



CHORUS

This is the accepted manuscript made available via CHORUS. The article has been published as:

Photonic Topological Insulating Phase Induced Solely by Gain and Loss

Kenta Takata and Masaya Notomi

Phys. Rev. Lett. **121**, 213902 — Published 20 November 2018

DOI: [10.1103/PhysRevLett.121.213902](https://doi.org/10.1103/PhysRevLett.121.213902)

Photonic Topological Insulating Phase Induced Solely by Gain and Loss

Kenta Takata^{1,2} and Masaya Notomi^{1,2,3}

¹*NTT Nanophotonics Center, NTT Corporation,
3-1 Morinosato-Wakamiya, Atsugi 243-0198, Kanagawa, Japan*

²*NTT Basic Research Laboratories, NTT Corporation,
3-1 Morinosato-Wakamiya, Atsugi 243-0198, Kanagawa, Japan*

³*Department of Physics, Tokyo Institute of Technology,
H-55, Ookayama 2-12-1, Meguro 152-8550, Japan*

(Dated: October 11, 2018)

We reveal a one-dimensional topological insulating phase induced solely by gain and loss control in non-Hermitian optical lattices. The system comprises units of four uniformly coupled cavities, where successive two have loss, the others experience gain and they are balanced under two magnitudes. The gain and loss parts are effectively *dimerized*, and a bulk bandgap, topological transition, midgap topological edge and interface states in finite systems can all be achieved by controlled pumping. We also clarify non-Hermitian topological invariants and edge states in gapless conditions.

Controlling optical properties with external signals is a major destination in photonics research [1], and it is largely associated with tailoring the refractive index. Recent studies have revealed that the imaginary part of the refractive index, namely gain and loss, can do much more than just tuning the optical intensity. The key concept, parity-time (\mathcal{PT}) symmetry [2, 3], was introduced for obtaining real spectra of quantum systems with non-Hermitian components. Its analogy in optics [4, 5] corresponds to complex refractive index profiles with symmetric real parts and antisymmetric imaginary parts, i.e. $n(\mathbf{r}) = n^*(-\mathbf{r})$. Such a system can show an exceptional point (EP) [6] where its eigen-detuning sharply changes from real to imaginary values [7, 8] (spontaneous \mathcal{PT} symmetry breaking). There are many interesting phenomena related to \mathcal{PT} symmetry including power oscillation [4, 9], double refraction [4, 10], Bloch oscillation [11, 12], mode-locking [13], coherent absorption [14–16], fast light [10, 17, 18], and unidirectional reflectivity [19–21]. Furthermore, nonlinearity-induced isolation [22, 23], single-mode lasing [24, 25], and beam steering [26] were achieved under controlled pumping.

To widen the scope of non-Hermitian optics [27, 28], there are growing attempts to incorporate topological features to photonic systems with gain and loss. While Hermitian photonic topological phases [29–31] are based on celebrated discoveries such as the quantum Hall effect [32, 33] and topological insulators [34, 35], non-Hermitian topological optics originates from the theoretical question as to whether or not stable topological quantum states exist in non-Hermitian systems [36–40]. For photonics based on classical electromagnetic waves, however, it has been clarified that there exist topological states even when their eigenvalues are not real [41]. Researchers applied Su-Schrieffer-Heeger (SSH) photonic lattices [42] with relevant loss and experimentally confirmed their topological interface states [43] and topological transition [44]. Moreover, a photonic topological bound state with global \mathcal{PT} symmetry was observed [45]. Even the

lasing of such topological modes has recently been shown to be feasible [46–49].

Then, another question may arise. *Can we create a topological insulating phase solely from gain and loss control?* In previous studies of non-Hermitian optics, the emergence of nontrivial topologies was attributed to Hermitian factors, namely the magneto-optic effect [49], and lattice and coupling profiles of host systems [37, 50–53]. Even though such systems are armed with non-Hermiticity, they take over original Hermitian topological characters predetermined by fabrication. Moreover, gain and loss in conventional non-Hermitian systems [27, 28, 54] only close frequency bandgaps. Thus, they were considered to destroy topological insulating phases. In contrast, our aim is to generate a topological bandgap solely by adding static gain and loss to a topologically trivial structure. Then, achieved topological features, including a well-defined topological number, should originate purely from non-Hermitian factors. Here, the gain and loss are readily tunable by multi-channel current injection or properly masked optical pumping in various laser systems [18, 26, 46–48]. We will hence have full manipulability over the topological properties in optical circuits, such as a topological transition, and the number and position of topological states, simply by changing the gain and loss.

Here, we show theoretically a one-dimensional photonic lattice with the gain- and loss-induced reconfigurable topological insulating phase. We consider unit cells of four uniformly coupled resonators, with loss introduced into two successive cavities and gain introduced into the other two. The system then forms a pair of dimers by effective decoupling between cavities with gain and loss. This can result in a bulk bandgap, topological transition and midgap edge states for a wide range of parameters. Topological interface states can also be achieved at a controlled boundary between the nontrivial and trivial lattices. Our scheme is unique in non-Hermiticity-based *midgap* topological states protected by

their isolation from bulk states. Although, we also clarify system topological features in non-Hermitian gapless conditions, which will be relevant with defect and edge states in gapless systems [37, 55–59].

Theoretical model.— The system comprises periods of four single-mode cavities with uniform couplings κ [Fig. 1 (a)]. We introduce an on-site imaginary potential profile $(ig_1, -ig_2, -ig_1, ig_2)$ to the cavities, where its positive and negative coefficients mean gain and loss, respectively. Here, we assume that κ , g_1 and g_2 are sufficiently small compared to the cavities' resonant frequency ω_0 and the cavity-mode Q-factor is high, so that we can safely neglect the effect of the imaginary index profile and radiation loss on κ , as expected in semiconductor lasers [18, 22, 23, 25, 26, 46–48, 60]. Within the linear analysis, the coupled mode equation describing the system is equivalent to the Schrödinger equation $i\partial_t |\Psi\rangle = \hat{\mathcal{H}} |\Psi\rangle$, where $|\Psi\rangle = (\{\Psi_n\})^T$ is the vector of the slowly-varying complex cavity-mode amplitudes (n : cavity index) and $\hat{\mathcal{H}}$ is a tight-binding lattice Hamiltonian. Considering the Bloch theorem and a dynamical factor $e^{-i\omega t}$, the analysis reduces to an eigenvalue problem for the four-component eigenvector $|\psi_B\rangle$ under the Bloch Hamiltonian $\hat{\mathcal{H}}(k)$,

$$\hat{\mathcal{H}}(k) = \begin{pmatrix} ig_1 & \kappa & 0 & \kappa e^{-ika} \\ \kappa & -ig_2 & \kappa & 0 \\ 0 & \kappa & -ig_1 & \kappa \\ \kappa e^{ika} & 0 & \kappa & ig_2 \end{pmatrix}, \quad (1)$$

where a is the spatial interval between the four-cavity units and k is the Bloch wavenumber. The eigenfrequency detuning $\omega(k)$ with reference to ω_0 is given by,

$$\omega(k) = \pm \frac{1}{\sqrt{2}} \sqrt{A \pm \sqrt{A^2 - B^2 - 16\kappa^4 \sin^2 \frac{ka}{2}}}, \quad (2)$$

where $A = 4\kappa^2 - g_1^2 - g_2^2$ and $B = 2g_1g_2$. We also find analytic forms of $|\psi_{B,s}\rangle$ (s : eigenstate index), which are given in Supplementary Materials [61]. In the following analysis, the gain and loss are measured with respect to the cavity coupling, i.e. $\kappa = 1$. We focus on the case where $g_1 > 0$ and $g_1 \geq |g_2|$ for studying the bulk properties, because the spatial and/or time reversal can map the system with this condition to that with the other parameter range. When $g_1 = g_2 = 0$, the system has a gapless four-fold cosinusoidal band structure with a degeneracy at $\omega(0) = 0$, because of the reduced first Brillouin zone [Fig. 1 (b)].

The system band structure is classified into five patterns via the value of the inside of the double radical sign of Eq. (2) for $k = 0$ [Fig. 1 (c)]. With $g_1, g_2 \geq 0$ for simplicity, the divided phase regions are, (I) $B = 0$, (II) $A + B > 0$, $A - B \geq 0$, (III) $A + B > 0$, $A - B < 0$, (IV) $A + B \leq 0$, $A^2 - B^2 - 16\kappa^4 < 0$ and (V) $A^2 - B^2 - 16\kappa^4 \geq 0$. Here, the phase boundaries are symmetric to $g_1 = g_2$. The systems in phase (II) and (III) have complete frequency bandgaps, while those in phase (I), (IV) and (V)

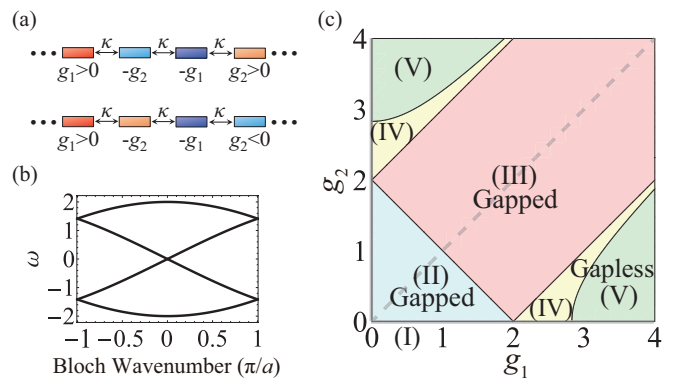


FIG. 1. (Color online) (a) Schematic of considered system. Upper and lower lattices for $g_2 > 0$ and $g_2 < 0$ are topologically nontrivial and trivial, respectively. (b) Folded cosinusoidal band structure for $g_1 = g_2 = 0$. (c) Phase diagram for the system band structure. $\kappa = 1$, $g_1, g_2 \geq 0$.

are gapless. System band structures for phase (IV) and (V) are shown elsewhere (Fig. S1 [61]). Note that the diagram for $g_2 < 0$ is obtained by the mirror inversion of Fig. 1 (c) with regard to $g_2 = 0$.

We see from Eq. (2) that $\omega(k)$ is real as long as $A > 0$ and $A^2 - B^2 - 16\kappa^4 \sin^2(ka/2) > 0$. Such real eigenvalues are obtained because $\hat{\mathcal{H}}(k)$ has a pseudo-Hermiticity [62], $\hat{\mathcal{S}}(k)\hat{\mathcal{H}}(k)\hat{\mathcal{S}}(k) = \hat{\mathcal{H}}^\dagger(k)$. Here, the k -dependent linear operator $\hat{\mathcal{S}}(k) = \hat{\mathcal{S}}(k)^{-1} = \hat{\sigma}_x \otimes (\cos \frac{k}{2})\hat{I}_2 + \hat{\sigma}_y \otimes (\sin \frac{k}{2})\hat{I}_2$ means a half-period translation. $\hat{\sigma}_{x,y,z}$ are Pauli matrices and \hat{I}_2 is the 2×2 identity matrix. The pseudo-Hermiticity guarantees an associated antilinear symmetry [63]. Although the system does not respect \mathcal{PT} symmetry, the bulk antilinear symmetry can instead give at least partially real spectra by balancing net gain and loss. Meanwhile, the operation to which $\hat{\mathcal{H}}(k)$ shows the invariance is implicit due to its complexity.

$\hat{\mathcal{H}}(k)$ also satisfies a pseudo-anti-Hermiticity [36, 37, 61], $\hat{\mathcal{H}}(k) = -\hat{\eta}\hat{\mathcal{H}}^\dagger(k)\hat{\eta}$, where $\hat{\eta} = \hat{I}_2 \otimes \hat{\sigma}_z = \text{diag}(1, -1, 1, -1)$ and $\hat{\eta}^{-1} = \hat{\eta}^\dagger = \hat{\eta}$ in our model. It is known that this symmetry can lead to a nontrivial topology via *chirality* in terms of pairwise eigenvalues, $\omega(k)$ and $-\omega^*(k)$. Since $\hat{\eta}$ is local (diagonal), the resultant topological protection covers all the system parameters. We notice that the symmetry is equivalent to a particle-hole symmetry, $-\hat{\mathcal{H}}(k) = \hat{\eta}\hat{\mathcal{H}}^*(-k)\hat{\eta}$.

Bulk properties.— Figure 2 shows the real and imaginary band structures and eigenmode distributions for different g_1 and g_2 values. When $g_1 < 2$ and $g_2 = 0$ [phase (I) with $A > 0$], Dirac-like dispersion in $\text{Re}\omega(k)$ [Fig. 2 (a)] appears around $\omega(0) = 0$, with cancelled net gain and loss [$\text{Im}\omega(k) = 0$, Fig. 2 (b)]. This implies a topological transition point and reflects the antilinear symmetry. The gapless feature is attributed to $B = 0$ in Eq. (2). The band structure also has two EPs meaning the spontaneous antilinear symmetry breaking. The in-

tensity distributions for $|\psi_{B,s}\rangle$ [Fig. 2 (c)] show that the fields are evenly distributed in the gain and loss cavities ($n = 1, 3$) before this transition ($k = 0.05\pi/a$), while the eigenmodes with the broken symmetry ($k = 0.95\pi/a$) exhibit localization at either of them, and hence complex $\omega(k)$. Note that $g_1 > 2$ and $g_2 = 0$ [phase (I), $A < 0$] give a Dirac cone in $\text{Im}\omega(k)$, with a degeneracy at $\omega(0) = 0$ (Fig. S1 [61]).

For $g_1, g_2 \neq 0$ under phase (II), a bandgap opens due to $B^2 > 0$ [$|g_2| \leq 2\kappa - g_1$, Fig. 2 (c)], while the real eigenvalues remain around $k = 0$ [Fig. 2 (d)]. Here, the effective couplings between gain and loss cavities become *weaker* than that between the two gain cavities and that between the loss cavities [61], as anticipated from the mode localization by \mathcal{PT} symmetry breaking [7, 8, 27, 28]. This decoupling results in dimerization of the successive cavities with gain and those with loss [61].

Bulk mode patterns in the insulating conditions confirm this non-Hermitian effect. Before crossing the EPs in phase (II), the eigenmodes spread over the entire unit cell to cancel the net gain and loss for real $\omega(k)$ [$k = 0.05\pi/a$, Fig. 2 (f)]. After the symmetry breaking, however, they eventually turn into couples of states with localization at the gain cavities ($n = 1, 4$) and loss cavities ($n = 2, 3$) for complex $\omega(k)$ ($k = 0.95\pi/a$). In phase (III) ($|g_2| > 2\kappa - g_1$), the complete antilinear symmetry breaking makes the pairs of upper and lower real bands overlap and gives the split imaginary bands [Fig. 2 (g), (h)]. Thus, only the dimerized eigenstates are allowed all over the Brillouin zone [Fig. 2 (i)]. Here, reversing the signs of g_1 and g_2 does not affect the band structure [Eq. (2)], while a topological transition between systems with $g_2 > 0$ and $g_2 < 0$ is expected at the gap closing with $\omega(0) = 0$. The non-Hermiticity-based bandgap broadens continuously as (g_1, g_2) gets toward the inside of phase (II) and (III) from their boundaries (Fig. S2 [61]).

We introduce the normalized global Berry phase [64] in k -space, $W = \sum_s \frac{i}{4\pi} \oint dk \langle \langle \psi_{B,s} | \partial_k | \psi_{B,s} \rangle \rangle$ ($s = 1, \dots, 4$), as our topological number. W denotes the topological feature of the entire system [65], thus it resolves the problem that the Zak phase [66] of each band is not discretized in non-Hermitian systems with EPs. Here, $|\psi_{B,s}\rangle$ is the left eigenstate that forms a duality with $|\psi_{B,s}\rangle$, namely $\hat{\mathcal{H}}(k)|\psi_{B,s}\rangle = \omega(k)|\psi_{B,s}\rangle$ and $\hat{\mathcal{H}}^\dagger(k)|\psi_{B,s}\rangle = \omega^*(k)|\psi_{B,s}\rangle$ [67]. The biorthonormal basis ($\{|\psi_{B,s}\rangle\}, \{\langle\psi_{B,s}|\rangle\}$) enables the normalization $\langle\langle\psi_{B,s}|\psi_{B,t}\rangle\rangle = \delta_{s,t}$ and the extraction of pure geometric phases from non-Hermitian eigenvectors. W also reflects the 4π periodicity of the eigenvectors [52, 61]. We obtain integer values of $W = 1$ for $g_2 > 0$ and $W = 0$ for $g_2 < 0$, under $g_1 > 0$ (Fig. S3 [61]). W hence confirms the non-Hermiticity-induced nontrivial photonic topology and topological transition between the two conditions in Fig. 1 (a). Interestingly, the discrete change in W holds in the gapless phases, (IV) and (V). We discuss

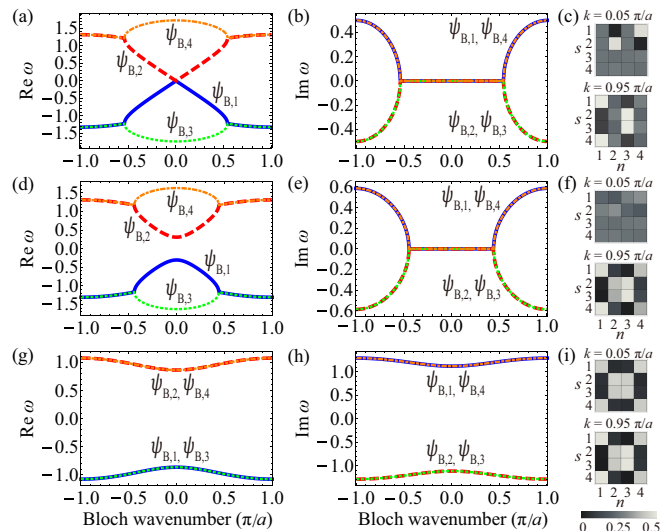


FIG. 2. (Color online) Band structures and mode patterns of system for different gain and loss profiles. $\kappa = 1$. (a) $\text{Re}\omega(k)$, (b) $\text{Im}\omega(k)$ and (c) $|\psi_{B,s,n}|^2 / \langle\psi_{B,s}|\psi_{B,s}\rangle$ for $g_1 = 1$, $g_2 = 0$, phase (I). (d) $\text{Re}\omega(k)$, (e) $\text{Im}\omega(k)$ and (f) $|\psi_{B,s,n}|^2 / \langle\psi_{B,s}|\psi_{B,s}\rangle$ for $g_1 = 1$, $g_2 = 0.5$, phase (II). (g) $\text{Re}\omega(k)$, (h) $\text{Im}\omega(k)$ and (i) $|\psi_{B,s,n}|^2 / \langle\psi_{B,s}|\psi_{B,s}\rangle$ for $g_1 = 2$, $g_2 = 1$, phase (III). s and n : eigenstate and cavity indices.

a geometric picture of W , which is based on coupling parameters relevant with dimerization and can illustrate the topological transition of both the SSH and our model, in Supplementary Materials [61] (Fig. S4 and S5).

Edge states.— Figure 3 shows the topological edge states in our finite systems with 40 cavities for $g_1 > 0, g_2 > 0$. Here, there is relatively weaker effective coupling between gain and loss cavities at each edge [Fig. 3 (a)], like the edge-state generation condition in the Hermitian SSH model [42, 50]. Displayed eigenfrequencies show a pair of midgap states for both phase (II) [Fig. 3 (b)] and (III) [Fig. 3 (d)]. For Fig. 3 (d), we consider an offset absorption potential $i\gamma$ for every cavity, which only shifts all the eigenfrequencies by $i\gamma$ and cancels $\text{Im}\omega$ of a midgap state. Each eigenmode with $\text{Re}\omega = 0$ is localized at the left or right edge. Their $\text{Im}\omega$ reflect the imaginary potential around the relevant edge cavities. A remarkable difference is whether the localization is unit-based [phase (II), Fig. 3 (c)] or cavity-based [phase (III), Fig. 3 (e)], corresponding to whether the bulk without the imaginary offset is in the *exact* phase [$\text{Im}\omega(0) = 0$] or *broken* phase [$\text{Im}\omega(0) \neq 0$]. In phase (III), a topological edge mode has the largest $\text{Im}\omega$ and can be the only state that oscillates ($\text{Im}\omega = 0$) via the loss offset, $\gamma < 0$ [Fig. 3 (d)]. Note that the edge states are also found when each side is terminated by a loss cavity ($g_1 < 0, g_2 < 0$). However, they disappear if a cavity on one edge has gain and one on the other undergoes loss ($g_1 g_2 < 0$), as indicated by $W = 0$. In Supplementary Materials [61], we

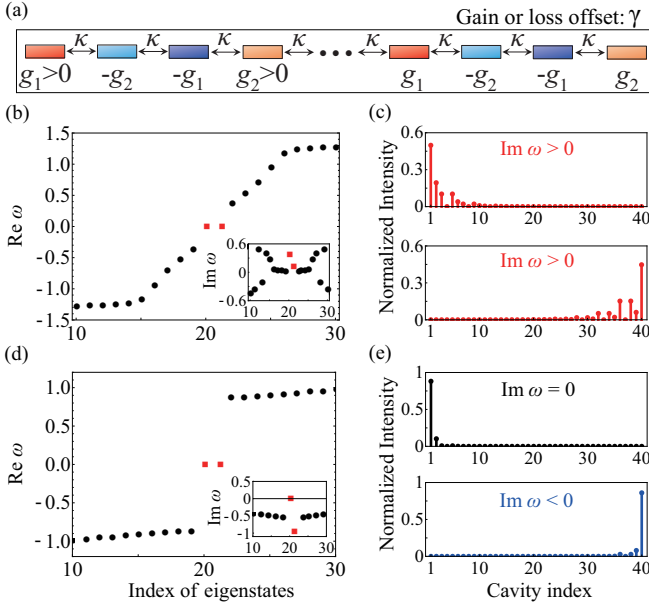


FIG. 3. (Color online) (a) Illustration of our finite non-Hermitian topological lattice. (b), (d) Sorted and selected real eigenvalues for forty-cavity systems. Insets: corresponding imaginary eigenvalues. Squares: topological edge states. (c), (e) Intensity distributions for the edge states. (b), (c) $g_1 = 1, g_2 = 0.5, \gamma = 0$. (d), (e) $g_1 = 2, g_2 = 1, \gamma \sim -1.569$.

show detuned edge states for $g_1 = g_2$, striking robustness of the edge states to disorder, and edge states in the gapless phases (Fig. S6, S7, S8 and S9).

The topological vortex charge of non-Hermitian bulk eigen-detuning [57], $V = \frac{1}{2\pi} \int \partial_k \text{Arg}[\omega(k)] dk$, can be evaluated for our model [61]. Here, finite fractional charges $V = \pm 1/2$ are found around two degenerate EPs with $\omega = 0$ in the gapless phase (IV) [Fig. S10 (a)]. They indicate anomalous edge states with non-Hermitian chirality emerging at $\text{Re } \omega = 0$, and such states are compatible with both the topologically nontrivial ($W = 1$) and trivial ($W = 0$) systems. In phase (V), $V = \pm 1/2$ is held by each eigenstate of the non-Hermitian flatband with $\text{Re } \omega = 0$ [Fig. S10 (b)].

We can discuss the origin of the topological edge states with a particle-hole symmetry [68] equivalent to our pseudo-anti-Hermiticity. Because our lattice Hamiltonian $\hat{\mathcal{H}}$ with a finite number of cavities and the open boundary condition is a symmetric matrix ($\hat{\mathcal{H}} = \hat{\mathcal{H}}^T$), its global pseudo-anti-Hermiticity, $\hat{\mathcal{H}} = -\hat{\eta}' \hat{\mathcal{H}}^\dagger \hat{\eta}'$, easily reduces to a particle-hole symmetry, $-\hat{\mathcal{H}} = \hat{\eta}' \hat{\mathcal{H}}^* \hat{\eta}'$. Here, $\hat{\eta}' = \text{diag}(1, -1, 1, -1, \dots, 1, -1, 1, -1)$ is again local, $\hat{\eta}'^{-1} = \hat{\eta}'^\dagger = \hat{\eta}'$ and $\hat{\mathcal{H}}^*$ is the complex conjugate of $\hat{\mathcal{H}}$. It indicates that the number of states with $\text{Re } \omega = 0$ at each edge can change only by two [69, 70]. Thus, a single isolated edge state with $\text{Re } \omega = 0$ on each side, based on $(1, 0, 0, \dots, 0)^T$ and $(0, \dots, 0, 0, 1)^T$ for $g_1 = g_2 = 0$, is topologically protected by this symmetry under proper bandgap-opening conditions [71].

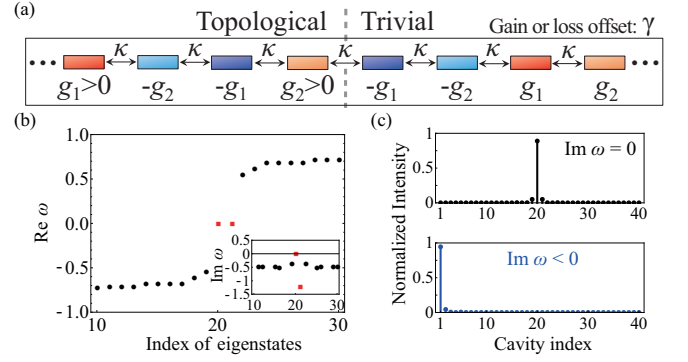


FIG. 4. (Color online) (a) Interface between topologically nontrivial (left) and trivial (right) lattices, with 20 cavities for each. (b) Sorted and selected $\text{Re } \omega$ of the system. Inset: corresponding $\text{Im } \omega$. Left and right squares: eigenvalues for topological interface and edge states. (c) Intensity profiles for the interface (upper) and edge (lower) modes. $g_1 = 1.5, g_2 = 3, \gamma \sim -2.526$.

Interface states.— We apply our non-Hermitian scheme for a controllable midgap topological interface state (Fig. 4). Here, we prepare butting of topologically nontrivial and trivial lattices, effectively forming an “long-long defect” at their boundary, as in an SSH system [72] [Fig. 4 (a)]. Both lattices are adjusted to be in phase (III), and larger gain is applied to the interface cavity of the nontrivial array, than its edge ($g_2 > g_1$). In consequence, a topological interface state with $\text{Re } \omega = 0$ and strong boundary localization obtains the largest $\text{Im } \omega$ as the abovementioned edge state. Including a global loss bias $i\gamma$, we can hence expect single-mode lasing ($\text{Im } \omega = 0$) of such a state. The system eigenfrequency profile [Fig. 4 (b)] confirms a pair of midgap states and cancellation of $\text{Im } \omega$ in one of them. The intensity distributions of the eigenmodes with $\omega = 0$ and $\text{Re } \omega = 0, \text{Im } \omega < 0$ certainly indicate topological interface and edge states confined at the right ($n = 20$) and left ($n = 1$) sides of the nontrivial lattice, respectively [Fig. 4 (c)]. A topological bound state with global \mathcal{PT} symmetry, which systematically satisfies $\text{Re } \omega = \text{Im } \omega = 0$, can also be demonstrated (Fig. S11 [61]).

In conclusion, we have shown that the topological insulating properties of the one-dimensional resonator array can be controlled by the gain and loss. Our scheme is experimentally feasible by modifying the existing laser arrays [46–48] and valid for coupled waveguides [44, 45]. Moreover, it can be explored as an extension of \mathcal{PT} -symmetric systems in photonics [10, 18, 22–26, 43], phononics [73–75], and circuit electronics [76, 77]. It would pave the way for various possibilities of non-Hermitian topological photonics, such as reconfigurable lasing modes, non-Hermiticity-based topological pumping [41, 78], superstructures [79] and Floquet systems [40, 80]. Developing the topological controllability in two dimension is another important direction.

We thank Hiroshi Yamaguchi, Kensuke Inaba and Koji Muraki for fruitful discussions. We acknowledge financial support from the Japan Science and Technology Agency (JST) through the CREST program under Grant No. JPMJCR15N4.

-
- [1] A. Yariv and P. Yeh, *Photonics: optical electronics in modern communications*, 6th ed. (Oxford University Press, New York, 2007).
- [2] C. M. Bender and S. Boettcher, *Phys. Rev. Lett.* **80**, 5243 (1998).
- [3] C. M. Bender, D. C. Brody, and H. F. Jones, *Phys. Rev. Lett.* **89**, 270401 (2002).
- [4] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, *Phys. Rev. Lett.* **100**, 103904 (2008).
- [5] Z. H. Musslimani, K. G. Makris, R. El-Ganainy, and D. N. Christodoulides, *Phys. Rev. Lett.* **100**, 030402 (2008).
- [6] S. Klaiman, U. Günther, and N. Moiseyev, *Phys. Rev. Lett.* **101**, 080402 (2008).
- [7] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, *Phys. Rev. Lett.* **103**, 093902 (2009).
- [8] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, *Nat. Phys.* **6**, 192 (2010).
- [9] I. V. Barashenkov, S. V. Suchkov, A. A. Sukhorukov, S. V. Dmitriev, and Y. S. Kivshar, *Phys. Rev. A* **86**, 053809 (2012).
- [10] A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, *Nature* **488**, 167 (2012).
- [11] S. Longhi, *Phys. Rev. Lett.* **103**, 123601 (2009).
- [12] M. Wimmer, M.-A. Miri, D. Christodoulides, and U. Peschel, *Sci. Rep.* **5**, 17760 (2015).
- [13] S. Longhi, *Opt. Lett.* **41**, 4518 (2016).
- [14] Y. D. Chong, L. Ge, H. Cao, and A. D. Stone, *Phys. Rev. Lett.* **105**, 053901 (2010).
- [15] S. Longhi, *Phys. Rev. A* **82**, 031801 (R) (2010).
- [16] Y. Sun, W. Tan, H.-Q. Li, J. Li, and H. Chen, *Phys. Rev. Lett.* **112**, 143903 (2014).
- [17] A. Szameit, M. C. Rechtsman, O. Bahat-Treidel, and M. Segev, *Phys. Rev. A* **84**, 021806 (R) (2011).
- [18] K. Takata and M. Notomi, *Phys. Rev. Applied* **7**, 054023 (2017).
- [19] Z. Lin, H. Ramezani, T. Eichelkraut, T. Kottos, H. Cao, and D. N. Christodoulides, *Phys. Rev. Lett.* **106**, 213901 (2011).
- [20] L. Feng, Y.-L. Xu, W. S. Fegadolli, M.-H. Lu, J. E. B. Oliveira, V. R. Almeida, Y.-F. Chen, and A. Scherer, *Nat. Mater.* **12**, 108 (2013).
- [21] H. Ramezani, H.-K. Li, Y. Wang, and X. Zhang, *Phys. Rev. Lett.* **113**, 263905 (2014).
- [22] B. Peng, S. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, *Nat. Phys.* **10**, 394 (2014).
- [23] L. Chang, X. Jiang, S. Hua, C. Yang, J. Wen, L. Jiang, G. Li, G. Wang, and M. Xiao, *Nat. Photonics* **8**, 524 (2014).
- [24] L. Feng, Z. J. Wong, R.-M. Ma, Y. Wang, and X. Zhang, *Science* **346**, 972 (2014).
- [25] H. Hodaei, M.-A. Miri, M. Heinrich, D. N. Christodoulides, and M. Khajavikhan, *Science* **346**, 975 (2014).
- [26] Z. Gao, S. T. M. Fryslie, B. J. Thompson, P. S. Carney, and K. D. Choquette, *Optica* **4**, 323 (2017).
- [27] L. Feng, R. El-Ganainy, and L. Ge, *Nat. Photonics* **11**, 752 (2017).
- [28] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, *Nat. Phys.* **14**, 11 (2018).
- [29] F. D. M. Haldane and S. Raghu, *Phys. Rev. Lett.* **100**, 013904 (2008).
- [30] L. Lu, J. D. Joannopoulos, and M. Soljačić, *Nat. Photonics* **8**, 821 (2014).
- [31] L. Lu, J. D. Joannopoulos, and M. Soljačić, *Nat. Phys.* **12**, 626 (2016).
- [32] K. v. Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).
- [33] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, *Phys. Rev. Lett.* **49**, 405 (1982).
- [34] C. L. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 146802 (2005).
- [35] M. Z. Hasan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).
- [36] M. Sato, K. Hasebe, K. Esaki, and M. Kohmoto, *Prog. Theor. Phys.* **127**, 937 (2012).
- [37] K. Esaki, M. Sato, K. Hasebe, and M. Kohmoto, *Phys. Rev. B* **84**, 205128 (2011).
- [38] Y. C. Hu and T. L. Hughes, *Phys. Rev. B* **84**, 153101 (2011).
- [39] C. Yuce, *Phys. Lett. A* **379**, 1213 (2015).
- [40] C. Yuce, *Eur. Phys. J. D* **69**, 184 (2015).
- [41] H. Schomerus, *Opt. Lett.* **38**, 1912 (2013).
- [42] W. P. Su, J. R. Schrieffer, and A. J. Heeger, *Phys. Rev. Lett.* **42**, 1698 (1979).
- [43] C. Poli, M. Bellec, U. Kuhl, F. Mortessagne, and H. Schomerus, *Nat. Commun.* **6**, 6710 (2015).
- [44] J. M. Zeuner, M. C. Rechtsman, Y. Plotnik, Y. Lumer, S. Nolte, M. S. Rudner, M. Segev, and A. Szameit, *Phys. Rev. Lett.* **115**, 040402 (2015).
- [45] S. Weimann, M. Kremer, Y. Plotnik, Y. Lumer, S. Nolte, K. G. Makris, M. Segev, M. C. Rechtsman, and A. Szameit, *Nat. Mater.* **16**, 433 (2017).
- [46] P. St-Jean, V. Goblot, E. Galopin, A. Lemaitre, T. Ozawa, L. L. Gratiet, I. Sagnes, J. Bloch, and A. Amo, *Nat. Photonics* **11**, 651 (2017).
- [47] M. Parto, S. Wittek, H. Hodaei, G. Harari, M. A. Bandres, J. Ren, M. C. Rechtsman, M. Segev, D. N. Christodoulides, and M. Khajavikhan, *Phys. Rev. Lett.* **120**, 113901 (2018).
- [48] H. Zhao, P. Miao, M. H. Teimourpour, S. Malzard, R. El-Ganainy, H. Schomerus, and L. Feng, *Nat. Commun.* **9**, 981 (2018).
- [49] B. Bahari, A. Ndao, F. Vallini, A. E. Amili, Y. Fainman, and B. Kanté, *Science* (2017), 10.1126/science.aao4551.
- [50] N. Malkova, I. Hromada, X. Wang, G. Bryant, and Z. Chen, *Opt. Lett.* **34**, 1633 (2009).
- [51] S. Aubry and G. André, *Ann. Isr. Phys. Soc.* **3**, 133 (1980).
- [52] T. E. Lee, *Phys. Rev. Lett.* **116**, 133903 (2016).
- [53] G. Q. Liang and Y. D. Chong, *Phys. Rev. Lett.* **110**, 203904 (2013).
- [54] H. Kogelnik and C. V. Shank, *J. Appl. Phys.* **43**, 2327

- (1972).
- [55] S. Malzard, C. Poli, and H. Schomerus, *Phys. Rev. Lett.* **115**, 200402 (2015).
- [56] H. Zhao, S. Longhi, and L. Feng, *Sci. Rep.* **5**, 17022 (2015).
- [57] D. Leykam, K. Y. Bliokh, C. Huang, Y. D. Chong, and F. Nori, *Phys. Rev. Lett.* **118**, 040401 (2017).
- [58] B. Qi, L. Zhang, and L. Ge, *Phys. Rev. Lett.* **120**, 093901 (2018).
- [59] M. Pan, H. Zhao, P. Miao, S. Longhi, and L. Feng, *Nat. Commun.* **9**, 1308 (2018).
- [60] J. K. S. Poon and A. Yariv, *J. Opt. Soc. Am. B* **24**, 2378 (2007).
- [61] See Supplementary Materials for the system gapless phases, bulk symmetry, effective decoupling, bandgap opening, topological number and its geometrical picture, possible finite size effects, robustness of the topological edge states, chiral edge states in gapless phases, vortex charges of eigendetuning with band coalescence, and non-Hermiticity-based topological bound state with \mathcal{PT} symmetry, which include Ref. [81–88].
- [62] A. Mostafazadeh, *J. Math. Phys.* **43**, 205 (2002).
- [63] A. Mostafazadeh, *J. Math. Phys.* **43**, 3944 (2002).
- [64] S.-D. Liang and G.-Y. Huang, *Phys. Rev. A* **87**, 012118 (2013).
- [65] Y. Hatsugai, *J. Phys. Soc. Jpn.* **73**, 2604 (2004).
- [66] J. Zak, *Phys. Rev. Lett.* **62**, 2747 (1989).
- [67] F. Keck, H. J. Korsch, and S. Mossmann, *J. Phys. A: Math. Gen.* **36**, 2125 (2003).
- [68] S. Ryu, A. P. Schnyder, A. Furusaki, and A. W. W. Ludwig, *New J. Phys.* **12**, 065010 (2010).
- [69] D. I. Pikulin and Y. V. Nazarov, *JETP Lett.* **94**, 693 (2012).
- [70] D. I. Pikulin and Y. V. Nazarov, *Phys. Rev. B* **87**, 235421 (2013).
- [71] S. Ryu and Y. Hatsugai, *Phys. Rev. Lett.* **89**, 077002 (2002).
- [72] A. Blanco-Redondo, I. Andonegui, M. J. Collins, G. Harari, Y. Lumer, M. C. Rechtsman, B. J. Eggleton, and M. Segev, *Phys. Rev. Lett.* **116**, 163901 (2016).
- [73] H. Jing, S. K. Özdemir, X.-Y. Lü, J. Zhang, L. Yang, and F. Nori, *Phys. Rev. Lett.* **113**, 053604 (2014).
- [74] X. Zhu, H. Ramezani, C. Shi, J. Zhu, and X. Zhang, *Phys. Rev. X* **4**, 031042 (2014).
- [75] X.-Y. Lü, H. Jing, J.-Y. Ma, and Y. Wu, *Phys. Rev. Lett.* **114**, 253601 (2015).
- [76] J. Schindler, A. Li, M. C. Zheng, F. M. Ellis, and T. Kottos, *Phys. Rev. A* **84**, 040101 (2011).
- [77] B. Lv, J. Fu, B. Wu, R. Li, Q. Zeng, X. Yin, Q. Wu, L. Gao, W. Chen, Z. Wang, Z. Liang, A. Li, and R. Ma, *Sci. Rep.* **7**, 40575 (2017).
- [78] Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and O. Zeitlinger, *Phys. Rev. Lett.* **109**, 106402 (2012).
- [79] A. A. Burkov and L. Balents, *Phys. Rev. Lett.* **107**, 127205 (2011).
- [80] M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, *Nature* **496**, 196 (2013).
- [81] B. Zhen, C. W. Hsu, Y. Igarashi, L. Lu, I. Kaminer, A. Pick, S.-L. Chua, J. D. Joannopoulos, and M. Soljačić, *Nature* **525**, 354 (2015).
- [82] M. Berry, *Czech. J. Phys.* **54**, 1039 (2004).
- [83] W. D. Heiss, *J. Phys. A: Math. Theor.* **45**, 444016 (2012).
- [84] F. H. M. Faisal and J. V. Moloney, *J. Phys. B* **14**, 3603 (1981).
- [85] T. E. Lee, “Reply to comment on “Anomalous Edge State in a Non-Hermitian Lattice”,” arXiv:1611.00355.
- [86] P. Delplace, D. Ullmo, and G. Montambaux, *Phys. Rev. B* **84**, 195452 (2011).
- [87] C. Yin, H. Jiang, L. Li, R. Lü, and S. Chen, *Phys. Rev. A* **97**, 052115 (2018).
- [88] X.-F. Zhu, *Opt. Express* **23**, 22274 (2015).