Missing Satellites Problem: Completeness Corrections to the Number of Satellite Galaxies in the Milky Way are Consistent with Cold Dark Matter Predictions

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There is No Missing Satellites Problem

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A critical challenge to the cold dark matter (CDM) paradigm is that there are fewer satellites observed around the Milky Way than found in simulations of dark matter substructure. We show that there is a match between the observed satellite counts corrected by the detection efficiency of the Sloan Digital Sky Survey (for luminosities \( L \gtrsim 340 \, L_\odot \)) and the number of luminous satellites predicted by CDM, assuming an empirical relation between stellar mass and halo mass. The “missing satellites problem”, cast in terms of number counts, is thus solved. We also show that warm dark matter models with a thermal relic mass smaller than 4 keV are in tension with satellite counts, putting pressure on the sterile neutrino interpretation of recent X-ray observations. Importantly, the total number of Milky Way satellites depends sensitively on the spatial distribution of satellites, possibly leading to a “too many satellites” problem. Measurements of completely dark halos below \( 10^9 \, M_\odot \), achievable with substructure lensing and stellar stream perturbations, are the next frontier for tests of CDM.

INTRODUCTION

One outstanding problem for the cold dark matter (CDM) paradigm is the missing satellites problem (MSP). When originally formulated, the MSP highlighted the discrepancy between the number of satellites predicted in CDM simulations, numbering in the 100s, and observed in the Milky Way (MW), numbering \( \sim 10 \) \([1, 2]\). Since then, increasingly sensitive surveys have pushed the observed satellite count to \( \sim 50 \) (e.g., Ref. 3). Simultaneously, however, improved resolution in numerical simulations has also increased the number of predicted satellites (e.g., 1).

A crucial step towards resolving the MSP is to correct for those satellites that have not yet been detected. Only a fraction of the MW’s virial volume has been surveyed \([5]\). The Sloan Digital Sky Survey (SDSS), by which ultra-faint dwarfs with luminosities as low as 340 \( L_\odot \) (Segue I) were discovered, covered only about a third of the sky. For the faintest dwarfs, SDSS was complete to \( \sim 10\% \) of the MW’s virial radius \([6, 10]\). The observed count is thus a lower bound on the luminous MW satellite population. Completeness corrections must be applied to derive the total number of luminous MW satellites.

Fully resolving the MSP requires that the completeness-corrected galaxy count match the predicted luminous satellite abundance. This depends on the physics of an additional key component: baryons. There is growing evidence that not all dark matter subhalos host an observable galaxy. Galaxy evolution models \([11]\) and star-formation histories of ultra-faint dwarfs \([12]\) indicate that feedback processes and reionization prevent star formation. In fact, subhalos below \( \sim 10^9 \, M_\odot \) are inefficient in forming a luminous component \([13, 14]\). In CDM, most MW subhalos are dark.

In this work, we compare completeness corrections of the observed MW luminous galaxy population to theoretical predictions for the luminous galaxy population. We use an analytic approach to highlight specific physics, and provide a roadmap for future MW-based DM constraints. Our completeness correction is inspired by Refs. 6, 15, 18, which used simulations or Bayesian techniques to estimate that the MW hosts hundreds of luminous satellites. We calculate the total number of luminous galaxies down to 340 \( L_\odot \) based on the satellites observed by SDSS. For comparison, we predict the number of luminous satellites expected in CDM based on empirical scaling relations between halos and galaxies.

Successful dark matter models cannot produce just enough dark matter subhalos to match the corrected galaxy count—they must produce enough luminous galaxies. This places stringent constraints on warm dark matter (WDM) and sterile neutrino models, competitive with Lyman-\( \alpha \) forest constraints \([18]\).

Successful galaxy formation models must produce enough luminous galaxies to match the corrected galaxy count. This has implications for the mass threshold for the subhalos that host the faintest galaxies, the redshift of reionization, and the tidal stripping of subhalos.

COMPLETENESS CORRECTIONS

The total number of luminous satellites within the MW virial radius \( (R_{\text{vir}} = 300 \, \text{kpc}) \) can be extrapolated from
the number of observed satellites by calculating the correction factor \( c \) that converts

\[
N_{\text{tot}} = c(\Phi)N_{\text{obs}},
\]

where \( \Phi \) represents the set of parameters the correction depends on. This includes the survey area, survey sensitivity, and the spatial distribution of satellites.

Recasting in luminosities \( L \), and given either a continuous observed luminosity function \( dN_{\text{obs}}/dL \), or set of \( N_{\text{obs}} \) satellites we can express Eq. 1 as

\[
N_{\text{tot}} = \int c(L) \frac{dN_{\text{obs}}}{dL} dL \approx \sum_{i=1}^{N_{\text{obs}}} c(L_i),
\]

i.e. we integrate over the luminosity function or sum together the correction for each observed satellite. The correction is

\[
c(L) = \frac{\int_{V_{\text{vir}}} n(r) \, dr}{\int_{V_{\text{obs}}(L)} n(r) \, dr}
\]

where \( n(r) \) is the 3D satellite distribution, \( V_{\text{vir}} \) the MW virial volume, and \( V_{\text{obs}}(L) \) the volume over which a satellite of luminosity \( L \) has been surveyed. Note that the normalization to the spatial distribution cancels, and thus the correction depends only on the shape of the spatial distribution function—not the absolute number of satellites. Eq. 3 naturally accounts for anisotropies in the satellite distribution.

Although there are hints that the luminous satellite distribution is anisotropic [18, 19, 23], we assume it is sufficiently isotropic to be separable. The correction factor is thus

\[
c(L) = c_r(L) \ c_\Omega(L)
\]

where \( c_r \) and \( c_\Omega \) are the radial and angular corrections, respectively, and

\[
c_r(L) = \frac{\int_0^{r_{\text{vir}}} \frac{dN}{dL} \, dr}{\int_0^{r_{\text{obs}}(L)} \frac{dN}{dL} \, dr} \text{ and } c_\Omega = \frac{4\pi}{\int_0^{r_{\text{vir}}} L^2 \, d\Omega}
\]

and \( r_{\text{vir}}(L) \) is the radius out to which a survey covering an area \( \Omega_c \) of the sky is complete for a galaxy with luminosity \( L \). To predict the number of satellites out to a given detection limit for other surveys based on counts from an earlier survey like SDSS, one can replace \( r_{\text{vir}} \) with the radius out to which those surveys are complete. For SDSS, we adopt the completeness radius derived by Ref. [10], for which

\[
r_{\text{c}}(L) = 15.7 \text{ kpc } \left( \frac{L}{100 \ L_\odot} \right)^{0.51}.
\]

The angular correction dominates for the brightest galaxies, while the radial correction dominates for the faintest galaxies. The turnover between the two occurs at roughly \( L = 500 - 2000 \ L_\odot \) (depending on the satellite distribution), and rapidly becomes large at lower luminosities.

The radial distribution of luminous satellites, which is highly uncertain, has a significant impact on the corrected galaxy count. Well-motivated radial profiles from the literature, spanning the range of uncertainty on tidal stripping and subhalo-galaxy identification, are shown in Fig. 1. The centrally concentrated NFW (with concentration \( c_\odot = 9 \) [25] and SIS (singular isothermal sphere) [26] models correspond to the smooth dark matter distribution of the host. These profiles include satellites that are severely tidally stripped [8, 27], which may be considered destroyed in other contexts (or unresolved in numerical simulations). The light red line is representative of distributions generated from dark matter only (DMO) simulations, but with assumptions on which subhalos host galaxies [16, 25]. The black dotted line shows how tidal stripping by a baryonic disk reduces the number of satellites close to the center of the MW as in Ref. [24] (hereafter GK17; see also [29]). In contrast, the most severely stripped halos in Ref. [30] are shown in green. This corresponds to the hypothesis that the SDSS satellites are highly stripped remnants of larger galaxies. For our fiducial distribution, we adopt that derived in Ref. [25] (hereafter D17; shown by the dark red line), which matches the distribution of observed classical MW satellites.

We correct the number of galaxies observed through SDSS Data Release 8 (DR8) with these radial pro-
files. Our results are shown in the left panel of Fig. 2 and are listed in Tab. 1. The width of the bars denote the uncertainty due to anisotropy, as measured in [15] (see supplementary material for a detailed discussion of anisotropies). In agreement with Refs. [8, 16], we find that the correction is insensitive to the MW halo mass.

Radial distributions corresponding to the hypothesis that satellites survive extreme tidal stripping (NFW and SIS) are more centrally concentrated, resulting in smaller corrected counts. Accounting for the effects of tidal stripping due to the presence of a baryonic disk as predicted by [24] produce the largest corrections.

These results are a lower limit to the number of luminous satellites of the MW. We have not included the satellites of the Large Magellanic Cloud, dwarfs with surface brightnesses $\mu \leq 30$ mag arcsec$^{-2}$ (“stealth galaxies”, e.g. [32, 33]), which are below the detection limit of SDSS DR8, although they have been found by new surveys [4, 33], and dwarfs with luminosities below Segue I’s. Segue I itself accounts for $\sim 40\%$ of the correction; accounting for even fainter galaxies will increase the total number significantly. The inferred luminosity function of satellites is shown in the supplementary material.

<table>
<thead>
<tr>
<th>TABLE I. Completeness corrected satellite counts</th>
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<tr>
<td>Predictions</td>
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<tr>
<td>-------------</td>
</tr>
<tr>
<td>NFW</td>
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<tr>
<td>SIS</td>
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<tr>
<td>ELVIS, stripped</td>
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<tr>
<td>D17</td>
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<tr>
<td>DMO + gal</td>
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<tr>
<td>DMO + gal + GK17 stripping</td>
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</tbody>
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Predictions for DES, when complete after year 5, and sensitive down to apparent magnitudes $V = 24.7$; and for LSST after year 1, down to $V = 26$.

CONSTRAINTS ON GALAXY EVOLUTION AND DARK MATTER MODELS

The calculations above set only the total number of luminous MW satellites that we can infer exists based on the observed dwarfs. Do the corrected counts imply that the MSP is solved? We present our fiducial calculation here, and provide details on choices and variants in the supplementary material.

The number of dark matter subhalos hosted by the MW is derived by integrating the CDM mass function, which follows the form

$$\frac{dN}{dM} = K_0 \left( \frac{M}{M_*} \right)^{-\alpha} \frac{M_{\text{host}}}{M_*},$$

(7)

where $M$ denotes the mass of a subhalo at infall. The
mass function based on present day (e.g. \( z = 0 \)) subhalo masses is lower due to tidal stripping (\( M(z = 0) < M \)). We adopt \( \mathcal{K}_0 = 1.88 \times 10^{-3} \, \text{M}_\odot^{-1} \) and \( \alpha = 1.87 \) as in D17. The total number of subhalos above a threshold \( M_{\text{min}} \) is thus

\[
N_{\text{sub}} = \int_{M_{\text{min}}}^{M_{\text{host}}} \frac{dN}{dM} \, dM.
\]  

(8)

Not all subhalos are believed to host galaxies \[34\,37\]. Given the fraction of subhalos of a given mass that host a luminous galaxy, \( f_{\text{lum}}(M) \), we can derive the total number of luminous galaxies

\[
N_{\text{gal}} = \int_{M_{\text{min}}}^{M_{\text{host}}} \frac{dN}{dM} f_{\text{lum}}(M) \, dM.
\]

(9)

The luminous fraction is a strong function of reionization redshift, \( z_{\text{re}} \), and the survival criterion \[38\]. We adopt the relation by D17 (see their Fig. 3), which assumes \( z_{\text{re}} = 9.3 \) and a generous baryon survival criterion, which requires \( v_{\text{max}} = 9.5 \, \text{km/s} \) (the peak of the circular velocity curve) at \( z < z_{\text{re}} \) and \( v_{\text{peak}} = 23.5 \, \text{km/s} \) at \( z > z_{\text{re}} \). Adopting a less generous criterion to match other work in the literature \[39\,41\] drops the predicted number of luminous satellites by a factor 2—our results thus represent an upper bound.

For comparison with our completeness correction, which only includes galaxies brighter than Segue I, we adopt \( M_{\text{min}} = M_{\text{SegI}} \). We derive its total stellar mass by assuming a stellar mass-to-light ratio of 2 (i.e. \( M_\star \sim 680 \, \text{M}_\odot \)) expected for an ancient metal-poor stellar population with a Kroupa initial mass function \[12\,13\]. To derive Segue I’s halo mass, we make use of the fact that a galaxy’s stellar mass is empirically tightly correlated with halo mass \[44\,45\], a relation known as the stellar-mass–halo-mass (SMHM) relation. SMHM relations have only been calibrated for stellar masses greater than \( M_\star \sim 10^9 \, \text{M}_\odot \) \[40\,47\], but hydrodynamic simulations indicate that extrapolations to lower masses are reasonable \[45\]. We adopt three SMHM relations that capture the diversity of SMHM relations and their scatter \[46\,47\,49\], which gives a large range for \( M_{\text{SegI}} = 8 \times 10^6 \) to \( 7 \times 10^8 \, \text{M}_\odot \). The SMHM of Moster et al. \[47\] best matches hydrodynamic simulations of isolated galaxies \[48\,50\].

The resultant number of subhalos and galaxies assuming a MW mass of \( 1.5 \times 10^{12} \, \text{M}_\odot \) is shown in Fig. 3. The solid line denote the number of luminous satellites predicted by CDM; the dashed line shows the total number of subhalos. Down to \( M_{\text{min}} = 10^7 \, \text{M}_\odot \), there exists \( \sim 2600 \) total subhalos and 280 galaxies, implying that only \( \sim 10\% \) of such subhalos are luminous. The number of luminous satellites down to \( M_{\text{SegI}} \) for our range of SMHM relations is shown by the gray-shaded regions in Fig. 3 and the left-hand panel of Fig. 2. The SMHM relation of Moster et al. \[47\] predicts \( \sim 120 \) galaxies.

![Figure 3](https://example.com/figure3.png)

**FIG. 3.** The number of luminous (solid) and total (dashed) subhalos allowed by CDM and WDM mass functions for a given lower bound on the lowest mass subhalo, assuming a MW mass of \( 1.5 \times 10^{12} \, \text{M}_\odot \). The number of luminous subhalos is modeled as in D17, assuming a reionization redshift \( z_{\text{re}} = 9.3 \) \[51\]. The grey band shows SMHM predictions for the infall mass of Segue I.

We now have all the tools required to match the theoretical and completeness-corrected galaxy counts. Our key result is shown in the left-hand plot of Fig. 2. The number of galaxies more luminous than Segue I predicted in CDM matches the completeness-corrected observations for even the most conservative radial profile models (i.e. lies inside the gray-shaded region). Moreover, some radial profiles lead to corrected counts that exceed the predicted range. This is exacerbated if the reionization of the Local Group occurs earlier (see supplementary material). We call this the “too many satellites” problem.

These results have implications for galaxy formation theory and dark matter physics.

**Subhalo minimum mass.** Matching the corrected counts to CDM predictions gives a minimum subhalo mass for galaxies as faint as Segue I, as suggested by Ref. \[17\]. This is shown on the right panel of Fig. 2. The bars denote the uncertainty on the lowest mass galaxy halo due to uncertainties on the MW mass, which we allowed to range from \((1-2) \times 10^{12} \, \text{M}_\odot \). As the transition from mostly bright to mostly dark subhalos (e.g. \( f_{\text{lum}} = 0.5 \)) occurs at \( \sim 10^8 \, \text{M}_\odot \) in our reionization model, the lowest mass galaxy is near that mass threshold. Counts accounting for tidal stripping as in GK17 predict even smaller masses. The tidal-stripping-induced uncertainty on the completeness correction is the single-biggest driver of uncertainty in the subhalo minimum mass.

**Dark matter model.** Dark matter models with suppressed matter power spectra \[52\,54\], such as WDM, must reproduce at a minimum the completeness-
corrected counts. We briefly sketch the constraints we can place on WDM models with our corrected counts. This calculation is not intended to be a rigorous derivation of WDM constraints, but an illustration of possible limits when corrected counts are taken into account.

The radial distribution of WDM satellites closely follows CDM [63], and thus the corrected counts derived above applies. To obtain the number of luminous satellites predicted by WDM, an identical analysis as in the previous section can be performed with the WDM mass function, which can be obtained from the CDM mass function by multiplying the factor

$$\frac{dn_{\text{WDM}}}{dn_{\text{CDM}}} = \left(1 + \frac{M_{\text{hm}}}{M}\right)^{-\beta},$$

where $\beta = 1.16$, $M$ is the infall mass, and $M_{\text{hm}}$ is the half-mode mass quantifying the suppression scale of the matter power spectrum [63]. We again use D17’s reionization cutoff to estimate the number of luminous galaxies. This is a conservative overestimate, as WDM halos tend to form, and form stars, later than in CDM [67], and because some of the satellites will be fainter than Segue I. In Fig. 3, we show the number of satellites predicted by WDM mass functions for thermal relic particle masses ranging from 1-8 keV. If the MW has 120-150 galaxies brighter than Segue I, thermal relics below \(~4\) keV are ruled out, although this depends on the MW halo mass [63], and see supplementary material). This implies that the 7 keV sterile neutrino is in tension with satellite counts. More robust limits require the machinery of Ref. [17], who find a 95% lower limit of 2.9 keV.

**CONCLUSIONS**

Since the MSP was first identified, several advances in our understanding of dwarf galaxy evolution have reduced the severity of the missing satellites problem. Star formation in low-mass halos has been demonstrated to be suppressed by reionization and feedback. The discovery of many new dwarfs below the luminosity limit of the classical dwarfs have also closed the gap, as has the understanding that completeness corrections for the new dwarfs are large. In this Letter, we show that such corrections imply that the number of satellite galaxies that inhabit the Milky Way is consistent with the number of luminous satellites predicted by CDM down to halo masses of \(~10^8\) M☉. There is thus no missing satellites problem. If anything, there may be a “too many satellites” problem. The major remaining uncertainty is the radial distribution of satellites, stemming from the uncertainty in tidal stripping. Our result pushes the scale for tests of CDM below \(10^8\) M☉ in infall mass, or \(~10^7\) M⊙ in present day subhalo mass. Methods that do not rely on baryonic tracers, like substructure lensing [68][70] or stellar stream gaps [71], are required to test the predictions of CDM below this scale. The implications for dark matter models are significant. WDM theories equivalent to having thermal relic particle masses below 4 keV are in tension with MW satellite counts.

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