Observation of Nonreciprocal Wave Propagation in a Dynamic Phononic Lattice

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Observation of non-reciprocal wave propagation in a dynamic phononic lattice

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Abstract

Acoustic waves in a linear time-invariant medium are generally reciprocal, however, reciprocity can break down in a time-variant system. In this paper, we report on an experimental demonstration of non-reciprocity in a dynamic one-dimensional phononic crystal, where the local elastic properties are dependent on time. The system consists of an array of repelling magnets, and the on-site elastic potentials of the constitutive elements are modulated by an array of electromagnets. The modulation in time breaks time-reversal symmetry and opens a directional bandgap in the dispersion relation. A theoretical explanation of the observed non-reciprocal behavior is provided as well. This work provides a prototype for developing acoustic diode that can serve in acoustic circuits for rectification applications .

Phononic crystals and metamaterials control acoustic waves through the geometry of their building blocks, engineered with periodic impedance mismatches and/or local resonances [1-7]. The majority of current realizations focus on designing metamaterials in their spatial dimensions, while the material properties remain unchanged over time. This design framework restricts the application of metamaterials in scenarios where material's tunability and adaptivity is required [8,9]. More importantly, in these time-invariant metamaterials, reciprocity holds as a fundamental principle in wave propagation, requiring the transmission of information or energy between any two points in space to be symmetric for opposite propagating directions [10].

However, non-reciprocal materials or devices, i.e., diodes, are usually required for rectification and control of the associated energy flow. Unlike electric diodes, mechanical or acoustic diodes are just starting to be explored [11-18]. Achieveing non-reciprocity in mechanical systems through intrinsic time-reversal symmetry breaking has been demonstrated in strongly nonlinear networks [11,13,14], selective acoustic circulators [15], and topological mechanical insulators [16-18]. In nonlinear systems, the non-reciprocal behavior is a function of the nonlinear potential and may be tuned by the wave amplitude [19,20]. Recently, theoretical proposals [21-24] suggested the use of external, spatio-temporal modulation of material's properties as a mean to achieve non-reciprocity within the linear operating regime.

Here we demonstrate realization of a dynamic phononic lattice in which the elastic properties can vary over time with spatiotemporal modulation. This time dependence leads to novel wave propagation behaviors such as non-reciprocity [21-24], which is very difficult to achieve in time-invariant systems. Though we focus on elastic waves in a magnetically coupled lattice, the concept extends to other types of waves such as thermal diodes [25] and photonic systems [26]. For instance, non-reciprocal propagation in photonic systems was observed in coupled, modulated waveguides [27] where modulation leads to irreversible mode conversion between the two waveguides. As for our system, it behaves as a mechanical diode operating at tunable frequency ranges.

Such device may serve in acoustic circuits, like circulators, transducers and imaging systems to rectify mechanical or acoustic energy flows [11].

Experimental realizations of modulation-induced non-reciprocity in a single phononic waveguide require (i) a dynamic lattice with controllable elastic properties, and (ii) a dynamic modulation with speed comparable to the wave propagation velocity. We meet these requirements by building a mass-spring chain of repelling magnets modulated by externally driven coils. The chain consists of 12 ring magnets ($m = 9.8$ g) free to slide on a supporting smooth cylindrical rail as shown in Fig. 1a. The first and last magnets are fixed to the rail (fixed boundary conditions). To dynamically modulate the chain, we introduce electrical coils around the 8 central ring magnets (masses 3 to 10). The electrical coils are positioned coaxially with the magnets and rest at the same center positions $x_{0,n}$ as shown in Fig. 1a. When a current flows through the electrical coils, they create local magnetic fields that couple to the ring magnets. When the ring magnets are at rest ($x_{0,n}$ position), they sit at the apex of the magnetic potential created by the coils and their coupling forces vanish. When the ring magnets displace, they experience either restoring or repelling forces from the coils, depending on the current direction. The coupling between each pair of ring magnet and coil is similar to a grounding spring. When the grounding spring stiffness is modulated spatiotemporally, time-reversal symmetry is broken leading to the formation of a non-reciprocal bandgap in the dispersion diagram [21-24] as shown in Fig. 1b.

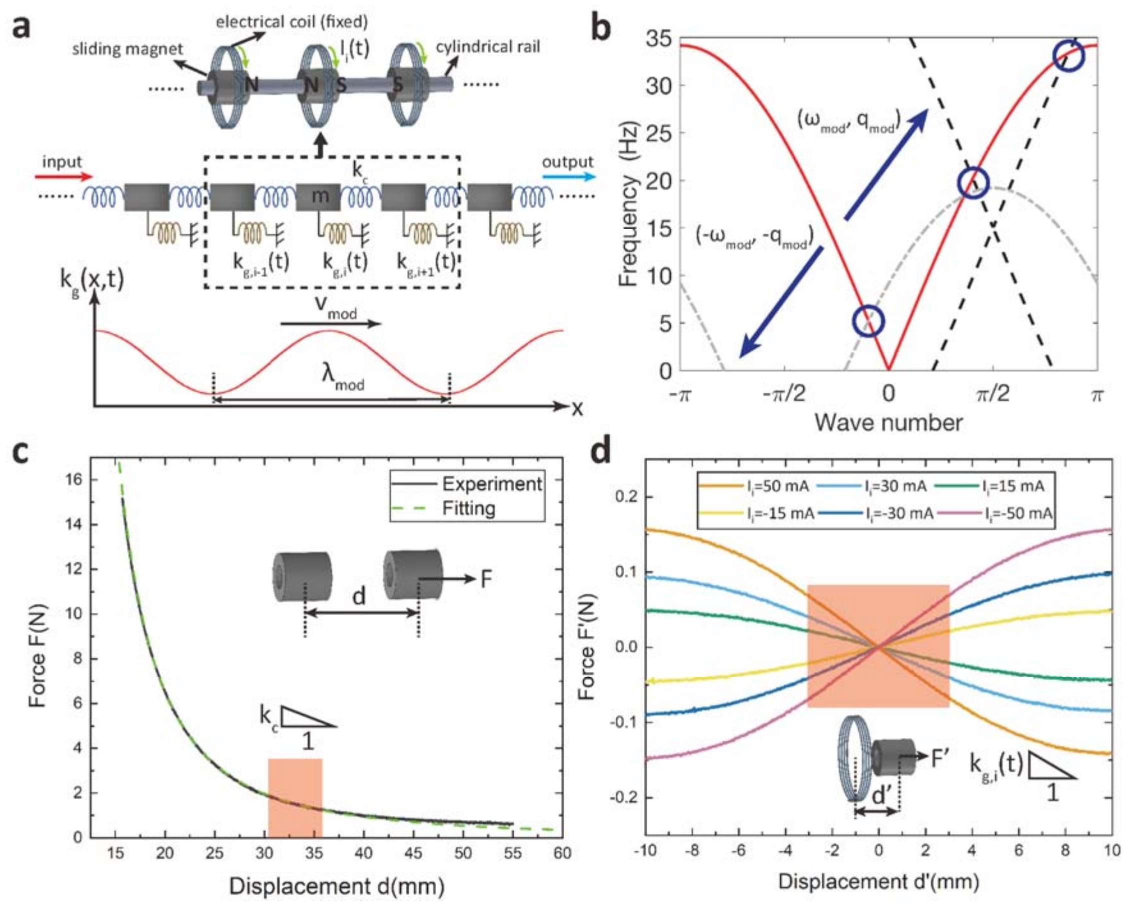

Figure 1 Experimental setup for the non-reciprocal dynamic phononic lattice. (a) Top: Schematic of the experimental setup. Middle: Discrete mechanical representation of the system with masses and springs. Bottom: Schematic illustration of the modulation concept by changing the gounding spring stiffness (k_g) in a wave-like fashion. (b) Scattering analysis: The red solid curve describes the original dispersion relation of the un-modulated monatomic lattice. The black dashed and grey dash-dotted curves correspond to Floquet-Bloch replicas of the original curves obtained by translation along the solid blue arrows $\pm(\omega_{mod}, q_{mod}) = \pm(15\text{Hz}, \pi/2)$. Parity-breaking crossings (circled) are where Bragg's condition is satisfied and non-reciprocal wave scattering is anticipated. (c) Force-displacement curve for neighboring magnetic masses, measurement (solid) and fitted curve (dashed). (d) Measured force-dispacement curve between the ring magnet and its surrounding coil at different currents. The red shaded

regions in both (c) and (d) corresponds to the dynamic operating regime of our experiments.

To characterize the mechanical parameters of our system, we measure the repelling force between neighboring masses as a function of their displacement (see Supplemental Material). The resulting force-displacement curve exhibits a nonlinear force that is characteristic of dipole repulsion shown in Fig. 1c. We also measure the force between the magnets and the surrounding coils at different applied currents in Fig. 1d. To measure the dynamic response of the system, we drive the 2nd mass with a sinusoidal force of frequency f_{dr}, and the velocity of mass 11 is monitored with a laser vibrometer (output signal). The velocity response is measured using a lock-in amplifier as a function of different f_{dr} for different modulation parameters. Due to the small vibration amplitude of the driving signal (≤ 5 mm), the coupling between masses can be approximated by a linear response in the red shaded area of Fig. 1c. The linearized coupling stiffness between adjacent magnets obtained from experiments is $k_c \approx 113$ N/m. Similarly the coupling between the electromagnets and the masses can be linearized in the dynamic regime of interest in Fig. 1d. We consider only nearest neighbor interactions between masses and mass-coil pairs, since non-nearest neighbour interactions decay to a negligible amount (see Supplemental Material).

The spatiotemporal modulation of the system can be achieved by applying sinusoidal AC currents through the coils. Each coil is subjected to a current of the same frequency, f_{mod}, but with a phase shift of $\pi/2$ or $-\pi/2$ between neighbours. The equivalent grounding stiffness for the n-th mass thus can be modelled as:

$$k_{g,n} = k_{g,DC} + k_{g,AC}\cos\left(2\pi f_{mod}t \mp \frac{\pi x_{0,n}}{2a}\right) = k_{g,DC} + k_{g,AC}\cos(2\pi f_{mod}t \mp q_{mod}n) \quad (1)$$

where $k_{g,DC}$ is the small time-independent grounding stiffness added by the on-site electromagnetic force, $k_{g,AC}$ is the modulation amplitude of the grounding stiffness, $x_{0,n}$ is the equilibrium position of each unit, and $q_{mod} = \pm\pi/2$ is the normalized wave number. Equation (1) describes a traveling wave with wavelength $\lambda_{mod} = 4a$ and speed $v_{mod} = 4af_{mod}$. The modulation amplitude measured in our experiments is $k_{g,AC} = 24$ N/m, which is 21% of the coupling stiffness, k_c. The constant part of the grounding stiffness is $k_{g,DC} = 2.4$ N/m, which is one order of magnitude smaller than the oscillatory component.

In the absence of modulation ($k_{g,AC} = 0$), the dispersion relation for an incident small-amplitude plane wave $u_0(n,t) = U_0\exp(i(qn - \omega t))$ is described by $D(\omega, q) = k_{g,DC} - m\omega^2 + 4k_c\sin^2\left(\frac{q}{2}\right) = 0$. Modulating the lattice harmonically with (f_{mod}, q_{mod}) generates an additional scattered field $u_s(n,t) = U_s\exp\big(i(q_s n - \omega_s t)\big)$ whose mode is *shifted* by an amount (ω_{mod}, q_{mod}) due to spatiotemporal periodicity: $(\omega_s, q_s) = (\omega_0, q_0) \pm (\omega_{mod}, q_{mod})$. The scattered field is negligible however $(U_s \ll U_0)$ except when it is resonant with the incident field; i.e., when the modified Bragg's condition $D(\omega_s, q_s) = D(\omega_0, q_0) = 0$ is met [22]. Graphically, scattered modes are located at crossings between the original ($D(\omega, q) = 0$) and shifted ($D(\omega_s, q_s) = 0$) dispersion curves. Note that the crossings are non-symmetrically distributed in a way that breaks parity of the dispersion diagram and, ultimately, reciprocity of wave propagation. Depending on whether $q_0 q_s$ is positive or negative, the scattered mode propagates either with or against the incident wave, i.e., is either transmitted or reflected. In both cases however, its frequency is shifted away from the incident frequency ω_0. This translates into a one-way dip in the transmission spectrum around ω_0.

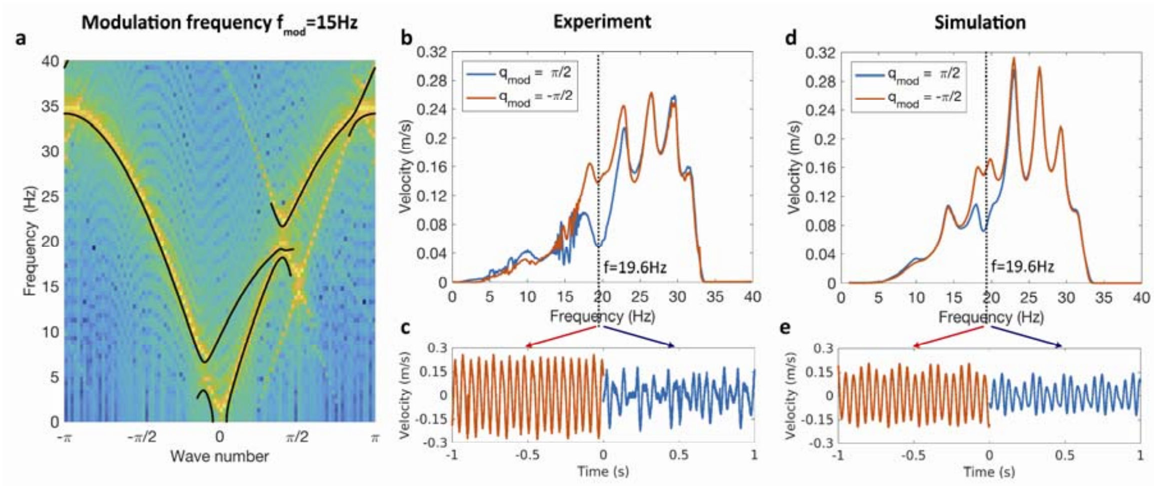

Figure 2 Non-reciprocal wave propagation for $f_{mod} = 15$ Hz. (a) Dispersion diagram of the modulated lattice calculated by Fourier analysis of simulated velocity fields (color map) and analytically by coupled mode theory (solid black line). (b) Measured velocity response function. The amplitude ratio at 19.6 Hz is $r = 2.9$. (c) Measured velocity time series at $f_{dr} = 19.6$ Hz. The time series for $q_{mod} = -\pi/2$ is shown along the negative time axis for better illustration. (d) and (e) are the simulation results corresponding to (b) and (c), respectively. The simulated amplitude ratio at 19.6 Hz is $r = 1.9$ in panel (d).

We first set the modulation frequency to $f_{mod} = 15$ Hz, which falls within the pass band of the monoatomic lattice. For this modulation frequency, three crossings exist at 5 Hz, 19 Hz and 33 Hz and non-reciprocal wave characteristics are anticipated for neighboring driving frequencies f_{dr} as shown in Fig. 2a. We measure the velocity of the last mass in the array as a function of the driving frequency, f_{dr} in Fig. 2b. The velocity profiles differ when the acoustic waves are traveling in the same (red) or opposite (blue) direction to the modulation wave, at driving frequencies close to $f_{dr} = 19.6$ Hz. We define the co-directional/contradirectional bias ratio as $r = U^-/U^+$ where $U^\mp$ denotes the velocity response amplitude for $q_{mod} = \mp\pi/2$. At $f_{dr} = 19.6$ Hz, the measured velocity response profile in time shows that waves traveling in opposite directions have different amplitudes and profiles, with a bias of $r \approx 2.9$, shown in Figs. 2b, c. The time-domain amplitudes are lower than the amplitudes obtained from the velocity response functions. This is due to the anharmonic nature of the response in the modulated lattice. However, results demonstrate that the signal transfer around $f_{dr} = 19.6$ Hz is strongly enhanced when traveling along the modulation direction and suppressed in the other direction, thus exhibiting a non-reciprocal behavior.

We developed a mathematical model to capture the dynamic characteristics of the modulated lattice. The system can be described as:

$$m\ddot{u}_n + F_{loss} + k_{g,n}u_n + F_{coupl} = \delta_{2,n}A\cos(2\pi f_{dr}t) \qquad (2)$$

for $1 \leq n \leq 12$. Here, $u_n(t) = 0$ at the two boundaries $n = 1,12$. $F_{loss} = b\dot{u}_n + \mu\,\mathrm{sign}(u_n)$ represents dissipative forces within the chain, with viscous damping coefficient $b = 0.056$ kg/s and Coulomb friction coefficient $\mu = 0.012$ N (see Supplemental Material). The coupling force term is $F_{coupl} = P(a - u_n + u_{n+1}) - P(a - u_{n-1} + u_n)$, where we use the approximation $P(x) = c_1/(x - c_2)^2$ with $c_1 = 0.9788$ mNm^2 and $c_2 = 7.748$ mm obtained from a fitting based on Fig. 1c. $\delta_{2,n}$ is the Kronecker delta which is 1 for $n = 2$ and zero everywhere else. The forcing amplitude $A = 0.21$ N is obtained as a fitting parameter. At this value of the forcing amplitude, the response of the system is well approximated by the linearized equations of motion (the contribution from nonlinearity is discussed in the Supplemental Material). The experimental and numerical velocity response functions for a non-modulated lattice agree well [28] (see Supplemental Material). When the modulation is turned on, the velocity profiles obtained in experiments and simulations show a similar nonreciprocal response in Figs. 2d, e. However, the non-reciprocal behavior at $f_{dr} = 19.6$ Hz is less pronounced in simulations than in measurements ($r \approx 1.9$).

We computed dispersion curves from space-time Fourier analysis of the velocity field and compared them with the ones obtained with the plane-wave expansion method in Fig. 2a. The observed non-reciprocal wave characteristics, at $f_{dr} = 19.6$ Hz, agree well with the dispersion characteristics. The dispersion curves in Figs. 1b & 2a predict non-reciprocal behavior also near 5 Hz and 33 Hz. However, the experimental velocities are too small at these frequencies to capture the effect. Note that the analyses (numerical and theoretical) on an infinite lossless lattice (Fig 2a) predicted the same frequency range for non-reciprocal wave propagation as the experiments (Fig 2b) and simulations (Fig 2d) on a finite lossy lattice. The effects of energy loss and finite number of units are therefore secondary to modulation effects; see Supplemental Material for discussions of finite-size and loss effects.

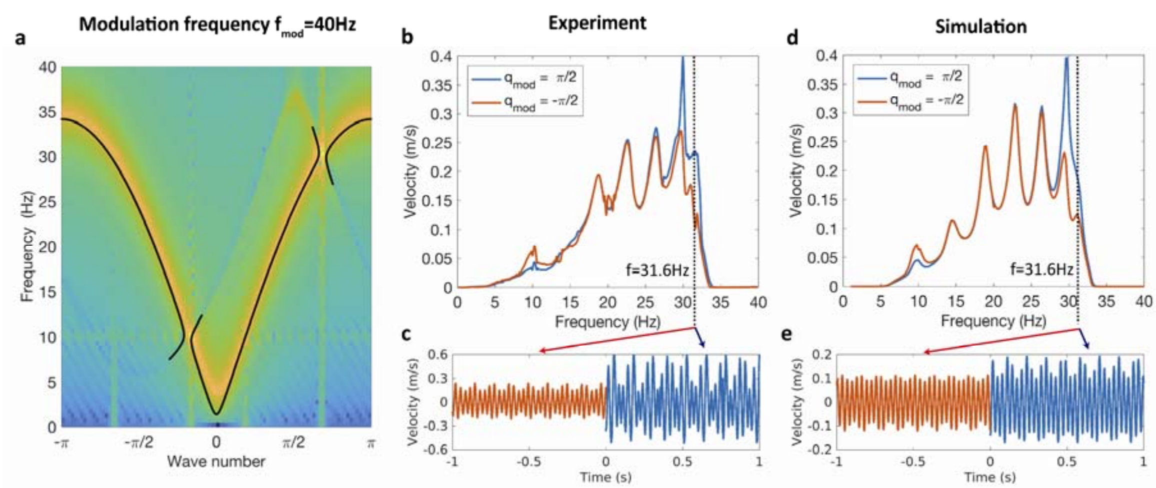

Figure 3. Non-reciprocal wave propagation for $f_{mod} = 40$ Hz. (a) Dispersion diagram of the modulated lattice calculated by Fourier analysis of simulated velocity fields (color map) and analytically by coupled mode theory (solid black line). (b) Measured velocity response function. The amplitude ratios are $r = 1.8$ at 9.8 Hz and $r = 0.4$ at 31.6 Hz. (c) Measured velocity time series at $f_{dr} = 31.6$ Hz. The time series for $q_{mod} = -\pi\ /2$ is shown along the negative time axis for better illustration. (d) and (e) are the simulation results corresponding to (b) and (c), respectively. The simulated bias ratios are $r = 1.6$ at 9.8 Hz and $r = 0.7$ at 31.6 Hz in panel (c).

In order to demonstrate the tunability of the non-reciprocal frequency range in our system, we next set the modulation frequency to $f_{mod} = 40$ Hz, within the band gap of the underlying monatomic lattice. Our model predicts non-reciprocal wave behavior for driving frequencies near the crossings at 10 Hz and 30 Hz as shown in Fig. 3a. This is also captured in the measured velocity responses in Fig. 3b and time domain profiles at $f_{dr} = 31.6$ Hz in Fig. 3c. Corresponding numerical simulations in Figs. 3d,e agree very well with the measurements.

The dispersion curve of the modulated lattice in Fig. 3a obtained from numerical calculation corroborates the observed non-reciprocal characteristics for $f_{mod} = 40$ Hz. It reveals two crossings located near 30 Hz and 10 Hz and visible as small bright yellow regions lying on a main dispersion branch. At these points, the modulation-induced scattered field is strong enough to change the overall wave field. This manifests in the velocity response functions as $r > 1$ near 10 Hz and $r < 1$ near 30 Hz. For other points along the main dispersion branch, the scattered wave is too weak compared to the incident field to induce any noticeable non-reciprocal effects. In contrast to the case for $f_{mod} = 15$ Hz, the crossing here occurs between a positive and a negative branch of the dispersion curve ($\omega_0 \omega_s < 0$) and leads to the opening of a couple of "vertical" bandgaps as shown in Fig. 3a. Such crossings in infinite loss-less systems are characteristic of unstable interactions caused by supersonic modulation velocities, where the velocity field is continuously amplified by drawing energy from the modulation [31,32]. However, our experimental system is intrinsically lossy and finite, and remains stable in the studied regime. The presence of losses is known to quench instabilities [33]. In our system, this translates in the presence of a sharp peak around 30 Hz in the transmission spectrum, shown in Figs. 3b,d.

In conclusion, our results provide an experimental demonstration of modulation-induced non-reciprocity in a linear phononic lattice. The operating range of our lattice is beyond the asymptotic limits that are typically enforced in the existing theoretical work. The experimental realization of dynamically modulated nonreciprocal systems opens new opportunities for sound and vibration insulation [11,12,15], phononic logic [13,14] and energy localization and trapping [34]. In the future, the phononic waveguide developed in our work could be employed to study the nonlinear dynamics of modulated lattices, a regime that has not been explored before. The design could also be miniaturized into micro- or nano-scale electromechanical systems [35-37] with tunable frequencies as basic elements for acoustic rectifying circuits.

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Author contributions

Y.W. and C.D. designed the experiment. Y.W. performed the experiments. B.Y. performed analytical and numerical modelling of the system. H.C., H.N. and G.H. performed analytical calculations on the dispersion curves. Y.W., B.Y., H.N., G.H. and C.D. wrote the manuscript. All authors interpreted the results and reviewed the manuscript.

Competing Financial Interests

Nothing to report.

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