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Brownian dynamics of particles “dressed” by chiral director configurations in lyotropic chromonic liquid crystals

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We study Brownian dynamics of colloidal spheres, with planar anchoring conditions, suspended in the nematic phase of the lyotropic chromonic liquid crystal disodium chromoglycate (DSCG). Unlike typical liquid crystals, the unusually small twist elastic modulus of DSCG permits two energetically distinct helical distortions (twisted tails) of the nematic director to “dress” the suspended spheres. Video microscopy is used to characterize the helical distortions versus particle size and to measure particle mean-square displacements. Diffusion coefficients parallel and perpendicular to the far-field director, and their anisotropy ratio, are different for the two twisted tail configurations. Moreover, the cross-over from sub-diffusive to diffusive behavior is anomalously slow for motion perpendicular to the director (>100 seconds). Simple arguments using Miesowicz viscosities and ideas about twist relaxation are suggested to understand the mean-square displacement observations.

The investigation of Brownian dynamics boasts a spectacular history that has produced deep insights about statistical mechanics and about the microscopic environments of diffusing particles [1–4]. Although most of this work concerns the motion of spheres in isotropic media, the effects of anisotropy in Brownian dynamics are proving to be fascinating as well, including the diffusion of anisotropic particles in isotropic fluids [5–10] and particles of all types in anisotropic fluids [11–17].

In this contribution, we explore diffusion of spherical particles in the lyotropic chromonic liquid crystal (LCLC) disodium cromoglycate (DSCG). This background fluid is a nematic liquid crystal (LC) composed of self-assembled achiral rod-like molecular assemblies aligned uniaxially along a director, \mathbf{n} (Figure 1a) [18, 19]. As in all LC systems, particles in DSCG are expected to exhibit different coefficients for diffusion parallel versus perpendicular to the director field [11, 13, 14, 16, 20]. Lyotropic chromonic liquid crystals are different from typical LCs, however, because they easily form twisted or chiral configurations due to their unusually small twist elastic modulus [21–23], an effect also seen in polymer nematic LC systems [24, 25]. Thus the local environment for a sphere diffusing in DSCG has a chiral character that depends sensitively on the interplay of bulk elasticity, local geometry, and director boundary conditions. Indeed, it has been shown [26] that spherical colloidal particles with planar anchoring in the nematic phase of DSCG induce localized helical distortions, *i.e.*, “twisted tails” that extend away from the particle parallel to the uniform far-field director \mathbf{n}_0 (Figure 1b-h). The energetics and symmetries of these “twisted tail” configurations have been studied [26, 27], but to our knowledge the effects of chiral distortion of the director on unconstrained particle diffusion have not been considered nor investigated. Moreover, the influence of giant twist elastic anisotropy on relaxation dynamics, broadly defined, has never been explored.

To this end we employ video and polarized optical

microscopy (POM) to investigate diffusion of spheres “dressed” by two classes of chiral director configurations with twisted tails. Diffusion coefficients, parallel and perpendicular to the far-field director, as well as diffusion anisotropy, are revealed to be different for the two classes. In addition, the short-time diffusion dynamics show sub-diffusive scaling for motion parallel and perpendicular to the far-field director, and they exhibit anomalously long relaxation times (>100 seconds) for diffusion perpendicular to \mathbf{n}_0 . We offer qualitative arguments to understand the measured diffusion coefficient anisotropy based on the Miesowicz viscosities of LCLCs, and we show how giant twist elastic anisotropy can introduce differences in the diffusion behavior of the dressed particles. Our results complement and differ from diffusion measurements, reported in Ref. [16], of heavy silica spheres very close to the substrate without director configuration class identified (see Supplementary Information (SI), Section 4). By positioning particles farther from the substrates, our experiments ameliorate surface drag effects and probe the bulk LC response more effectively. These improvements enable us to uncover substantial variation in subdiffusivity parallel versus perpendicular to \mathbf{n}_0 , and to distinguish diffusion of particles dressed by different director field configurations. Broadly, the present work elucidates how chiral structures that readily form in LCLCs can affect the dynamics of embedded micro- and nano-particles which, in turn, influences our ability to control particles in LCs for potential device applications [28, 29].

Our experiments (see also, SI) employ polystyrene divinylbenzene (DVB) spheres in DSCG, with diameters, d , ranging from 6.5 to 7 μm (unless otherwise specified); typical polydispersity in the source samples is $\sim 2.4\%$. DSCG molecules are plank-like (inset, Figure 1a). The suspensions consist of ~ 16 wt.% DSCG and ~ 0.015 wt.% X100 surfactant in DI water to prevent the formation of particle clusters. The sample suspensions are loaded into rectangular cells with two rubbed glass substrates, or with one rubbed glass substrate and one

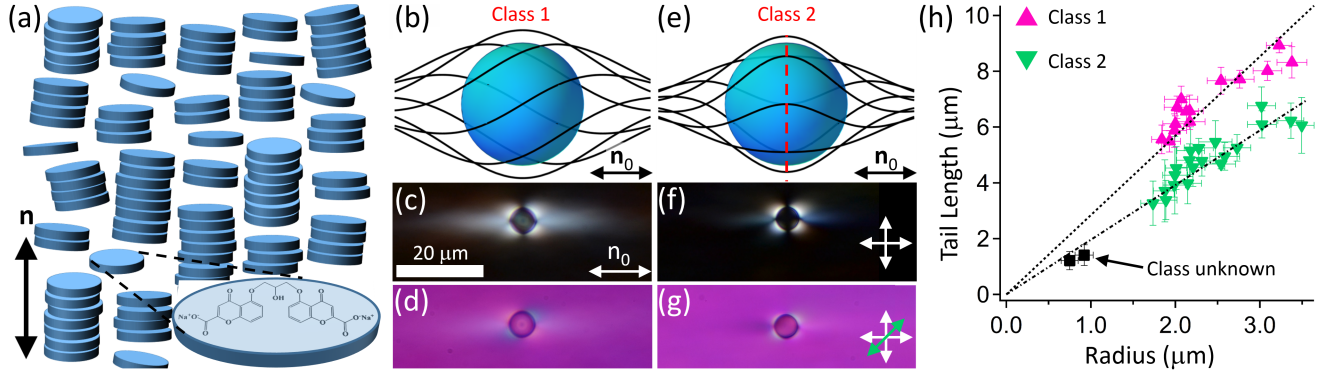


FIG. 1. (a) Illustration of the nematic phase of DSCG. The nematic mesogens are stacks of plank-like molecules. (b-g) A spherical particle with planar boundary conditions suspended in the uniaxially aligned nematic phase of DSCG induces two distinct twisted tail director configurations: class 1 (b, c, d) and class 2 (e, f, g). (b) and (e) are schematics of twisted director fields (black lines) around the particle. (c) and (f) are crossed-polarized (white arrows) microscopy images. (d) and (g) are crossed-polarized microscopy images with an inserted full waveplate (green arrow). (h) Tail length plotted as a function of particle radius for both symmetry classes. Dotted lines show best proportional fits for each class (see text).

rubbed polyimide-coated (SE-7511L; Crystal Diagnostics) substrate. Since the particle density (1.05 g/cm^3) is slightly lower than the density of the DSCG solution ($1.0784 \pm 0.0032 \text{ g/cm}^3$), the particles are weakly driven toward the upper substrate by gravity. We permit the suspensions to equilibrate for 1.5 days before imaging to ensure that particle proximity to the cell substrate is consistent and does not influence measured diffusion (SI Section 4). The sphere number density is also kept very low to maintain interparticle separations >10 diameters and thereby minimize long-range elastic interactions. Once equilibrated, sample cells are placed within a home-built, temperature-controlled chamber mounted on a microscope stage (SI Figure 2). The temperature is held to within 0.1°C , and one hour is allowed for temperature equilibration before experiments begin.

Samples are imaged in bright-field and POM using a Leica DM IRB Inverted Microscope with $100\times$ oil objective (N.A. = 1.4). Typically, digital imaging at 60 frames per second over a period of ~ 15 minutes is carried out using a CMOS camera (Mikrotron EoSens CL MC1362). Particle tracking uses standard IDL routines [30]. The mean-square displacement (MSD) experimental results are typically generated from ~ 30 video trajectories (~ 10 trajectories for 3 particles in each class at each temperature). The MSD curves shown represent an average of these trajectories; error bars reported for diffusion coefficients are derived from fits to the average curves.

The resultant particles in the nematic LCLCs are dressed by twisted tails. In contrast to conventional LC systems, wherein spherical particles with planar anchoring generate topological quadrupoles with large splay and bend distortions in the surrounding nematic [15], the splay and bend elastic energies in LCLCs are reduced by twisting the director. As a result, the character of the elastic distortion in the particle vicinity is novel and

varied (Figure 1b-g). One configuration symmetry (class 1), with the director spiraling around the central axis in the same sense on opposite sides of the colloidal particle, is called a chiral dipole and is characterized as an elastic multipole composed of both quadrupole and chiral dipole terms [27, 33, 34]. The other director configuration (class 2) is globally non-chiral, because the director spirals in an opposite sense on opposite sides of the particle and thereby creates a mirror plane of symmetry (red dashed line in Figure 1e). Generation of either symmetry class is spontaneous and reversible via melting and re-quenching of the nematic background phase (SI Figure 3). The class 1 configuration is calculated to have the lowest energy [26]. In our experiments, we observe both configurations with probabilities $\sim 64\%$ for class 1 and $\sim 36\%$ for class 2 (more details, SI Section 2), which supports the calculations. We do not observe spontaneous transformation of one class to another without reheating the sample.

Director distortions near the particle are predicted to diminish as the sphere radius is decreased [26]. We measured this effect for the first time and report it in Figure 1h for both classes. An estimate for tail length in each class is obtained by fitting the POM image intensity data to a decaying exponential (SI Section 3). Even for the smallest particles ($d \approx 1.5 \mu\text{m}$), the distortions never vanish completely. The tail length, defined as the exponential decay length, is $\sim 8 \mu\text{m}$ ($\sim 6 \mu\text{m}$) for the class 1 (class 2) particles employed in the diffusion measurements; the tail length is ~ 2.4 (~ 1.8) times the particle radius. Thus our data for large diameter particles ($d \gtrsim 4 \mu\text{m}$) are consistent with a theoretical prediction that the director distortion scales with particle radius [26] if the particle is far from the substrate and anchoring at all surfaces is infinitely strong. In short, spherical particles in the LCLC effectively become anisotropic and twisted composite objects when dressed by the surrounding ne-

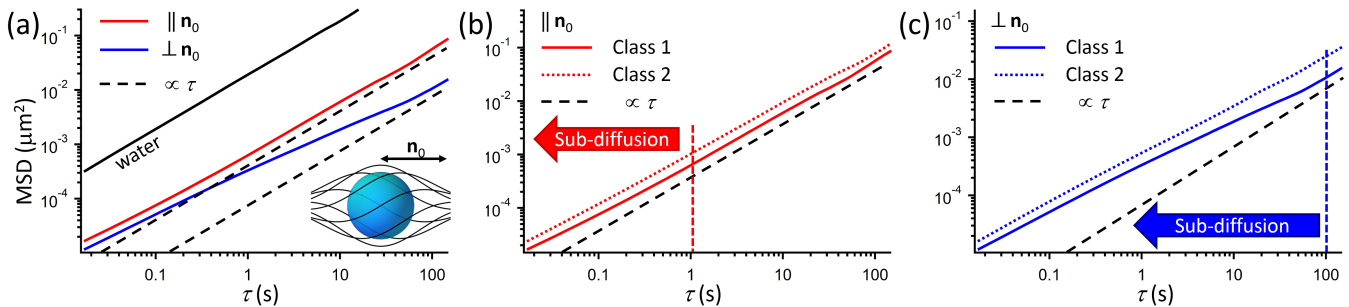


FIG. 2. (a) Mean-squared-displacement (MSD) parallel (red) and perpendicular (blue) to \mathbf{n}_0 plotted versus lag time for class 1 particles at 23 °C. The black solid line is the MSD for a similar particle in pure water. (b) Comparison of MSD versus lag time for class 1 and 2 particles diffusing parallel to \mathbf{n}_0 at 23 °C. (c) Comparison of MSD versus lag time for class 1 and 2 particles diffusing perpendicular to \mathbf{n}_0 at 23 °C. Black dashed lines in all plots show normal diffusion (*i.e.*, $\text{MSD} \propto \tau$).

matic field.

Figure 2 shows our primary mean-squared-displacement (MSD) versus lag time (τ) results. Figure 2a exhibits the salient features of all trajectories. Motion parallel to \mathbf{n}_0 is more substantial than perpendicular to \mathbf{n}_0 , with a comparatively fast cross-over from sub-diffusive to diffusive behavior. By contrast, this same cross-over occurs much more slowly for motion perpendicular to \mathbf{n}_0 . Figure 2b,c compares the MSD curves parallel and perpendicular to \mathbf{n}_0 for class 1 and 2 particles. These data clearly demonstrate that nearly identical colloidal spheres, dressed with the two types of twisted tails, diffuse differently: particles with longer, globally chiral helical tails (class 1) diffuse more slowly than particles with shorter, globally achiral tails (class 2). Long-time diffusion coefficients parallel and perpendicular to \mathbf{n}_0 are given in Table I. We also studied diffusivity temperature-dependence and found that (1) diffusivity at long lag times increases with temperature as expected, and (2) sub-diffusivity persists at all temperatures (SI Section 5).

The diffusion coefficient anisotropy ratios for the two classes are also shown. Although the dressed spheres do not rotate like ellipsoids, comparison to the diffusion of ellipsoids in isotropic fluids is revealing. Diffusion is slower for motion perpendicular to the ellipsoid long axis, as is the case for the dressed particles. Moreover, the diffusion anisotropy is larger for ellipsoids with larger major/minor axis ratios, just as class 1 particles with larger effective aspect ratios have larger diffusion anisotropies than class 2 particles. However, as expected, the dressed particles are different from ellipsoids. The class 1 and class 2 diffusion anisotropy ratios in Table I, for example, are significantly larger than for ellipsoids diffusing in 3D, and comparable only to ellipsoids with axis ratios >8 and ~ 5 , respectively, diffusing in quasi-2D [7, 35, 36]; axis ratios for the dressed spheres based on their measured tail lengths and diameters are ~ 3.4 and ~ 2.8 , respectively.

Diffusion anisotropy, resulting from the direction-dependent viscous drag, is typical in nematics, but the

TABLE I. Diffusion coefficients and diffusion anisotropy ratio for class 1 and 2 particles at 23 °C. These constants are extracted from linear fits to the MSD curves in Figure 2b,c between lag times of 10 and 150 seconds for D_{\parallel} and between 100 and 150 seconds for D_{\perp} .

	Diameter (μm)	$D_{\parallel} \times 10^{-16}$ (m^2/s)	$D_{\perp} \times 10^{-16}$ (m^2/s)	D_{\parallel}/D_{\perp}
Class 1	6.70 ± 0.16	2.67 ± 0.18	0.515 ± 0.070	5.19
Class 2	6.59 ± 0.14	3.90 ± 0.25	1.12 ± 0.11	3.48

Stokes drag on a sphere in a nematic LC is very challenging to calculate [15, 20, 31, 32]. It is governed by Ericksen-Leslie equations that connect fluid flow to the director reorientation via elastic and viscous stresses [13, 14, 37, 38]. The coefficients of the viscous stress tensor, the Leslie coefficients, combine to give three Miesowicz viscosities, η_a , η_b and η_c , for the three shear geometries shown in Figures 3a (a-type geometry), 3b (b-type), and 3c (c-type), respectively. When considering particle diffusion through a uniaxially aligned nematic director configuration, different Miesowicz viscosities become important. For the case of pure (planar) quadrupoles (Figure 3d,e and SI Figures 7a, 8a,d), when the particle moves parallel to \mathbf{n}_0 , the nematic fluid in the shaded regions of Figure 3d (and SI Figure 7a) experiences mostly b-type shear, since the larger component of the director is parallel to the fluid velocity and perpendicular to the velocity gradient. The smaller director component is perpendicular to the fluid velocity and parallel to the velocity gradient, resulting in a small contribution of c-type shear, so $\eta_{\parallel} \sim \eta_b + \epsilon \eta_c$ ($\epsilon \ll 1$). When the same particle moves perpendicular to \mathbf{n}_0 , as in Figure 3e (also, SI Figure 8a,d), the nematic fluid in the shaded regions experiences mostly c-type shear, since the larger component of the director is perpendicular to the fluid velocity and parallel to the velocity gradient. However, in regions in front and behind the plane of Figure 3e, the larger component of the director is perpendicular to both the

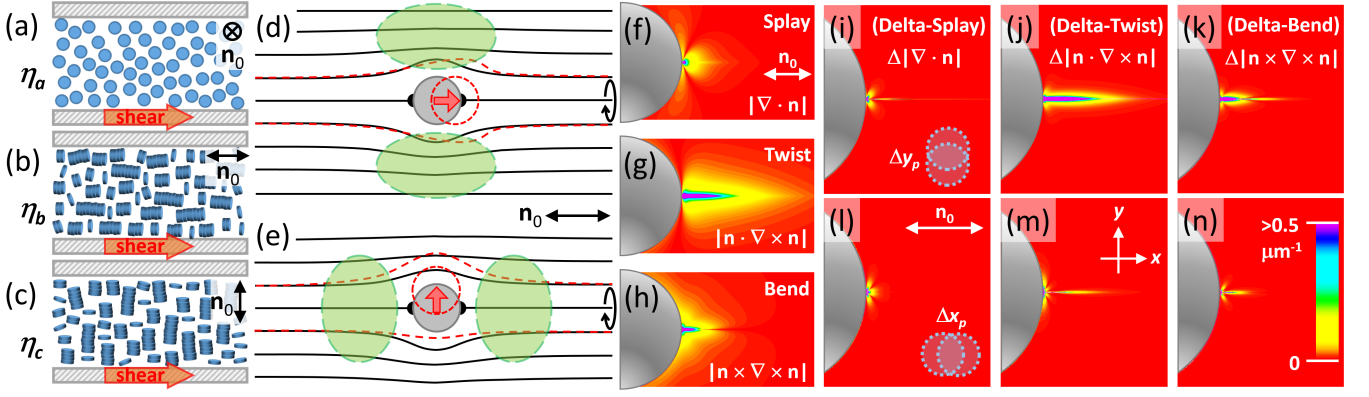


FIG. 3. (a-c) Three shear geometries of a nematic LC between parallel plates corresponding to the three Miesowicz viscosities (a) η_a , (b) η_b and (c) η_c . (d, e) Cross-sectional schematic of a spherical particle with tangential surface anchoring suspended in a uniaxially aligned nematic. Black lines represent the equilibrium director field for a stationary particle. Red dashed lines represent perturbations in the director field due to colloidal displacements (depicted by red arrows) either (d) parallel or (e) perpendicular to \mathbf{n}_0 . The green shaded regions are regions of interest when considering the effects of the Miesowicz viscosities (see text and SI). The round arrows to the right indicate that the equilibrium field is axially symmetric. (f-h) Calculated (f) splay, (g) twist and (h) bend distortions as predicted by the theoretical ansatz from ref. [27]. (i-n) Calculated changes in the director fields shown in panels (f-h), due to small particle translations ($\Delta x_p, \Delta y_p \approx 4.5$ nm) both (i-k) perpendicular and (l-n) parallel to \mathbf{n}_0 . The reference frame and color scale insets in panels (m) and (n), respectively, apply to panels (f-n).

fluid velocity and velocity gradient, so a-type shear is also present. The smaller director component is parallel to the fluid velocity and perpendicular to the velocity gradient so b-type shear gives a small contribution, so $\eta_{\perp} \sim (\eta_a + \eta_c)/2 + \epsilon\eta_b$.

Typically, for rod-like nematics [14, 20, 25], $\eta_b < \eta_a < \eta_c$; thus, we expect $D_{\parallel} > D_{\perp}$ as observed. These qualitative estimates are useful because they resemble predictions of simple analytical models using the Ericksen-Leslie equations [20]. However, numerical models addressing planar anchoring at the particle surface show that the effective viscosities are generally not simple combinations of the Miesowicz viscosities [20]. Furthermore, the relationships between the Miesowicz viscosities and the effective viscosities are further complicated by the helicity and large anisotropy of the chiral director distortions (SI Figures 7, 8). Nevertheless, based on the field symmetry and the size of the overall distortions around the spheres, we expect and observe that class 2 particles (SI Figures 7a,c, 8d,f) experience an effective viscosity comparatively more similar to that of a pure (planar) quadrupole than the class 1 particles.

Finally, following a phenomenological analysis similar to that in Ref. [16], we consider the origin of the timescales which characterize the cross-over from sub-diffusive to diffusive behavior. Sub-diffusive behavior at short lag times derives from LC-mediated elastic forces due to displacement- and thermally-driven fluctuations of the surrounding director field. Director field perturbations due to colloidal displacements cost elastic free energy and generate restoring forces on the particle. Ultimately, they relax back to equilibrium configurations.

The surrounding director perturbations are expected

to relax with a characteristic time $\tau_i \sim l_i^2 \eta_i / K_i$, where η_i is an effective viscosity, K_i is an elastic constant, and l_i is a characteristic length scale of the director distortions that must relax back to equilibrium [16, 37, 38]; the subscript indicates the type of distortion that is relaxing (twist, bend, or splay). Strictly speaking this analysis applies to the relaxation of small periodic fluctuations in the director orientation about the equilibrium field, \mathbf{n}_0 [21, 37, 38]. Previously, for diffusing particles, l_i has been assumed to be the particle diameter d [16], but better estimates might consider the regions around the particle wherein distortions change most due to small particle displacements (see below). We posit that cross-over from sub-diffusion to normal diffusion will occur on timescales comparable to τ_i . To derive these timescales, we use measured estimates for the effective viscosities and elastic constants of DSCG at 23 °C [21], and we choose, for simplicity, $l_i = 2 \mu\text{m}$ for all cases. These assumptions provide an estimate τ_i for each distortion mode: ($\tau_{\text{splay}}, \tau_{\text{twist}}, \tau_{\text{bend}}$) \sim (8 s, 133 s, 0.002 s). Notice, the twist relaxation time is very long compared to bend and splay as long as the characteristic length scales of the director distortions are comparable; moreover, the timescale of ~ 100 seconds for twist relaxation is in rough agreement with the observed cross-over time for particle motion perpendicular to \mathbf{n}_0 .

To understand more deeply how the director field distortions can affect particle diffusion dynamics, we compute the variation of each pure distortion (splay, twist and bend) due to a small particle displacement ($\Delta x_p, \Delta y_p \ll d$) either parallel or perpendicular to \mathbf{n}_0 ; here we utilize class 1 spheres because a model describing the distortions is available [27]. The calculated splay,

twist and bend distortions are plotted in Figure 3f-h for a stationary sphere. It is apparent that all modes of distortion are present. In Figure 3i-n, we plot the magnitude of the change in these distortions due to small particle displacements ($\Delta x_p, \Delta y_p \sim 4.5$ nm) perpendicular (Figure 3i-k) and parallel (Figure 3l-n) to \mathbf{n}_0 . These are the director fluctuations that must relax back to equilibrium after the displacement. Notably, the plots (Figure 3i-n) suggest that the largest changes in \mathbf{n} are associated with twist for particle motion perpendicular to \mathbf{n}_0 ; furthermore, significantly less overall change in \mathbf{n} is observed for motion parallel to \mathbf{n}_0 . These findings, and the relative timescales for relaxation estimated above, provide strong evidence that long-lived twist distortions play a dominant role in affecting diffusion dynamics in LCLCs.

In summary, the diffusion of spheres in LCLCs is quite different from diffusion in thermotropic LCs. The primary origin of these differences is the unusually small twist elastic constant of LCLCs. Because twist distortions are energetically cheap, the director configuration on each side of the particle is chiral, leading to twisted tails that extend much farther away from the particle than the splay and bend distortions of pure quadrupoles. We showed that these differences in twist uniquely affect particle dynamics, both diffusive and sub-diffusive motions, as well as the timescales for these processes. Further, differences in diffusion constants and diffusion anisotropy are found for the two chiral director configurations. All of these phenomena are manifestations of the giant twist elastic anisotropy characteristic of LCLCs. Thus, twist effects can be expected to influence our ability to control micro- and nano-particles in LCs.

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