundamental physics [10]. One of the ultimate fundamental questions one would like to answer relates to the full theory of quantum gravity. What is the best framework for unification? How are the Einstein equations corrected at the Planck scale? It is natural then to ask whether GWs could inform us about these questions [11, 12], as future detectors become more sensitive.

In a set of pioneering studies, it was recently shown that future observations of GWs could be used to distinguish between BHs and other exotic compact objects (ECOs), i.e. BH mimickers that do not possess a horizon [13, 21–23]. The main idea is that as compact objects coalesce, the tidal Love number imprints on the GWs emitted. Therefore, given a sufficiently sensitive observation, one could extract this tidal Love number and determine whether it is compatible with that of a BH with zero Love number or that of a horizonless object with non-zero Love number. One can thus think of GWs acting as a *gravitational microscope* of near horizon physics.

We here build on this work and ask whether the observation of the tidal Love number of an ECO could reveal information about the structure of spacetime with Planckian resolution. That is, we wish to determine whether the gravitational microscope can achieve Planckian resolution of near horizon physics. In particular, we explore in detail two potential limitations. First, we consider how the statistical error in the measurement of the tidal Love number propagates into error on the extraction of near horizon physics. Second, we study whether or not the uncertainty principle of quantum gravity can be evaded with the gravitational microscope.

Before proceeding, let us stress that the work we do here is completely unrelated to recent work on gravita-

tional waves echoes [14, 15, 17–20]. The latter involve the ringdown phase of the BH merger (not the inspiral), and moreover, they do not *directly* probe Planckian distances close to the ECO’s surface, but rather Planckian effects become amplified through reflection.

The Love number and the gravitational microscope. Consider a binary with masses m_1 and m_2 in the inspiral phase. This system can be modeled in post-Newtonian (PN) theory [24], a weak-field/slow-velocity expansion of the field equations, provided the two objects are sufficiently far from each other, so that non-linear relativistic corrections can be treated perturbatively. In this regime, one can safely model the response of an interferometer to an impinging GW in the frequency domain as

$$\tilde{h}(f) = \mathcal{A}(f)e^{i\psi_1(f)+i\psi_2(f)+i\psi_3(f)}, \quad (1)$$

where f is the GW frequency, $\mathcal{A}(f)$ is the GW Fourier amplitude, $\psi_1(f)$ is the contribution to the GW Fourier phase when treating the objects as spinning test particles, $\psi_2(f)$ is the contribution due to tidal heating, and $\psi_3(f)$ is the contribution due to their tidal deformability.

Let us focus on this last contribution. To leading PN order, one can show that this contribution is

$$\psi_3(f) = -\psi_N \frac{\Lambda}{6m^5} v^{10} \frac{(1+q)^2}{q}, \quad (2)$$

where $v = (\pi m f)^{1/3}$ is the velocity, with $m = m_1 + m_2$ the total mass, $\psi_N = (3/128)\eta^{-1}v^{-5}$ is the leading part of ψ_1 , with $\eta = m_1 m_2 / m^2$ the symmetric mass ratio, and

$$\Lambda = (1 + 12/q)m_1^5 k_1 + (1 + 12q)m_2^5 k_2, \quad (3)$$

with $k_{1,2}$ the ($\ell = 2$, electric-type) tidal Love numbers and $q = m_1/m_2$ the mass ratio. For two compact objects of the same type and the same mass, then $\Lambda = 26M^5 k$ where $k_1 \equiv k \equiv k_2$ and $m_1 \equiv M \equiv m_2$.

The Love number depends on the internal structure of the compact object. For the BHs of GR, the Love number vanishes [25, 26], although this does not mean the horizon does not deform [27, 28]. Compact objects that

are not BHs, however, do have a non-zero Love number. Neutron stars, for example, have Love numbers of $\mathcal{O}(10^2)$ depending on their equation of state [29–32], while the boson stars so far constructed [33–36] have Love numbers of $\mathcal{O}(10)$ [22]. ECOs, on the other hand, have Love numbers that can scale as $1/|\log(\delta)|$, where $\delta = r_0 - r_H$, with r_0 the location of the ECO’s surface and r_H the location of the horizon if the ECO had been a BH of mass M .

Given this, can the Love number be measured accurately enough to distinguish between a BH coalescence (which would have $\Lambda = 0$) from an ECO coalescence (which would have $\Lambda \neq 0$ but possibly small) [13, 21–23]? A Fisher analysis assuming GW detections of comparable mass binaries by LISA [37] suggests that this is possible. More specifically, an ECO coalescence with Love numbers of $\mathcal{O}(10^{-2})$ could be measured with a statistical accuracy of 10%–50% [13] using highly-spinning “golden binaries,” i.e. the cleanest and loudest signals observed. This analysis employed multiple approximations, but they should be well-justified for golden binaries. Therefore, one concludes that GWs can be used as a gravitational microscope to distinguish between BHs and ECOs, provided the latter have a sufficiently large Love number.

Resolving near horizon structure. Given a GW measurement of the Love number of an ECO, can one infer additional near horizon physics? Since the ECO Love number $k \propto 1/|\log(\delta)|$, can δ be inferred given a measurement of k ? Inverting the k - δ relation, one finds that

$$\delta = r_0 - r_H = r_H e^{-1/k}, \quad (4)$$

which then suggests that a measurement of k and of the mass M of the ECO, which determines r_H via $r_H = 2GM$ if the ECO is not spinning, yields a measurement of δ . For most of the remainder of this note, we assume Eq. (4) is valid, but this assumption is not obvious and we will return to it in the discussion section.

Let us pause for a second to scrutinize the conclusion above. The quantity δ as defined above is coordinate dependent, and thus, it is not clear whether it is observable. One possibility is to declare that δ is indeed a physical quantity related to some fundamental scale in the quantum gravity theory that leads to Eq. (4). A perhaps better possibility is to think of this quantity as a proxy for an invariant measure of length, such as one constructed from a curvature invariant. For example, if one uses the Kretschmann invariant, one can construct the curvature measure $\mathcal{R} = (R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})^{-1/6}$, which then yields

$$\frac{\mathcal{R}_0}{\mathcal{R}_{BH}} \approx \left(\frac{M^2}{r_0^6}\right)^{-1/6} (M^4)^{-1/6} = \frac{r_0}{M}, \quad (5)$$

in a specific coordinate system where r_0 is the ECO surface and M is its mass. This idea is appealing because if the quantity δ determines the quantity k , the latter of which is observable through its imprint on GWs, then δ ought to be describable in terms of invariant quantities.

Given the ECO mapping between k and δ in Eq. (4), how small a value of δ can be inferred from a measurement of k ? As mentioned earlier, LISA has been projected to measure $k \sim 10^{-2}$, given a supermassive BH binary signal [13]. Equation (4) then implies one can infer near-horizon physics to lengths of $\mathcal{O}(Me^{-100})$, which for supermassive ECO coalescences yields lengths of $\mathcal{O}(10^{-35})$ meters for $M = 10^6 M_\odot$. Such a resolution is of $\mathcal{O}(\ell_{\text{Pl}})$, where ℓ_{Pl} is the Planck length, near the ECO surface. Pushing this further, similar observations with lower mass binaries or higher signal-to-noise ratio, using e.g. U-DECIGO [38], BBO [39], Tian-Qing [40] or TAIJI [41], would probe sub-Planckian distances.

Statistical uncertainty in measurements of near horizon structure. We now consider the accuracy to which the length difference δ can be measured, given a measurement of the Love number k . The best-fit value of k and of the ECO masses possess statistical uncertainty. The latter can be estimated from the diagonal components of the variance-covariance matrix Σ^{ab} , where the superscripts run over the model parameters $\lambda^a = (M, k)$. The Σ^{ab} matrix can be estimated through a Fisher analysis, as we explain in the appendix, or alternatively obtained from a Markov-Chain Monte-Carlo exploration of the likelihood probability distribution in a Bayesian analysis.

Given the variance-covariance matrix, simple error propagation can be used to find the statistical uncertainty in the inferred parameter δ , namely

$$\sigma_\delta^{\text{stat}} = \sqrt{\left(\frac{d\delta}{d\lambda^a}\right) \left(\frac{d\delta}{d\lambda^b}\right) \Sigma^{ab}}. \quad (6)$$

Using the k - δ mapping for ECOs in Eq. (4), this yields

$$\frac{\sigma_\delta^{\text{stat}}}{\hat{\delta}} = \sqrt{\left(\frac{\sigma_M^{\text{stat}}}{\hat{M}}\right)^2 + \frac{1}{\hat{k}^2} \left(\frac{\sigma_k^{\text{stat}}}{\hat{k}}\right)^2}, \quad (7)$$

where $\hat{\lambda}^a = (\hat{M}, \hat{k})$ are the best-fit values of the parameters, and we have neglected covariances between the mass and the Love number. We then see clearly that for small measurements of k , the statistical uncertainty in δ scales as $\sigma_\delta^{\text{stat}} \propto 1/\hat{k}$. The value of the measured Love number at which the statistical uncertainty equals the inferred values of $\hat{\delta}$ is $\hat{k} \approx 0.2$ for the expected statistical accuracy in the estimation of M and k . Thus, any inferred value of $\hat{\delta}$ derived from $\hat{k} < 0.2$ will be dominated by statistical uncertainty because the uncertainty in $\hat{\delta}$ is exponentially affected by the uncertainty in \hat{k} .

Systematic uncertainty in measurements of near horizon structure. Even if one could distinguish the ECO radius from the corresponding Schwarzschild radius with Planckian precision, one would be in the resolution limit $\delta \rightarrow \ell_{\text{Pl}}$. As a consequence of the quantum uncertainty principle, this corresponds to a momentum resolution

$$\Delta p > \hbar \ell_{\text{Pl}}^{-1} = \hbar \sqrt{c^3/G\hbar}, \quad (8)$$

which leads to an uncertainty in the four-momenta of the inspiraling bodies, including both their rest mass and their inspiral velocities. This, in turn, implies a fundamental quantum uncertainty in the interaction or binding energy of the two objects, and thus, in the acceleration of the bodies and the GWs they emit. Considering the astrophysical masses and the relativistic velocities that are involved, this uncertainty might seem completely irrelevant to the dynamics of the merging processes. Nonetheless, when Planckian precision is sought, this uncertainty becomes extremely important, as we show below.

Let us pause again to scrutinize the above argument. Lacking a complete quantum gravity theory, one may argue that perhaps the uncertainty principle should not apply here. Spacetime, however, is defined by a manifold, which by definition reduces to flat spacetime in a small neighborhood about any point (this is in particular justified by our working assumption of following the framework of [13]). The 2-sphere that defines the location of the ECO surface is not special, and curvature effects are relatively weak on it for supermassive objects, as the curvature scales inversely with the mass. Therefore, one can choose any point on or near this 2-sphere and consider a small neighborhood about it larger than the Planck length, in which spacetime will look flat. In this neighborhood, quantum principles, like the uncertainty principle, should continue to hold.

The percolation of quantum uncertainty into GWs implies a fundamental limitation in the accuracy to which any GW model parameter can be extracted because the signal becomes quantum fuzzy. For example, in the $\delta \rightarrow \ell_{\text{Pl}}$ limit, the uncertainty in the mass $\Delta M \rightarrow \sqrt{\hbar c/G}$, which corresponds to the Planck scale. This, in turn, percolates into the gravitational interaction, since the uncertainty in the gravitational binding energy $\Delta E_b \rightarrow -G\eta m\Delta M/r$ at leading PN order. But the binding energy affects the rate at which the orbital and the GW frequency changes via the balance law

$$\frac{dF}{dt} = \left(\frac{dE_b}{dt}\right) \left(\frac{dE_b}{dF}\right)^{-1} = - \left(\frac{dE_{\text{GW}}}{dt}\right) \left(\frac{dE_b}{dF}\right)^{-1}, \quad (9)$$

where F is the orbital frequency and dE_{GW}/dt is the rate at which energy is removed from the system by GW emission. Therefore, quantum uncertainty in the binding energy translates into a quantum uncertainty in the orbital frequency, which then percolates into an uncertainty in the GW frequency and its phase of the signal itself, preventing measurements beyond the Planck scale.

What is the fundamental limitation that quantum uncertainty in the signal places on the accuracy to which model parameters can be fitted? Quantum fluctuations in the signal of $\mathcal{O}(\ell_{\text{Pl}})$ blur or fuzz out its amplitude and phase, and so when one fits this quantum fuzzy signal to waveform templates, the accuracy to which model parameter can be estimated will be limited by a systematic

uncertainty of the same size as the quantum jitter itself. The total uncertainty in the extraction of any parameter in a waveform model is then the sum of the statistical error σ_{stat} (described in Eq. (7)) and a systematic error $\sigma_{\text{sys}} = \mathcal{O}(\ell_{\text{Pl}})$ in quadrature, leading to

$$\frac{\sigma_{\hat{\delta}}^{\text{tot}}}{\hat{\delta}} = \sqrt{\left(\frac{\sigma_{\hat{M}}^{\text{stat}}}{\hat{M}}\right)^2 + \frac{1}{\hat{k}^2} \left(\frac{\sigma_{\hat{k}}^{\text{stat}}}{\hat{k}}\right)^2 + \frac{a^2 \ell_{\text{Pl}}^2}{\hat{\delta}^2}}, \quad (10)$$

where we have set $\sigma_{\text{sys}} = a \ell_{\text{Pl}}$ for $a \in \mathfrak{R}$ and of $\mathcal{O}(1)$. The a -dependent term should not be thought of as an ansatz, but rather as a parametrization of different theoretical possibilities. The most conservative one relies on Eq. (8), and just assumes that for Planckian masses the uncertainty on the distances are Planckian, i.e. $a = 1$. Nonetheless, other scenarios can be envisaged in quantum gravity for which $a \sim 10^2 \div 10^3$ – see e.g. the case of loop quantum gravity [42] – or in string theory, for which the relation between the string scale M_s and the Planck scale M_{Pl} , namely $M_s = gM_{\text{Pl}}$ with g denoting the strength of the string-string interaction [43], is unconstrained by theoretical arguments. Regardless of these details, quantum uncertainty forces a floor for the uncertainty in the measurement of δ , as we can see in Fig. 1. Observe that the total 1σ error is $\mathcal{O}(10^2)$ times larger than the inferred value, saturating at the quantum uncertainty floor at high spins. This saturation would occur at lower spin values if we had chosen a smaller variance for the \hat{k} measurement.

Discussion. We have investigated the resolving power of the gravitational microscope to use a GW measurement of the Love number k to infer near horizon physics a distance δ from an ECO surface. For future observations with LISA, we have found that the resolution in δ is limited by statistical error when $\hat{k} < 0.2$. In particular, the statistical error is only controlled if the statistical uncertainty in the Love number is less than the squared of the inferred Love number, as shown in Eq. (7). For a measurement of the Love number of $\hat{k} = 10^{-2}$, this implies a fractional accuracy better than $\sigma_k^{\text{stat}}/\hat{k} = 1\%$, which would require signal-to-noise ratios above 10^4 .

The above statistical considerations neglect the effect of systematics in the modeling of the waveform itself. All models used to date assume compact binaries in isolation (i.e. in vacuum), but supermassive compact object binaries may have a circumbinary accretion disk, or they may be perturbed by a third body. The presence of such effects could impact the GW signal [49–52] and, the use of vacuum waveforms to fit this signal could incorrectly lead to non-zero measurements of Love numbers, which could be in turn incorrectly associated with ECOs. Mismodeling error [53] is a form of systematic uncertainty that becomes more severe for high signal-to-noise ratio events and must thus be included in the total error budget, which could further limit inferences made about ECOs from Love number measurements.

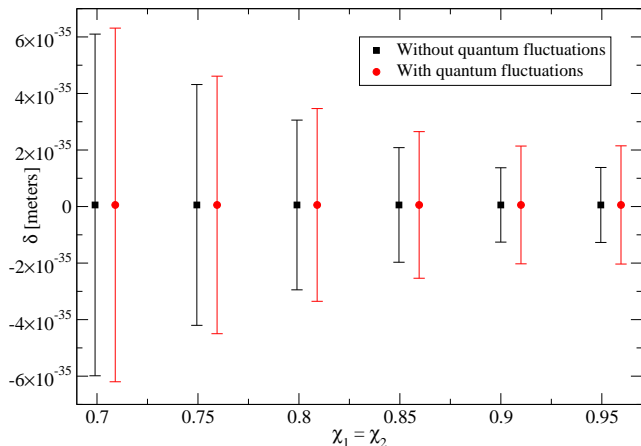


FIG. 1. (Color Online). Inferred value of δ and 1σ errors with (black, Eq. (7)) and without (red, Eq. (10)) quantum fluctuations. We have here assumed a GW observation of a compact ECO inspiral with $m_1 = 1.1 \times 10^6 M_\odot$ and $m_2 = 10^6 M_\odot$, dimensionless spin $\chi_{1,2}$, Love numbers $k_1 = k_2 = 0.02$ at a distance of 2Gpc. For the uncertainties in M and k , we used $\sigma_M^{\text{stat}}/\hat{M} = 10^{-5}$ and $\sigma_k^{\text{stat}}/\hat{k} \in (0.2, 1)$ (see the appendix), while for the quantum fluctuations we set $a = 1$. Observe that the error bars on the inferred value of δ are much larger than the measurement itself.

Putting mismodeling systematics aside, we have also found that the resolving power of the gravitational microscope will also be limited by systematics in the signal due to quantum fluctuations. If the uncertainty principle of quantum mechanics is valid near the horizon of ECOs, then quantum fluctuations in the four-momenta of the objects will percolate into a systematic uncertainty in the amplitude and phase of the GW signal. We have estimated this uncertainty to be of order the Planck scale, but in principle it could be larger, for example of order the string length, since typically the hierarchy between these scales is governed by the compactification volume and the string coupling. A better understanding of quantum gravity, for example through the completion of quantum gravity theories and the numerical study of the coalescence of quantum gravity compact objects, could aid in quantifying more precisely the impact of quantum fluctuations in GW observables.

But if quantum fluctuations are truly present in the gravitational measurement of distances at the Planck scale, then sub-Planckian measurements ought to be impossible. From the effective quantum gravity framework, at such scales quantum fluctuations become uncontrollable and one loses the very concept of spacetime continuity with the emergence of spacetime foaminess. From the quantum field theory perspective, this is re-

lated to the non-renormalizability of the theory, and at sub-Planckian scales one expects the emergence of different virtual spacetime topologies — for example virtual BH pairs that create and annihilate. Because of this, the very notion of a classical BH horizon as a Cauchy surface loses meaning at the Planck scale.

Unfortunately, the current status of quantum gravity models prevents us from going any further in this line of questioning. Without a full model, even the construction of an isolated compact object with quantum gravity corrections is missing. In this paper, we have studied the possibility of using the ECO relation between k and δ in Eq. (4) to see if a measurement of the former allows for microscopic measurements of the latter, but it is unclear whether this relation persists in quantum gravitational compact objects. The relation has only been shown to hold for wormholes [44] and gravastars [45, 46], which as [10] put it are both examples of cut-and-paste metrics: wormholes are Schwarzschild metrics glued together at a finite radius, while gravastars are an exterior Schwarzschild metric glued to an interior de Sitter metric. To our knowledge, neither of these classical metrics arises as a solution to a quantum gravity model, they have not been shown to arise generically from gravitational collapse, and even if they did, they would be at least unstable when spin is included, unless the ECO's surface is somehow sufficiently absorbing [47, 48].

These observations then suggest the question of whether the exponential relation between k and δ is also realized in other spacetimes for compact objects with quantum-gravity inspired modifications. Several insightful attempts have been made to construct such objects, e.g. fuzzball string condensates [54–56], graviton condensates [57–59], or string holes [60]. Alternative formulations of non-perturbative quantum gravity BHs also exist [61–63], as well as BH solutions in effective field theory expansions of heterotic string theory, as in Einstein-dilaton-Gauss-Bonnet gravity [64–66] and dynamical Chern-Simons gravity [67, 68] (see appendix). None of these is perfect, and in fact, they all have model-specific problems. But what they do have in common is that the k - δ mapping in Eq. (4) either does not hold (because the Love number vanishes [64–68]), or is not expected to hold at Planck scales. It is thus unclear how the subset of ECOs for which Eq. (4) holds is connected to compact objects with quantum gravity modifications.

This discussion then brings us back to the *generality* of the k - δ mapping in Eq. (4) and to the third finding mentioned in the abstract. Even if the quantum uncertainty and the statistical issues were not present, the ability to agnostically probe Planckian distances with a measurement of the Love number depends strongly on the validity of this mapping. As explained above, the mapping is not general since there exist counter-examples: some BHs in modified gravity have zero Love number (and thus no k - δ mapping) [64–68], and some ECOs, like the boson stars

so-far constructed, have a different k - δ mapping [22]. The k - δ mapping in Eq. (4) then seems to hold for a very special subset of compact objects, which introduce deviations from BH spacetimes that are not necessarily mediated by curvature corrections, but are rather generated through cut-and-paste procedures. Since the k - δ mapping in Eq. (4) is only valid for such specific models, the assumption that it is also valid for generic compact objects that arise in quantum gravity is very strong and not necessarily well-justified.

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* andrea.addazi@qq.com

† marciano@fudan.edu.cn

‡ nyunes@physics.montana.edu

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