Nonlocal Spin Transport Mediated by a Vortex Liquid in Superconductors

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Nonlocal Spin Transport Mediated by a Vortex Liquid in Superconductors

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Departing from the conventional view on superconducting vortices as a parasitic source of dissipation for charge transport, we propose to use mobile vortices as topologically-stable information carriers for spin transport. To this end, we start by constructing a phenomenological theory for the interconversion between spin and vorticity, a topological charge carried by vortices, at the interface between a magnetic insulator and a superconductor, by invoking the interfacial spin Hall effect therein. We then show that a vortex liquid in superconductors can serve as a spin-transport channel between two magnetic insulators by encoding spin information in the vorticity. The vortex-mediated nonlocal signal between the two magnetic insulators is shown to decay algebraically as a function of their separation, contrasting with the exponential decay of the quasiparticle-mediated spin transport. We envision that hydrodynamics of topological excitations, such as vortices in superconductors and domain walls in magnets, may serve as a universal framework to discuss long-range transport properties of ordered materials.

Introduction.—Superconductivity refers to collective charge transport, whose stability is rooted in the topology of the order-parameter field [1]. Its dissipation, which requires discrete changes in the superconducting phase winding, is impeded by the associated energy barrier. It has been one of the central topics in physics because of practical motivations, e.g., for long-distance power transmission, as well as fundamental interest in quantum phases of matter. A superconductor loses its ability for lossless transport when the phase coherence of the condensate wavefunction is destroyed by the proliferation of vortices, topological phase defects [2, 3]. Since the motion of vortices gives rise to a finite resistance in superconductors, a central goal in the materials engineering of superconductors has been to immobilize them by engineering pinning defects [4].

In contrast to the conventional, antagonistic view on vortices in superconductors for charge transport, in this Letter, we propose to use mobile superconducting vortices as robust information carriers for spin transport, which are endowed with a stability by their topological characteristics. Analogous research has been done previously for topological solitons in magnets, such as domain walls [5] and skyrmions [6]. Topological magnetic solitons can store information in their topological charges: the chirality of a domain wall and the winding number of a skyrmion. The stability associated with the topological characteristics allows them to transport information over relatively long distances, compared to quasiparticles, such as magnons with a finite lifetime, giving rise to an algebraically decaying nonlocal transport [7, 8].

In this Letter, we show that topological defects in superconductors, vortices, can transport spin information efficiently by encoding it in their topological charge that is referred to as vorticity. We are interested in vortex liquids (VLs), with a focus on planar geometries, which can be found in conventional superconductors such as Nb or high T_c superconductors such as $La_{2-x}Sr_xCuO_4$ [2, 9]. One promising material candidate is the borocarbides such as YNi_2B_2C, which are known to have weak pinning for vortex flow [10]. For example, the schematic mean-field phase diagram of bulk type-II superconductors is shown in Fig. 1(b) [2]. The mixed phase harbors superconducting vortices, and they can form various states of matter such as vortex lattice or vortex liquid [11]. Specifically, first, we develop a phenomenological theory for the interconversion between spin in a magnetic insulator and vorticity in a superconductor at the interface, which is schematically illustrated in Fig. 1(a).

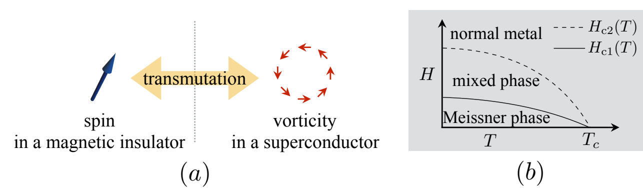

FIG. 1. (a) A schematic of spin-vorticity transmutation at the interface between a magnetic insulator and a superconductor. The blue arrow represents spin in a magnetic insulator and the red arrows depict the phase of a superconducting vortex. (b) A schematic of the mean-field phase diagram of bulk type-II superconductors [2], which comprises the Meissner phase at low magnetic fields $H < H_{c1}$ with complete expulsion of the magnetic flux, the mixed phase at intermediate fields $H_{c1} < H < H_{c2}$, where the magnetic flux penetrates into a superconductor in the form of vortices, and the normal-metal phase at high fields $H > H_{c2}$, where superconductivity is destroyed. T_c is the critical temperature for the onset of superconductivity. Vortices in the mixed phase can form several states of matter, including vortex liquid.

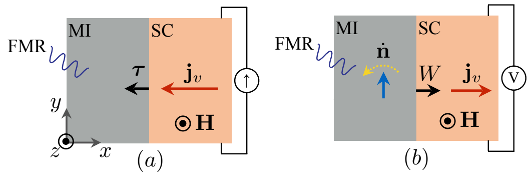

FIG. 2. Schematics of experimental setups for probing spin-vorticity transmutation at the interface between a magnetic insulator (MI) and a superconductor (SC) subjected to an external magnetic field $\mathbf{H}$. (a) The torque $\boldsymbol{\tau}$ [Eq. (2a)] on MI exerted by the vorticity flux $\mathbf{j}_v$ in SC can be probed by performing ferromagnetic resonance (FMR) measurements on MI, while applying a transverse current to SC, which generates a longitudinal vorticity flux $\mathbf{j}_v$ via the Lorentz force [13]. (b) The vorticity-dependent work W [Eq. (2b)] performed by magnetic dynamics $\dot{\mathbf{n}}$ on vortices creates the nonequilibrium vorticity accumulation at the interface and thereby induces a diffusive vorticity flux $\mathbf{j}_v$. The vorticity flux $\mathbf{j}_v$ can be probed by measuring a transverse voltage drop across the SC, which is induced by $\mathbf{j}_v$ via the Josephson effect [13], while driving FMR dynamics in the MI. In addition, the energy dissipation through vortex generation suggests a new channel for Gilbert damping of MI, which can be manifested through the enhancement FMR linewidth [14].

One process for the interconversion, which occurs via the interfacial spin Hall effect, is given for concreteness. Second, based on the aforementioned theory for spin-vorticity interconversion, we show that vortices in a superconductor can support algebraically-decaying nonlocal spin transport between two distant magnetic insulators sandwiching a superconductor. We will conclude the Letter by discussing other possible mechanisms for spin-vorticity interconversion and providing some future outlooks. We envision that the field of superconducting spintronics [12], in which the interaction between a magnet and a superconductor has been explored mainly focusing on spin-polarization of quasiparticles, can be enriched by incorporating the hitherto largely ignored objects—vortices—as active ingredients along with a natural spin-vorticity transmutation. The materials library of previously considered low-performance superconductors (with mobile vortices), for example those with small lower critical field H_{c1}, can be revisited for their potential as spin-vortex conversion layers.

Spin and vorticity.—We provide a phenomenological theory for the interconversion between spin in a magnetic insulator (MI) and vorticity in a superconductor (SC) harboring a vortex liquid [11]. For concreteness, we consider materials lying on the xy plane that share two-dimensional interfaces in the yz plane, and will assume that the magnetic order parameter and the superconducting wavefunction are uniform along the z direction. See Fig. 2 for schematics. A vortex in a superconductor is characterized by its vorticity,

$$q = \frac{1}{2\pi} \oint d\mathbf{r} \cdot \boldsymbol{\nabla}\phi \,, \tag{1}$$

an integer number measuring how many times the phase ϕ of the condensate wavefunction winds the unit circle when moving along a closed line encircling the vortex core. We focus on the elementary vortices with the unit vorticity, $q = \pm 1$, since the other vortices are suppressed energetically (while elementary vortices with the same charge repel each other).

When the wavefunction is well defined everywhere, the total vorticity is conserved due to its topological nature, which allows us to use the hydrodynamic theory to describe their macroscopic dynamics. The relevant hydrodynamic variables are the vorticity density $\rho_v = \rho_+ - \rho_-$ (per unit area) [15], and the vorticity-current density $\mathbf{j}_v = \mathbf{j}_+ - \mathbf{j}_-$ (per unit length), where ρ_q and $\mathbf{j}_q$ are the number density and the current density of vortices with the vorticity $q = \pm 1$. At temperature T, vortices are nucleated and annihilated by thermal fluctuations, giving $\rho_q^0 \propto \exp(-E_q/k_B T)$ at sufficiently low temperatures in equilibrium, where E_q is the energy of a vortex with the vorticity q and k_B is the Boltzmann constant. In equilibrium, the vorticity current vanishes, $\mathbf{j}_v \equiv 0$, but the vorticity density can be finite if the energy of a vortex depends on the vorticity, $E_+ \neq E_-$, due to, e.g., an external magnetic field.

Our main results for the spin-vorticity interconversion, which we obtain below, can be summarized as follows:

$$\boldsymbol{\tau} = (g' + g\mathbf{n}\times)(\mathbf{n} \times j_v \hat{\mathbf{z}}) \,, \tag{2a}$$
$$W = -q[(g' + g\mathbf{n}\times)\dot{\mathbf{n}}] \cdot \hat{\mathbf{z}} \,, \tag{2b}$$

which are related by Onsager reciprocity. The first equation describes the spin torque (per unit length) on the magnetic material induced by the vorticity-current density j_v toward the magnetic insulator in the perpendicular direction to the interface, which is equal to the annihilation rate of vortices per unit length. The phenomenological coefficients g and g' have the unit of the angular momentum, which quantify the amount of spin transferred to the magnetic material by annihilation of one unit of the vorticity in a dissipative and reactive fashion, respectively. The second equation describes the work W performed by slow magnetic dynamics $\dot{\mathbf{n}}$ on a single nucleated vortex with the vorticity $q = \pm 1$ entering the superconductor through the interface. Schematics of experimental setups for probing our results for the spin-vorticity interconversion are shown in Figs. 2.

One concrete toy model for the above results, which will be given below, connects spin in a magnetic insulator and vorticity in a superconductor via charge at the interface. The central step to understand the processes is to recognize that a region (denoted by SH in Fig. 3) of the superconductor interfacing with the magnetic insulator will be subject to the interfacial spin Hall effects [16, 17], by which a normal charge current can induce the spin-transfer torque on the magnetic insulator and, reciprocally, the magnetic dynamics can induce the charge current in the superconductor. The effective thickness of the region SH will be denoted by t. Here, we would like to emphasize that this is just a toy model, in

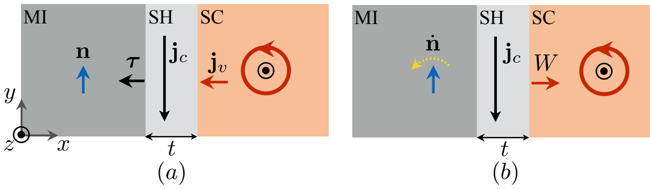

FIG. 3. (a) The vorticity-current density $\mathbf{j}_v$ induces the charge-current density $\mathbf{j}_c$ [Eq. (3)] in SH via the Josephson effect, which in turn exerts the torque $\boldsymbol{\tau}$ [Eq. (4)] on the MI via the spin Hall effect. (b) The dynamics of the MI $\dot{\mathbf{n}}$ induces the normal charge-current density $\mathbf{j}_c$ [Eq. (5)] in SH via the inverse spin Hall effect. The counter-propagating supercurrent then performs the vorticity-dependent work W [Eq. (6)] on a vortex via the Lorentz force.

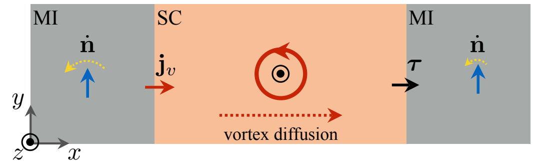

FIG. 4. A schematic of the nonlocal spin transport between two MIs mediated by a vortex liquid in the interconnecting SC. The dynamics of the magnetization $\mathbf{n}$ of the left MI pumps the vorticity current $\mathbf{j}_v$ into the SC. Some of the pumped vortices travel across the SC by thermal diffusion and reach the interface to the right MI, exerting the torque $\boldsymbol{\tau}$ to it. The width of the SC needs to be much larger than the length to prevent vortices from escaping through sides.

which the region SH is introduced conceptually to connect spin and vorticity via charge with separate accounts of spin-charge coupling and charge-vortex coupling. The phenomenology in Eqs. (2) should work generally, subject to any spin-orbit coupling at the interface.

Let us first describe the process for the torque on the magnetic insulator induced by the vorticity current in the superconductor. See Fig. 3(a) for the schematic geometry. First, the vorticity-current density $\mathbf{j}_v = -j_v \hat{\mathbf{x}}$ induces the transverse electric field, $\mathbf{E} = \mathbf{j}_v \times \Phi_0 \hat{\mathbf{z}}$ [13], manifesting the Josephson relation via the vortex flow. Here, $\Phi_0 = h/2e$ is the magnetic flux quantum, in terms of the Planck constant h and the magnitude of the electron charge $e > 0$. The induced diffusive charge-current density carried by the normal component in the region SH [18] is then given by

$$\mathbf{j}_c = \sigma \mathbf{E} = \sigma \Phi_0 j_v \hat{\mathbf{y}} \,. \tag{3}$$

Here, $\sigma\Phi_0$ has the unit of electric charge, parametrizing the interconversion efficiency from the vorticity current to the charge current. Due to the spin Hall effects, the charge current parallel to the interface gives rise to the torque on the MI, which can be written as

$$\boldsymbol{\tau} = (\sigma \Phi_0 t)(\eta + \vartheta \mathbf{n} \times)(\mathbf{n} \times j_v \hat{\mathbf{z}}) \,, \tag{4}$$

within the spin Hall phenomenology [16], where η and ϑ quantify the reactive and dissipative torques, respectively. Here, j_v is the rate of annihilation of vortices at the interface MI|SH per unit length. By comparing the obtained expression to the first equation in the main results [Eq. (2)], we can identify the coefficients: $g = \sigma\Phi_0 t \vartheta$ and $g' = \sigma \Phi_0 t \eta$ are the spin angular momentum transferred to the magnetic insulator during the annihilation of one vorticity via the dissipative and the reactive processes, respectively. The dissipative coefficient ϑ can be written as $\vartheta = (\hbar/2et)\tan\Theta$, with Θ identified as the effective spin Hall angle in the SH region. If we use the material parameters of platinum, $\sigma \sim 10^7\ (\Omega\mathrm{m})^{-1}$ and $\Theta \sim 0.1$ [19], we obtain $g \sim 600\,\hbar$; the annihilation of a single vortex can pump hundreds of spins (in units of $\hbar$).

Next, let us turn to the reciprocal process for the work on a vortex done by the magnetic dynamics. The dynamics of the MI gives rise to the electromotive force in the region SH, $\boldsymbol{\epsilon} = -[(\eta + \vartheta \mathbf{n}\times)\dot{\mathbf{n}}] \times \hat{\mathbf{x}}$, via the spin Hall effects [16]. Here, $\hat{\mathbf{x}}$ is the unit vector perpendicular to the interface plane. It subsequently induces the diffusive charge-current density in the region SH,

$$\mathbf{j}_c = -\sigma \left\{ [(\eta + \vartheta \mathbf{n}\times)\dot{\mathbf{n}}] \cdot \hat{\mathbf{z}} \right\} \hat{\mathbf{y}} \,. \tag{5}$$

In the steady state, where there must be no net charge flow, this diffusive current is counterbalanced by the super current, $\mathbf{I}_s = -t \mathbf{j}_c$.

The induced super current $\mathbf{I}_s$ then exerts the transverse Lorentz force on a vortex [13], which can be related to the Josephson relation mentioned above or, equivalently, Faraday's law by invoking Onsager reciprocity. The work performed by the Lorentz force on a vortex with vorticity q is given by

$$W = -q \left(\mathbf{I}_s \times \Phi_0 \hat{\mathbf{z}} \right) \cdot \hat{\mathbf{x}} = -q(\sigma \Phi_0 t)[(\eta + \vartheta \mathbf{n}\times)\dot{\mathbf{n}}] \cdot \hat{\mathbf{z}} \,, \tag{6}$$

yielding Eq. (2b), as expected from Onsager reciprocity.

Nonlocal spin transport by a vortex liquid.—The vorticity pumping by the magnetic dynamics and the reciprocal torque by the vorticity flux can be used to transport spin between two distant magnetic insulators via a vortex liquid in superconductors, which we shall study below based on our results in Eqs. (2). The geometry that we consider is schematically drawn in Fig. 4.

The left MI is a ferromagnet at resonance with a rf field under a magnetic field in the z direction, which serves as the spin battery [20] in our setup. The order parameter $\mathbf{n}$ of the left MI precesses with the cone angle θ around the z axis at frequency ω. According to Eq. (2b), the magnetic order-parameter dynamics performs the vorticity-dependent work, $W_\pm = \mp g \omega \sin^2\theta$, on a superconducting vortex when it enters the superconductor. Here, the reactive part $\propto g'$ vanishes after being averaged over time. This vorticity-dependent work pumps the vorticity into the superconductor through the interface according to the reaction-rate theory as follows [21, 22]. The nucleation rate for a vortex with vorticity $q = \pm 1$ per unit length along the y direction is described by $\Gamma_\pm = \nu \exp(-E_\pm/k_B T)$, where ν is the attempt frequency and $E_\pm$ is the energy barrier for the nucleation of a vortex. The annihilation rate per unit length is described by $\gamma\rho_\pm$, where γ is the annihilation rate per unit density that does not depend on the vorticity. In equilibrium, the nucleation and the annihilation rates must be equal, giving $\Gamma_\pm = \gamma\rho_\pm$.

The dynamics of the adjacent magnetic insulator breaks the balance as follows. The work done by the magnetic dynamics of the left MI modifies the energy barrier, $E_\pm = E_\pm^0 - W_\pm$, from its equilibrium value $E_\pm^0$ by which the nucleation rate changes as well, $\Gamma_\pm = \Gamma_\pm^0(1 \mp g\omega \sin^2\theta/k_B T)$ in linear order. Then, the net injection rate of the vorticity (per unit length) is given by $-(\Gamma_+^0 + \Gamma_-^0) g\omega \sin^2\theta/k_B T = -\gamma\rho^0 g\omega \sin^2\theta/k_B T$. Therefore, the vorticity-current density at the interface of MI|SC is given by

$$j_v(x=0) = -\gamma\rho^0 g\omega \sin^2\theta/k_B T - \gamma\delta\rho_v(x=0)\,, \tag{7}$$

where $\delta\rho_v \equiv \rho_v - \rho_v^0$ is the nonequilibrium vorticity density. Here and after, j_v represents the component of the vorticity-current density in the x direction, normal to the interfaces between MIs and SC. The first term on the right-hand side is the vorticity pumped by the magnetic dynamics; the second is the annihilation rate of the nonequilibrium vorticity density. The pumped vorticity diffuses through the bulk of SC, satisfing the continuity equation: $\partial_t \delta\rho_v + \partial_x j_v = 0$, which is rooted in the conservation of the topological charge, the net vorticity. We assume that the dynamics of vorticity is purely diffusive:

$$j_v = -D\partial_x \delta\rho_v\,, \tag{8}$$

where D is the diffusion coefficient of the vorticity. The vorticity-current density at the interface between SC and the right MI is given by

$$j_v(x=L) = \gamma\delta\rho_v(x=L)\,. \tag{9}$$

In the steady state, the vorticity density is constant and the vorticity-current density is uniform. By solving Eqs. (7), (8), and (9) for the uniform j_v, we obtain

$$j_v = -\frac{\gamma g\omega \sin^2\theta}{2+\gamma L/D}\frac{\rho^0}{k_B T}\,. \tag{10}$$

When SC is sufficiently short, $L \ll D/\gamma$, about half of the pumped vorticity $-\gamma\rho^0 g\omega\sin^2\theta/k_B T$ by the dynamics of the left MI passes through the superconductor and leaves the superconductor through the interface to the right MI. The vorticity annihilation at the interface SC|MI exerts the torque on the right MI, which can be obtained from Eq. (2a). The resultant antidamping torque can induce the dynamics of the right MI, e.g., by driving it into an auto-oscillation phase [23]. The modulation of the torque can also be detected magnetoresistively by a lock-in method.

Discussion.—In this Letter, we formulated symmetry- and topology-based principles for the spin-vortex interconversion at the edge of a superconductor. Focusing on spin injection and detection via coherent magnetic dynamics, we constructed a general phenomenology [cf. Eqs. (2)] for its coupling to the vortex flow through the interface. A specific, spin-Hall-effect-based microscopic model was constructed, as a proof of principle, to calculate the coupling coefficients, which are shown to naturally obey the Onsager reciprocity. There can be other mechanisms that can give rise to the spin-vortex interconversion. For example, a vortex in a superconductor can harbor spin-polarized normal quasiparticles in its core, due to Pauli susceptibility [24]. The spin angular momentum of quasiparticles in the vortex core can be directly transferred to the proximal magnetic order parameter when the vortex is annihilated at its interface, and vice versa. Recently, Vargunin and Silaev [25] invoked this spin polarization of vortices to predict that superconductors with the vortex Nernst effect could exhibit a spin Hall effect. In addition, although the present work is focused on the vortex-liquid phase of type-II superconductors, it can be extended to the other phases such as the vortex-solid phase as done for the previous work on nonlocal information transport by the elastic response of magnetic skyrmion crystals [26]. We remark here that spin-vorticity interconversion has been studied also for mechanical rotations of fluids [27].

To check the experimental feasibility, let us provide a numerical estimate for the spin torque induced by the vortex flow shown in Fig. 2(a). In Ref. [28], the vortex velocity $\sim 10^3$ m/s was reported in a high T_c material $YBa_2Cu_3O_{7-\delta}$ at 1 T, where the intervortex distance is about 20 nm. The corresponding vortex-current density (per unit length) is $j_v \sim 3\times 10^{18}$ $(\mathrm{ms})^{-1}$. For the MI, we consider yttirium-iron-garnet thin films with thickness 20 nm (along the x axis) and width 1 μm (along the z axis), which has the spin density $s \sim 10\hbar/\mathrm{nm}^3$ [29], the FMR frequency ~ 3 GHz [30], and the Gilbert damping constant $\alpha \sim 10^{-3}$ [30]. Then, the resultant damping-like torque [Eq. (2a)] with $g \sim 600$ $\hbar$ corresponds to the damping parameter $\delta\alpha \sim 5\times 10^{-4}$, which, being larger than the natural damping parameter of some common magnetic insulators, can be detected by the FMR linewidth measurements [31].

Let us make a few remarks regarding experiments. First, since the studied phenomenon of spin-vorticity transmutation is based on the vortex flow, the temperature- and the magnetic-field-dependences of the predicted experimental signals will be similar to those of the vortex Nernst effect caused by the thermal vortex flow, which can be found in, e.g., Fig. 3(b) of Ref. [32] and Fig. 2 of Ref. [33]. Secondly, cleaner superconductors with little pinning will be better for the efficiency of spin-vorticity transmutation, which is in contrast to the usual strategies for suppressing the vortex-mediated phase slips. Thirdly, we would like to mention that the vortex flow in a SC can be induced by applying a temperature gradient [32] and can be facilitated by engineering the geometry, e.g., creating a thickness gradient [34].

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