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Cooper-Pair Spin Current in a Strontium Ruthenate Heterostructure

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It has been recognized that the condensation of spin-triplet Cooper pairs requires not only the broken gauge symmetry but also the spin ordering as well. One consequence of this is the possibility of the Cooper-pair spin current analogous to the magnon spin current in magnetic insulators, the analogy also extending to the existence of the Gilbert damping of the collective spin-triplet dynamics. The recently fabricated heterostructure of the thin film of the itinerant ferromagnet SrRuO3 on the bulk Sr2RuO4, the best-known candidate material for the spin-triplet superconductor, offers a promising platform for generating such spin current. We will show how such heterostructure allows us to not only realize the long-range spin valve but also electrically drive the collective spin mode of the spin-triplet order parameter. Our proposal represents both a novel experimental realization of superfluid spin transport and a transport signature of the spin-triplet superconductivity therein.

Introduction: Harnessing spin rather than charge in electronic devices has been a major topic in solid state physics; it has been not only utilized for various memory devices but is also expected to play a key role in processing quantum information. In order for various spin devices to function robustly, the long-range spin transport needs to be achieved. Metallic wires, however, typically do not transport spins beyond the spin-diffusion length due to the single electron spin relaxation.

In recent years, it has been shown that the exponential damping can be circumvented in the spin transport via collective magnetic excitations. For example, easy-plane (ferro- and antiferro-)magnetic insulators are analogous to the conventional superfluid in being characterized by the U(1) order parameter. As Fig. 1 (a) illustrates, the planar spiraling of the order parameter in such magnetic insulators gives rise to the spin supercurrent, just as the phase gradient of the conventional superfluid gives rise to the mass supercurrent; in this sense these magnetic insulators can be regarded as spin superfluids.

Interestingly, there exists a class of superfluids and superconductors which can support both mass and spin supercurrent. Such superfluids and superconductors should break both spin rotational symmetry and gauge symmetry. Examples include the spin-1 boson condensate, the 3He superfluid, and the spin-triplet superconductor; in the two latter cases, the dissipationless spin current would be carried by the Cooper pairs. While the vortices with spin supercurrent circulate have been observed in all these systems, the bulk spin supercurrent has not been detected in the superconductor.

In this Letter, we will show how the spin superfluidity in the spin-triplet superconductor leads not only to the long-range spin current but also electrical excitation of the spin wave in the bulk. For realizing these phenomena, we propose a two-terminal setup with voltage bias between ferromagnetic metal leads in contact with the spin-triplet superconductor. While the static order-parameter case essentially reduces to the Blonder-Tinkham-Klapwijk type formalism for the interfacial transport, here we complement it with the appropriate equations of motion for the collective spin dynamics in the superconductor. Recently, a thin film of the itinerant ferromagnet SrRuO3 has been epitaxially deposited on the bulk Sr2RuO4, the best known candidate material for the spin-triplet superconductor, yielding, due to their structural compatibility, an atomically smooth and highly conductive interface with a strong Andreev conductance. This makes Sr2RuO4 and SrRuO3 the primary candidate materials for the bulk and the leads, respectively, of our setup. For the remainder of this paper, we will first show how the simplest effective spin Hamiltonian for the spin-triplet superconductor and the resulting spin dynamics are analogous to those of the antiferromagnetic insulator; then, we will discuss the magnetoresistance for the DC bias voltage and the coupling between the AC bias voltage and the spin wave.

General considerations: We first point out the close analogy between the spin order parameter of the antiferromagnet and the spin-triplet superconductor. Defined

$$i(d \cdot \sigma) \sigma = \begin{bmatrix} -i \partial_x + i \partial_y & \partial_z \cr \partial_z & i \partial_x + i \partial_y \end{bmatrix} \equiv \begin{bmatrix} \Delta_{\uparrow \uparrow} & \Delta_{\uparrow \downarrow} \cr \Delta_{\downarrow \uparrow} & \Delta_{\downarrow \downarrow} \end{bmatrix}$$ (1)
whose direction $\hat{d}$ parametrizes the Cooper-pair spin state, behaves similarly under spin rotations to the Néel order parameter of an antiferromagnet, i.e., $[S_i(r), d_j(r')]=i\hbar e_{ijk}\delta(r-r')d_k(r)$ and $[d_i, d_j]=0$ for the condensate spin $S$ (unlike the magnetization, neither the Néel order parameter nor the $\hat{d}$-vector generate the spin rotation in themselves) \[21, 11\]. Given that, in both cases, $S \times \hat{d}$ is the conjugate momentum to $\hat{d}$ by the commutation relations, it is natural that the simplest effective Hamiltonian for the spin-triplet superconductor $\hat{d}$-vector,

$$H = \frac{1}{2} \int d\tau [A(\nabla \hat{d})^2 + K \hat{d}_z^2 + \gamma_e^2 S^2/\chi],$$

where $\gamma_e$ is the electron gyromagnetic ratio, $A$ the $\hat{d}$-vector stiffness, and $\chi$ the magnetic susceptibility, should be equivalent to that of the antiferromagnet Néel order parameter, once we identify the $\hat{d}$-vector with the Néel order parameter [4]. This Hamiltonian, which can be constructed from the phenomenological approach of Ref. \[22\], applies to the electron pairing respecting the spin-rotational symmetry. Assuming a rigid $k$-space configuration of $\hat{d}$ at low energy, the low-energy manifold of the theory is parametrized by $\hat{d}(r)$, associated with smooth spatial variations of the triplet order. An easy-plane anisotropy $K$ for planar spin dynamics can be induced perpendicular to an applied magnetic field, analogously to the spin-flop transition in antiferromagnets [22].

In the case of antiferromagnet, a $(xy)$ planar texture of the orientational order parameter $\hat{n} \rightarrow (\cos \phi, \sin \phi, 0)$ is associated with a collective ($z$-polarized) spin current $J_z \propto \hat{z} \cdot (\hat{n} \times \partial_\tau \hat{n}) \rightarrow \partial_\tau \phi$ flowing in the $i$th direction. While this extends directly to our spin-triplet case, Eq. \[11\] gives the intuitive dual picture of Fig. \[11\] (c) for the planar spiraling of the $\hat{d}$-vector, i.e., $\hat{d} = (\cos \alpha, \sin \alpha, 0)$. Namely, as the phase of $\Delta_{\uparrow\uparrow}$ ($\Delta_{\downarrow\downarrow}$) is given by $\phi_e \mp \alpha$ (where $\phi_e$ is the overall phase of the superconductor), the spiraling of the $\hat{d}$-vector on the $xy$ plane as shown in Fig. \[11\] (b), or the gradient of $\alpha$, would imply the counterclockwise rotation of the spin up-down and down-up pairs. The resultant $(z$-polarized) spin current is $\propto -\nabla \alpha$ [22]. Given the same commutation relation and the same effective Hamiltonian, it is natural that, in absence of dissipation, the equations of motion for these two cases, the Leggett equations the $\hat{d}$-vector [8, 11, 22] and the Landau-Lifshitz type equation for the Néel order parameter, are identical.

We further argue that both cases have the same phenomenological form of dissipation as well. For the case of the Néel order parameter $\hat{n}$, such energy dissipation, at the rate $\propto a(\partial_t \hat{n})^2$ for low frequencies, known generally as Gilbert damping for collective magnetic dynamics, has been understood phenomenologically [11, 22, 27]. That such dissipation has not been featured in the $^3$He superfluid literature is due not to the intrinsic nature of the spin-triplet pairing but rather to the $^3$He spin-orbit coupling originating from the very weak nuclear dipole-dipole interaction [8]. In contrast, electrons in Sr$_2$RuO$_4$ are subject to the Ru atomic spin-orbit coupling [28] estimated $\approx 0.1eV$ [29]. In this work, we will consider the decay rate of $an\hbar^2\gamma_e^2/\chi$ for the condensate spin, the addition of which makes the Leggett equations of motion for spin [20] equivalent to the Landau-Lifshitz-Gilbert type equations for antiferromagnets:

$$\partial_t \hat{d} = -\hat{d} \times \frac{\gamma_e^2}{\chi} S,$$

$$\partial_t S = \hat{d} \times (AN^2\hat{d} - K\hat{d}_z \hat{z} - an\hbar\partial_\tau \phi),$$

where $\alpha$ is the dimensionless Gilbert damping parameter and $n$ the Cooper-pair density. Through this set of equations, we can obtain the local $\hat{d}$-vector dynamics, e.g., the spin-wave excitation and the collective dissipation, starting from the effective Hamiltonian of Eq. \[2\].

For the boundary conditions, at the interface between the ferromagnetic lead and the spin-triplet superconductor, we consider a two-channel interface conductance due to the spins aligned or anti-aligned to the lead magnetization. We note that the SrRuO$_4$ thin film has a 50% transport spin polarization [23, 26] with the magnetization enhanced in the heterostructure [17], promising a much higher spin injection/detection efficiency compared to graphene-based devices used in a recent long-range spin transport experiment [34]. In this Letter, we shall consider only the simple case of the collinear lead magnetizations. Furthermore, the $\hat{d}$-vector of the bulk spin-triplet superconductor will be taken to be perpendicular to the lead magnetization, i.e., the Cooper pairs are equal-spin paired along the magnetization direction; for the Sr$_2$RuO$_4$ superconductor, the $c$-axis magnetic field of 200G reportedly suffices for the $\hat{d}$-vector to flop into the $ab$-plane [32]. This interpretation is based on the model of the time-reversal symmetry broken $p$-wave superconductivity [11], which we will follow for the details of our experimental proposals; however the phenomena we predict can arise in any spin-triplet superconductor close to the SO(3) Cooper-pair spin rotational symmetry.

**Long-range spin valve:** The simplest physics that can arise in our two-terminal setup is the spin-valve magnetoresistance due to the lead magnetization alignment. We consider the case where the spin-triplet superconductor has the easy-plane anisotropy, that is, $K > 0$ in Eq. \[2\] (for which $a \geq 200$ G field is applied along the $c$-axis), with the lead magnetization perpendicular to this plane. In this case, we can take $\hat{d}_z$ to be a small parameter in $\hat{d} = (\sqrt{1 - \hat{d}_z^2} \cos \phi_z, \sqrt{1 - \hat{d}_z^2} \sin \phi_z, \hat{d}_z)$ and $|S_{x,y}| \ll |S_z|$. In such a case, $[\phi_z(r), S_z(r')] = i\hbar \delta(r-r')$ gives us the conjugate pair, leading to

$$\partial_t \phi_z = \frac{\gamma_e^2}{\chi} S_z, \quad \partial_t S_z = AN^2 \phi_z - an\hbar\partial_\tau \phi_z,$$

where the first equation is a spin analogue of the Josephson relation and the second is the spin continuity equa-
where the spin down-down condensate phase, conductor is given by the average of the spin up-up and shown that the overall (or charge) phase of the super-aging axis collinear with the lead magnetization and hence the bias voltage of the left (right) lead; this is due to the \( \partial_s \sigma \) where \( g \) the bias voltage of the left (right) lead, and as the easy-axis \( d \)-vector anisotropy favors the alignment along the \( c \)-axis, in the absence of an applied field, the AC bias voltage gives us the low-frequency standing wave of the \( d \)-vector oscillating around the \( c \)-axis in the \( bc \)-plane.

The current through the \( \text{Sr}_2\text{RuO}_4 \) bulk can be obtained from the interface boundary conditions and the continuity conditions above, with the larger magnitude for the parallel magnetization than the antiparallel magnetization. We define the total conductance \( g_{L,R} = \sum \sigma g_{\sigma L,R}^{\alpha} \) and the conductance polarization \( p_{L,R} = \sum \sigma g_{\sigma L,R}^{\alpha} \bar{g}_{L,R} \), which defines the relevant transport spin polarization. Applying the continuity conditions Eq. (3) on the interface boundary conditions Eq. (4) and setting \( V_L = -V_R = V/2 \), we obtain

\[
\left( \frac{g_{L} + g_{R}}{p_{L}g_{L} + p_{R}g_{R}} \right) \left( \frac{\omega_c}{\Omega} \right) = \frac{eV}{\hbar} \left( \frac{g_{L}g_{R} - g_{L}g_{R}}{p_{L}g_{L} - p_{R}g_{R}} \right),
\]

where \( g_\sigma \equiv \frac{4\alpha_{\sigma}e^2SL}{\hbar} \). We can now obtain the dependence of the charge current on the conductance polarization:

\[
I^e = \sum_\sigma I^\sigma = I_0 \left[ 1 - \frac{g_{L}g_{R}(p_{L} - p_{R})^2}{(g_{L} + g_{R})(g_{L} + g_{R} + g_\alpha) - (p_{L}g_{L} + p_{R}g_{R})^2} \right],
\]

where \( I_0 \equiv g_{L}g_{R}V/(g_{L} + g_{R}) \). Note that \( I^e \) is maximized at \( p_L = p_R \), when the steady-state angle \( \phi_{\sigma} \) remains static. Different spin polarizations at the two ends, on the other hand, would trigger spin dynamics and result in a nonzero dissipation rate of \( \frac{1}{4\alpha_{\sigma}eV}\Omega_{\sigma}^2 = R_0(1 - I^e/I_0)^2/(p_L - p_R)^2 \) per volume of the superconducting bulk, where \( R_0 = 8\alpha_{\sigma}(eV)^2/\hbar \). Given that \( p_{L,R} \) change sign on the magnetization reversal, the above results effectively give us the spin-valve magnetoresistance of our heterostructure, i.e., a larger conductance for the parallel magnetizations than for the antiparallel. Any effect that the spin-triplet pairing may have on the magnetization, hence the conductance polarization \([37]\), can be ignored when the Curie temperature of \( \text{SrRuO}_3 \) (\( \sim 160K \)) [38] is two orders of magnitude higher than the superconducting critical temperature (\( \sim 1.5K \)) \( \text{Sr}_2\text{RuO}_4 \).

We emphasize that the above magnetoresistance result is obtain solely for the current carried by Cooper pairs. At a finite-temperature, quasiparticle contribution would generally result in an exponentially-decaying magnetoresistance, negligible for the lead spacing beyond the spin-diffusion length. By contrast, the current of Eq. (6), which is carried by the Cooper pairs, gives us the \( \sim 1/L \) magnetoresistance for the large spacing limit. Therefore, any magnetoresistance beyond the quasiparticle spin-diffusion length should arise only below the superconducting transition at \( T_c \), upon the emergence of a Cooper-pair condensate. For our \( \text{Sr}_2\text{RuO}_4 / \text{SrRuO}_3 \) heterostructure, detection of magnetoresistance in the superconducting state for the lead spacing larger than the \( \text{Sr}_2\text{RuO}_4 \) spin-diffusion length can be taken as a trans-
port evidence for the spin-triplet superconductivity. The value of the spin-diffusion length itself can be extracted by measuring the exponential decay of the (normal) magnetoresistance, both above and below the transition.

Electrically driven spin collective mode: For the case of the easy-axis anisotropy of the d-vector, hence $K < 0$ in Eq. (2), the spin collective excitation of the Cooper pairs \[K \rho \approx \sqrt{\omega} \partial \phi \partial t,\] where $\phi$ is conjugate to $S_x$ and $\omega_0^2 = |K|/|\gamma_c|$ is the spin-wave energy gap. For the AC voltage bias $V = V_0 \exp(-i\omega t)$ at the frequencies far below the plasma frequency, the steady-state solution for the spin phase $\phi(x, t) = f(x) \exp(-i\omega t)$ and the charge phase $\phi_c(x, t) = g(x) \exp(-i\omega t)$ behave differently. Hence the spin equations of motion Eq. (9) gives us $f(x) = C_+ \cos kx \pm C_- \sinh kx$, where $\nu^2 \lambda_s^2 = \omega^2 - \omega_0^2 - i\omega \Gamma$, with $\nu \equiv \gamma_c \sqrt{\lambda_s/\chi}$ (the d-vector stiffness $\Lambda$ defined in Eq. (2)) being the spin-wave velocity and $\Gamma \equiv \alpha h \gamma_c^2 / \chi$ the damping rate. By contrast, the charge current $J(\partial^2) \phi_c(x, t) = \partial \partial \phi_c$, where $\rho$ is the charge stiffness, should be uniform, which means we can set $\phi_c(x, t) = \text{const.} - x(J_0^c/\rho) \exp(-i\omega t)$, with a constant $J_0^c$. By imposing boundary conditions of Eq. (6) and the dynamics of Eq. (4), we can solve for $J_0^c$ and $C_\pm$: Fig. 3 shows the numerical results for $J_0^c \equiv J_0^c S$ for the case of both $p_L = p_R$ and $p_L = -p_R$.

Our numerical results show that magnetoresistance becomes significant at $\omega \gtrsim \omega_0$, where the collective spin mode of the Cooper pairs is activated. For simplicity we have set $g_L = g_R = g$ and used the dimensionless parameters $\tilde{g} = g \nu / 2 \Lambda, \tilde{L} = \omega_0 L / 2 \nu$, and $\tilde{\Lambda} = \Lambda / \rho$. For $\omega < \omega_0$, in addition to barely noticeable magnetoresistance, the charge current amplitude does not oscillate with frequency; it remains close to the DC value $I_0$, unlike the complete transport suppression in the magnetic insulator. In contrast, for $\omega > \omega_0$, we see an oscillation with the $\omega/\omega_0$ period of about $\pi/\tilde{L}$, where the current amplitude maxima for the antiparallel lead magnetization occur at the current amplitude minima for the parallel lead magnetization and vice versa. As in the ferromagnetic insulator, we expect that for $\tilde{L} \ll 1$, i.e., much shorter than the d-vector relaxation length \[L_d \approx 0.2, 0.8\] for the solid and dashed lines, respectively.
The lower limit for this parameter is estimated to be in the order of $v/\omega_0 \sim 10\mu m$. Hence, our formalism for collective spin dynamics in spin-triplet superconductors could be extended analogously.


[28] A strong electron density nonuniformity may affect the spin relaxation of the Cooper-pair condensate, as the magnitude of the orbital hybridization induced by the Ru spin-orbit coupling at the Fermi surface depends on the chemical potential.


[30] Following [? ], we leave out in this work for simplicity any multi-orbital effects in Sr$_2$RuO$_4$. [? ].


[36] $I_s$ should be longer than the superconductor coherence length if the conductance at two leads were to be independent.


[41] $v/\omega_0 \sim 10\mu m$. 

[42] For general spin-triplet dynamics, the low-energy order-parameter manifold would consist of the SO(3) rotations, analogous to noncollinear antiferromagnets or spin glasses discussed in, e.g., Ref. [21].


[48] A strong electron density nonuniformity may affect the spin relaxation of the Cooper-pair condensate, as the magnitude of the orbital hybridization induced by the Ru spin-orbit coupling at the Fermi surface depends on the chemical potential.


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