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Dynamo Effect and Turbulence in Hydrodynamic Weyl Metals

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The dynamo effect is a class of macroscopic phenomena responsible for generation and maintaining magnetic fields in astrophysical bodies. It hinges on hydrodynamic three-dimensional motion of conducting gases and plasmas that achieve high hydrodynamic and/or magnetic Reynolds numbers due to large length scales involved. The existing laboratory experiments modeling dynamos are challenging and involve large apparatuses containing conducting fluids subject to fast helical flows. Here we propose that electronic solid-state materials – in particular, hydrodynamic metals – may serve as an alternative platform to observe some aspects of the dynamo effect. Motivated by recent experimental developments, this paper focuses on hydrodynamic Weyl semimetals, where the dominant scattering mechanism is due to interactions. We derive Navier-Stokes equations along with equations of magneto-hydrodynamics that describe transport of Weyl electron-hole plasma appropriate in this regime. We estimate the hydrodynamic and magnetic Reynolds numbers for this system. The latter is a key figure of merit of the dynamo mechanism. We show that it can be relatively large to enable observation of the dynamo-induced magnetic field bootstrap in experiment. Finally, we generalize the simplest dynamo instability model – Ponomarenko dynamo – to the case of a hydrodynamic Weyl semimetal and show that the chiral anomaly term reduces the threshold magnetic Reynolds number for the dynamo instability.

The dynamo effect is a beautiful astrophysical phenomenon, first proposed by Larmor in 1919 [1], that is believed to be responsible for generating and sustaining magnetic fields in galaxies, stars and planets including the Sun and the Earth [2]. There exist a large variety of different dynamo mechanisms [2–4] that all share the same key ingredient – hydrodynamic motion of an electrically conducting gas, fluid or plasma. The dynamo theory deals with the hydrodynamic motion of a conductive medium focussing on the possibility of self-generating and self-sustaining magnetic fields, whose presence has been observed in astrophysical bodies.

As detailed below, the underlying equations of the theory are the Navier-Stokes equations, describing the hydrodynamic motion of the medium, coupled to the Maxwell equations of electromagnetism. In the non-relativistic limit, they give rise to equations of magneto-hydrodynamics (MHD). These are complicated non-linear equations, and their exact solutions represent a great challenge. However, both the solutions of simplified MHD models [e.g., kinematic dynamos, with predetermined velocity fields $\mathbf{u}(\mathbf{r}, t)$] and qualitative arguments [2] suggest that the dynamo action is possible when the terms enhancing the magnetic field [e.g. the induction term, $\nabla \times (\mathbf{u} \times \mathbf{B})$] overwhelm the magnetic diffusion term, $\eta_m \Delta \mathbf{B}$ (where $\eta_m = c^2/4\pi\sigma$ where c is the speed of light and σ is the conductivity of the medium), which tend to suppress the self-generation. The respective figure of merit is the *magnetic Reynolds number* [5]

$$R_m = \frac{uL}{\eta_m} = uL \frac{4\pi\sigma}{c^2}, \quad (1)$$

where L is the characteristic system size and u is the typical velocity of the medium. The threshold value for a dynamo action to commence (usually lying in the range $R_m^{(cr)} \sim 10 - 100$, with $R_m^{(cr)} \approx 17.7$ for the simplest Ponomarenko dynamo [6] discussed below) depends on system's geometry and is rarely known exactly. It is clear, however, that the larger R_m , the

more likely and more effective the dynamo action. The conductivity of astrophysical media vary greatly from 10^{-11}Sm^{-1} for interstellar plasma to 10^3Sm^{-1} for the solar convection shell and 10^5Sm^{-1} for the Earth's core, but in all of these cases the large magnetic diffusion coefficient is compensated by literally astronomical distances resulting in large magnetic Reynolds numbers, however small the conductivities are. By contrast, laboratory dynamo experiments [7] deal with naturally limited system sizes and use the conductivity and the flow velocities as the only potentially tunable parameters.

Apart from large magnetic Reynolds numbers $R_m \gg 1$, the emergence of a dynamo requires a number of other conditions that need to be met. In particular, certain “no-go theorems” [8] have to be overcome, such as the impossibility of a two-dimensional dynamo effect or that in a planar three-dimensional flow (i.e., with one vanishing component of velocity). Finally, it is known the dynamo action is greatly helped by the helicity flow, which may arise either due to the geometry of an imposed flow or due to turbulence. The latter is possible if the second figure of merit, the *hydrodynamic Reynolds number*

$$R = \frac{uL}{\nu}, \quad (2)$$

where ν is the kinematic viscosity, is large. Separating both the velocity and magnetic field into a mean-field and fluctuating component - $\mathbf{u} = \bar{\mathbf{u}} + \delta\mathbf{u}$ and $\mathbf{B} = \bar{\mathbf{B}} + \delta\mathbf{B}$, and averaging over the small-scale fluctuations results in the Krause-Rädler equations [9, 10] of mean-field MHD, which in the simplest case of isotropic turbulence is given by

$$\frac{\partial \bar{\mathbf{B}}}{\partial t} = \nabla \times (\bar{\mathbf{u}} \times \bar{\mathbf{B}}) + \nabla \times (\alpha \bar{\mathbf{B}}) + \xi \Delta \bar{\mathbf{B}}, \quad (3)$$

where the second term in the right-hand-side is the “new” helicity term allowed in turbulent MHD (α -effect). If the veloc-

ity field is stationary, Eq. (3) or a similar MHD equation without helicity for non-turbulent flows becomes an eigenvalue problem for the magnetic field growth $\mathbf{B}(\mathbf{r}, t) \propto \mathbf{B}(\mathbf{r})e^{\gamma t}$. The existence of exponentially growing components ($\text{Re } \gamma > 0$) indicates an instability towards a self-generating magnetic field (were the imaginary part $\text{Im } \gamma > 0$ leads to the field oscillations, which have been suggested [11] by one of the authors to lead, e.g., to periodic cycles of solar magnetic activity).

Apart from the astrophysical context, there has been a tremendous interest in testing the predictions of dynamo theory and modeling a planetary-like or solar-like dynamo action in the laboratory [7, 12–14]. Several impressive laboratory experiments have been carried out and are currently under way that involve setting in motion a liquid metal – sodium or gallium – with the goal to achieve large Reynolds numbers to enable the dynamo mechanism. As obvious from Eqs. (1) and (2), this leads to the challenge of ultra-fast mechanical stirring or rotating the liquid metal.

Here we propose that electronic solid-state systems may provide an alternative platform for observing magnetohydrodynamic effects. Firstly, we list several necessary conditions of the dynamo effect in an electronic system: (i) Transport in the electron liquid should be governed by hydrodynamics, i.e. the primary momentum relaxation mechanism should be electron-electron collisions rather than impurity scattering. (ii) The system and the flow must be essentially three-dimensional. (iii) Large magnetic $R_m \gg 1$ and/or hydrodynamic $R \gg 1$ Reynolds numbers are required.

Hydrodynamic transport in solid state [condition (i)] has been a subject of intense recent studies [15–19], both theoretical and experimental. On the experimental side, two widely studied platforms for hydrodynamic phenomena are graphene [20] and Weyl semimetals (WSMs) [21–23]. Graphene, however, violates a “no-go dynamo theorem” - condition (ii) requiring 3D flows - and is thus of no relevance to the dynamo effect.

In what follows, we focus on magnetohydrodynamic phenomena in Weyl metals (doped Weyl semimetals). We note that in systems with the power-law quasiparticle dispersion [24–29] $\epsilon(\mathbf{p}) \propto |\mathbf{p}|^\beta$ with $\beta \leq 1$, the creation of electron-hole pairs is suppressed [30], because the energy and momentum conservation laws cannot be satisfied simultaneously for lowest-order processes. Weyl systems ($\beta = 1$) may, therefore, often be considered as electron-hole plasma with a linear particle dispersion.

A WSM generically has an even number of nodes, according to the fermion-doubling theorem [31], and electrons and holes near different nodes often behave as independent liquids. However, simultaneous application of external electric \mathbf{E} and magnetic \mathbf{B} fields results in the quasiparticle transfer from one node to another (chiral anomaly[32–36]). For simplicity, we assume in this paper that (a) the system has only two nodes, labeled by L and R , with the same quasiparticle dispersion, (b) the entire system is being kept at a constant temperature T and (c) the intranodal equilibration processes are significantly faster than the internodal particle-

transfer processes. This allows one to define the chemical potentials μ_α near each node $\alpha = L, R$ and the hydrodynamic velocity \mathbf{u} of the Weyl fluid. The distribution function of the linearly-dispersing quasiparticles near each node in the absence of electromagnetic fields is given by [37] $f_\alpha(\mathbf{k}) = \{\exp[\gamma(\mathbf{u})(\pm v_F|\mathbf{k}| - \mu_\alpha - \mathbf{u} \cdot \mathbf{k})/T] + 1\}^{-1}$, where “+” and “-” refer, respectively, to the conduction and valence bands; $\gamma(\mathbf{u}) = (1 - u^2/v_F^2)^{\frac{1}{2}}$; and $\gamma(\mathbf{u})(\pm v_F|\mathbf{k}| - \mu_\alpha - \mathbf{u} \cdot \mathbf{k})$ is the quasiparticle dispersion in the reference frame of the moving electron liquid.

The dynamics of charge densities ρ_α near node α , where $\alpha = L, R$, are described by the continuity equations

$$\partial_t \rho_\alpha + \nabla \cdot \mathbf{j}_\alpha - \chi_\alpha \frac{ge^3}{4\pi^2 \hbar^2 c} \mathbf{E} \cdot \mathbf{B} + \frac{\rho_\alpha - \rho_{\bar{\alpha}}}{\tau_{in}} = 0, \quad (4)$$

where $\chi_L = -1$ and $\chi_R = +1$ are the “chiralities” of quasiparticles near nodes L and R and g accounts for spin and possibly additional valley degeneracy; $\bar{\alpha}$ labels the node other than α ; hereinafter $e = -|e|$. The first two terms in Eq. (4) match the usual continuity equation for a liquid with density ρ_α ; the third term ($\propto \mathbf{E} \cdot \mathbf{B}$) accounts [35, 36] for the change of the electron concentration at node α due to the chiral anomaly; and the last term in Eq. (4) describes internodal scattering, e.g., due to short-range-correlated quenched disorder, with the internodal scattering time τ_{in} . The electric currents $\mathbf{j}_{L,R}$ of the charge carriers near the two nodes are given by

$$\mathbf{j}_\alpha = \sum_\beta \sigma_{\alpha\beta} \left[\mathbf{E} + \frac{1}{c} \mathbf{u} \times \mathbf{B} - \frac{1}{e} \nabla \mu_\beta \right] - \chi_\alpha \frac{ge^2}{4\pi^2 \hbar^2 c} \mathbf{B} \mu_\alpha, \quad (5)$$

where $\alpha, \beta = L, R$; μ_α is the chemical potential near node α , and \mathbf{u} is the hydrodynamic velocity of the Weyl fluid. In this paper we assume that the imbalance of the chemical potentials between the nodes, if any, is small $|\mu_L - \mu_R| \ll |\mu_{L,R}|, T$. The diagonal components $\sigma_{LL} = \sigma_{RR}$ of the conductivity tensor $\sigma_{\alpha\beta}$ describe the response of charge carriers near each node to the electromagnetic field; the off-diagonal entries $\sigma_{LR} = \sigma_{RL}$ account for the drag of the quasiparticles near each node by the current near the other node. The last term in Eq. (5) describes the chiral magnetic effect [38, 39], the generation of the charge current by an external magnetic field in the system in the presence of chirality imbalance, $\mu_L - \mu_R \neq 0$.

Equations (4)-(5), together with the relations [40]

$$\rho_{R,L} = ge \frac{\mu_{R,L}^3 + \pi^2 \mu_{R,L} T^2}{6\pi^2 v_F^3 \hbar^3} \quad (6)$$

for the charge density at node α and with Maxwell equations, which involve the total charge density $\rho = \rho_L + \rho_R$ and the current $\mathbf{j} = \mathbf{j}_L + \mathbf{j}_R$, constitute a closed system of equations which describes charge and current dynamics of the electron liquid in a WSM which moves with velocity \mathbf{u} in an external electromagnetic field. The motion of such a liquid may be generated by the electromagnetic fields, the temperature and chemical potential gradients, or even fast mechanical rotation of the sample.

To determine self-consistently the velocity field \mathbf{u} (which in practice is a tremendously difficult problem), the system of Eqs. (4)-(6) has to be complemented by the Navier-Stokes equation (derived in Supplemental Material [41])

$$\begin{aligned} \frac{w_\alpha}{v_F^2} \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = & -\nabla P_\alpha - \frac{\mathbf{u}}{v_F^2} \frac{\partial P_\alpha}{\partial t} + \rho_\alpha \mathbf{E} + \frac{1}{c} \mathbf{j}_\alpha \times \mathbf{B} \\ & + \frac{\mathbf{u}}{3} \left(\frac{\partial \varepsilon}{\partial \rho} \right)_\alpha \left(\chi_\alpha \frac{ge^3}{h^2 c} \mathbf{E} \cdot \mathbf{B} - \frac{\rho_\alpha - \rho_{\bar{\alpha}}}{\tau_{\text{in}}} \right) + \eta \nabla^2 \mathbf{u} + \zeta \nabla (\nabla \cdot \mathbf{u}), \end{aligned} \quad (7)$$

where $w_\alpha = \varepsilon_\alpha + P_\alpha$ is the the enthalpy of the charge carriers near node α per unit volume, with [42]

$$\varepsilon_\alpha \approx g \frac{7\pi^4 T^4 + 30\pi^2 \mu_\alpha^2 T^2 + 15\mu_\alpha^4}{120\pi^2 v_F^3 \hbar^3} \quad (8)$$

and $P_\alpha \approx \frac{\varepsilon_\alpha}{3}$ being, respectively, the contributions of node α to the internal energy and pressure; the current \mathbf{j}_α is given by Eq. (5); η and ζ are the shear and the bulk viscosities; the term $\propto \left(\frac{\partial \varepsilon}{\partial \rho} \right)_\alpha$ accounts for the change of the energy and pressure of the Weyl liquid near node α due to the internal scattering and the chiral anomaly, where $\left(\frac{\partial \varepsilon}{\partial \rho} \right)_\alpha = \frac{3\mu_\alpha}{e} \frac{\mu_\alpha^2 + \pi^2 T^2}{3\mu_\alpha^2 + \pi^2 T^2}$ for the case of an isothermal flow considered in this paper [see Supplemental Material [41] for the discussion of the assumptions about thermalization].

In this paper, we neglect the so-called chiral vortical effect [42], i.e. contributions to the current from the interplay of global rotations of the system and chirality imbalance ($\mu_L - \mu_R \neq 0$). In the Navier-Stokes equation (7) we also neglect terms of higher orders in u^2/v_F^2 . Equations (4)-(7), together with the Maxwell's equations and the equations of state, in the form of Eq. (8) and $P_\alpha = \frac{\varepsilon_\alpha}{3}$, constitute a closed system of equations describing the dynamics of the electromagnetic fields and the electron liquid in a WSM.

Using Eqs. (5), together with the Maxwell's equations $\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$ and $\mathbf{j}_L + \mathbf{j}_R \equiv \mathbf{j} = \frac{c}{4\pi} \nabla \times \mathbf{B}$, where we neglected the displacement current under the assumption of a quasi-stationary flow, we arrive at the equation for the dynamics of the magnetic field:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B} + \frac{ge^2}{4\pi^2 \hbar^2 \sigma} \nabla \times [(\mu_L - \mu_R) \mathbf{B}], \quad (9)$$

where $\sigma = 2\sigma_{LL} + 2\sigma_{LR}$ is the conductivity of the WSM and we have taken into account that the quasiparticles have the same dispersion near the two nodes. Apart from solid-state WSMs, an equation of the form (9) with phenomenologically introduced coefficients describes the dynamics of ultrarelativistic chiral particles [43].

Equation (9) indicates that Weyl liquids allow for the helicity term for macroscopic fields without turbulence, in contrast with the conventional α -dynamo of Krause and Rädler [9]. However, it can only appear in the presence of an already existing field, and while, as shown below, it can further enhance

magnetic field “bootstrap,” it can not lead to generation of the field in and by itself if there is no seed field to begin with. For that, the magnetic Reynolds number (1), R_m , has to be large enough, as discussed in the introduction.

To estimate, R_m , we use the equation for the Coulomb-interaction dominated conductivity of a Weyl semimetal [44]

$$\sigma \sim \frac{e^2}{\hbar} \frac{k_B T}{\hbar v_F} \frac{1}{\alpha^2}, \quad (10)$$

where the Weyl’s “fine-structure constant” is $\alpha = e^2/(\hbar v_F \kappa)$ and κ is the dielectric constant, which crucially may be rather large. While Eq. (10) has been derived neglecting screening effects [44], it should be adequate for estimates. For these purposes, we have also dropped logarithmic renormalisation factors.

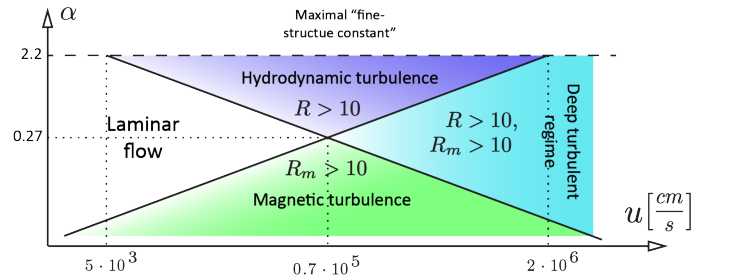


FIG. 1. (Colour online) Flow regimes for the electron liquid in a Weyl semimetal on the diagram “fine-structure constant” $\alpha = \frac{e^2}{\kappa \hbar v_F}$ vs. flow velocity u (log-log scale) for the room temperature $T = T_{\text{room}} = 300\text{K}$ and the Fermi velocity $v_F = 10^8 \frac{\text{cm}}{\text{s}}$. The maximum value of the “fine-structure constant” is $\alpha_{\text{max}} = \frac{e^2}{\hbar v_F} \approx 2.2$.

Let us emphasize that the dynamo effect is a *macroscopic classical phenomenon*. The effect is favoured by large system sizes L , which lead to large R_m . In experiments with solid-state systems the size L is rather limited, with centimeter-size samples being at the upper end of the range accessible for WSMs. Since the effect is not sensitive to quantum interference effects, higher temperatures T are much preferable to maximize R_m ; the room temperature, T_{room} , thus represents a reasonable comparison scale. We emphasize that even at room temperature Weyl semimetals are not Maxwell gases and quantum statistics and quantum nature of the electron-electron scattering are important, but quantum coherence is not essential for the dynamo effect. Using these length and temperature scales, we obtain the following estimate for the main figure of merit in the dynamo theory:

$$R_m \sim \frac{1}{\alpha^2} \frac{e^2}{\hbar} \frac{4\pi k_B T}{\hbar v_F c^2} u L \sim \frac{10^{-6}}{\alpha^2} \left(\frac{T}{T_{\text{room}}} \right) \times u \left[\frac{\text{cm}}{\text{s}} \right] L[\text{cm}], \quad (11)$$

where u is the typical velocity of the flow.

Now, we turn to estimates of the hydrodynamic Reynolds number [2]. The viscosity of the quasiparticles in a Weyl semimetal at temperature T may be estimated as $\eta \sim$

$n(T)T\tau_{\text{rel}}$, where $n(T)$ is the concentration of the thermally excited quasiparticles and $\tau_{\text{rel}} \sim \hbar(\alpha^2 k_B T)^{-1}$ is the momentum relaxation time. Note that this result follows from second-order perturbation theory in Coulomb interaction and neglects screening effects. This leads to

$$\eta \sim \frac{(k_B T)^3}{\alpha^2 \hbar^2 v_F^3}. \quad (12)$$

The motion of a Weyl-semimetal liquid is turbulent in the hydrodynamic sense when the term $\frac{w}{v_F}(\mathbf{u} \cdot \nabla)\mathbf{u}$ in the Navier-Stokes equation (7) dominates the dissipative terms $\sim \eta \nabla^2 \mathbf{u}$ that come from the viscosity of the Weyl fluid. This yields the following estimate

$$R = \frac{w u L}{\eta v_F^2} \sim \alpha^2 \frac{k_B T}{\hbar} \frac{u L}{v_F^2} \sim 4\alpha^2 10^{-3} \left(\frac{T}{T_{\text{room}}} \right) \times u \left[\frac{\text{cm}}{\text{s}} \right] L[\text{cm}], \quad (13)$$

where we have used the estimate $w \approx \frac{7g\pi^2 T^4}{90\hbar^3 v_F^3} \sim \frac{(k_B T)^4}{\hbar^3 v_F^3}$ for the specific enthalpy at high temperatures.

We note in this context that the viscosity of a Fermi liquid at temperature T may be estimated as $\eta \sim \varepsilon_F^5 / (T^2 \hbar^2 v_F^3)$, where ε_F and v_F are the Fermi energy and velocity, respectively. Because the hydrodynamic Reynolds number $R \sim \frac{T^2}{v_F \varepsilon_F \hbar}$ gets rapidly suppressed with increasing the Fermi energy ε_F , topological semimetals are indeed a favourable platform for achieving electronic turbulence as compared to “conventional” hydrodynamic metals.

Naturally, the geometry and the magnitude of the velocity field u much depends on the mechanism to stir up hydrodynamic motion and follows from the solution of the Navier-Stokes equations, which is a challenging task in most cases. Furthermore, since observation of a phenomenon of this kind has never been attempted in solid-state materials, specific experimental techniques for achieving high hydrodynamic flows in the most efficient way still deserve further investigation – pulsed fields (in particular orbital-angular-momentum pulses [45] in a cylindrical geometry), crossed electric and magnetic fields or just a rapid rotation of the sample are all possibilities to consider. While below we consider in detail one of the standard and simplest dynamo models, we emphasize immediately that the estimates (11) and (13) are not prohibitive; and it is conceivable that relatively large magnetic Reynolds numbers, necessary for the dynamo to commence, are achievable for realistic flow velocities with u of order one kilometer/second or greater (especially considering that the dielectric constant may be as high as $\kappa \sim 50$ in WSMs), cf. Fig. 1.

Now, we discuss a specific model of dynamo effect – the so-called kinematic Ponomarenko dynamo [6, 8], with an eye on how the terms in MHD equations, descending from the chiral anomaly, change the effect. The Ponomarenko dynamo does not necessarily represent the most experimentally realistic setup, but it does represent the simplest textbook model,

which contains the key qualitative features of a dynamo mechanism and is amenable to analytical analysis.

In order for a dynamo action to occur, the magnetic Reynolds number must exceed a critical value R_m^c [46]. The purpose of the calculation below is to obtain the dependence of the critical Reynolds number, R_m^c , on the helicity term. For simplicity, we neglect the time dependence of the chemical-potential difference $\mu_L - \mu_R$ on the times we consider.

We re-write Eq. (9) as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \mathbf{B} + \xi \nabla \times \mathbf{B}, \quad (14)$$

where $\xi = ge^2(\mu_L - \mu_R)/(4\pi^2 \hbar^2 \sigma)$. We consider a cylindrical geometry of the sample with a flow field $\mathbf{u} = (0, r\Omega, u_0)$, where Ω and u_0 are constants, for $r \leq a$, and $\mathbf{u} = 0$ for $r > a$ [46]. Plugging the ansatz $\mathbf{B}(r, \theta, z, t) = \mathbf{B}(r)e^{i(n\theta - kz) + \gamma t}$ into (9), the components of the magnetic field $B_{\pm} = B_r \pm iB_{\theta}$ satisfy the equations

$$y^2 B_{\pm}'' + y B_{\pm}' = [q^2 y^2 + (n \pm 1)^2] B_{\pm} - \delta [ny B_{\mp}' \mp n(n \mp 1) B_{\mp} \pm k^2 a^2 y^2 B_{\pm} \mp q^2 y^2 B_r] \quad (15)$$

for $y = r/a \leq 1$ and

$$y^2 B_{\pm}'' + y B_{\pm}' = [s^2 y^2 + (n \pm 1)^2] B_{\pm} \quad (16)$$

for $y > 1$, where B_{\pm}' (B_{\pm}'') is the first (second) derivative with respect to y ; $\delta = 4\pi\sigma\xi/kc^2$, $q^2 = k^2 a^2 + \gamma\tau_R + i(n\Omega - ku_0)$, $s^2 = k^2 a^2 + \gamma\tau_R$, where $\tau_R = 4\pi\sigma a^2/c^2$ is the time scale of the magnetic field diffusion.

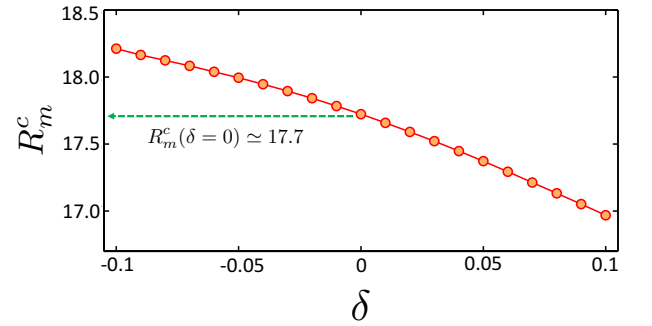


FIG. 2. (Colour online) The critical magnetic Reynolds number R_m^c of the $n = 1$ kinematic Ponomarenko dynamo as a function of the helicity parameter $\delta = 4\pi\sigma\xi/kc^2$, with k being the wave-vector of the dynamo instability. $R_m^c \approx 17.7$ is the critical value for the dynamo in the absence of helicity. We note that a self-exciting dynamo will always correspond to the chirality with a lower critical Reynolds number. The chiral anomaly, thus, always aids the dynamo effect.

For each mode n , the magnetic field starts to grow exponentially when $\text{Re}(\gamma) > 0$, which occurs if the magnetic Reynolds number exceeds a critical value R_m^c . In the absence of helicity ($\delta = 0$), Eq. (14) reduces to the conventional dynamo equation and the $n = 0$ mode is not excited for an arbitrary intensity of the flow [46]. For non-zero helicity, we solved the inhomogeneous equations (15) and (16) with appropriate boundary

conditions imposed [41] to obtain the dispersion relation for the dynamo mode. The obtained values of R_m^c for a dynamo with $n = 1$ and a particular direction of wavevector \mathbf{k} (the z axis) are shown in Fig. 2. The $n = 1$ mode is the leading mode, where the dynamo action commences first, and for which the critical magnetic Reynold number is the smallest and potentially within reach for actual Weyl systems. In the absence of helicity (i.e., if $\delta = 0$), it is known to be $R_m^c \approx 17.7$ [46]. Interestingly enough, the helicity $\delta > 0$ reduces the critical value of the magnetic Reynold number for the $n = 1$ mode and helps the dynamo action to occur for $R_m^c < 17.7$. Because dynamo flows with various directions of \mathbf{k} may emerge spontaneously in a turbulent liquid, the presence of helicity (a consequence of the chiral anomaly) would generically aid the dynamo bootstrap in any geometry of the flow.

In conclusion, this paper proposes hydrodynamic Weyl semimetals as a host to electronic turbulence and/or dynamo effect. We derived the Navier-Stokes equations (7) and equations of magnetohydrodynamics (9) and estimated two key figures of merit – the hydrodynamic and magnetic Reynolds numbers. Fig. 1 summarizes our findings and shows that both turbulence and dynamo mechanism are in principle experimentally achievable. However, many interesting questions remain, such as experimental signatures of the turbulent electronic motion and the role of “new” terms in the Navier-Stokes equations, descending from the quantum chiral anomaly. Finally, we mention that while three-dimensional Dirac materials are indeed interesting from the perspective of realizing the dynamo bootstrap, a number of other electronic materials may also serve as platforms to realize the effect. For example, electronic metals near critical points (e.g., right above a superconducting transition) represent a promising system to look at in this context (both from the perspective of achieving hydrodynamic flows and large Reynolds numbers) and could pave the way to simulating in solid-state materials the effect of magnetic field’s self-excitation – a remarkable phenomenon, usually delegated to the fields of geophysics, astrophysics and cosmology.

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