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Nonreciprocal Photon Blockade

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We propose how to create and manipulate one-way nonclassical light via photon blockade in rotating nonlinear devices. We refer to this effect as nonreciprocal photon blockade (PB). Specifically, we show that in a spinning Kerr resonator, PB happens when the resonator is driven in one direction but not the other. This occurs because of the Fizeau drag, leading to a full split of the resonance frequencies of the counter-circulating modes. Different types of purely quantum correlations, such as single- and two-photon blockades, can emerge in different directions in a well-controlled manner, and the transition from PB to photon-induced tunneling is revealed as well. Our work opens up a new route to achieve quantum nonreciprocal devices, which are crucial elements in chiral quantum technologies or topological photonics.

Nonreciprocal devices, allowing the flow of light from one side but blocking it from the other, are indispensable in a wide range of practical applications, such as invisible sensing or cloaking, and noise-free information processing [1]. To avoid the difficulties of conventional magnet-based devices (e.g., bulky and quite lossy at optical frequencies), nonreciprocal optical devices have been demonstrated in recent experiments based on nonlinear optics [2, 3], optomechanics [4–6], atomic gases [7, 8], and non-Hermitian optics [9–11]. Similar advances have also been achieved in making acoustic and electronic one-way devices [12–17]. However, previous studies have mainly focused on the *classical* regimes, i.e., one-way control of transmission rates instead of quantum noises. Nonreciprocal quantum devices have been explored very recently, including one-way quantum amplifiers [18–24] and routers of thermal noises [25]. Such devices can find applications for quantum control of light in chiral and topological quantum technologies [26–28].

Here we propose how to induce and control nonreciprocal *quantum* effects with rotating nonlinear devices. Specifically, we show that photon blockade (PB), which is a purely quantum effect, can emerge nonreciprocally in a spinning Kerr resonator. We note that single-photon blockade (1PB), i.e., blockade of the subsequent photons by absorbing the first one [29–32], has been demonstrated experimentally in diverse systems from cavity or circuit QED [33–40] to cavity-free devices [41]. In view of its important role in achieving single-photon devices, optomechanical PB [42–45] have also been explored, offering a way to test, e.g., the quantumness of massive objects [46–50]. In a very recent experiment [51], two-photon blockade (2PB) [31, 52–59] has also been observed, opening a route for creating two-photon devices. Thus, nonreciprocal PB devices, as studied here, together with other nonreciprocal quantum devices [18–23, 25], are expected

to play a key role in quantum engineering [60–62], metrology [63–65], and quantum information processing [66, 67] at the single- or few-photon levels.

In a very recent experiment [68], an optical diode with 99.6% isolation has been demonstrated by using a spinning resonator. Inspired by this experiment [68], here we study nonreciprocal PB in a spinning Kerr resonator. We find that light with *sub-* or *super-Poissonian* photon-number statistics can emerge when driving the resonator from its left or right side. Also, by varying the parameters of the system, different quantum correlations (i.e., 1PB or 2PB) can be achieved for the clockwise (CW) or counter-clockwise (CCW) modes, for a resonator spinning along the CCW direction. We note that the main idea of nonreciprocal PB is analogous to the classical nonreciprocity induced by the Doppler effect, which has been studied extensively in various areas of physics (see, e.g., [7, 8, 69]). Here we focus on *quantum* nonreciprocity induced by the Fizeau light-dragging effect. This opens up the prospect of engineering nonreciprocal PB devices for applications in e.g., unidirectional quantum sensing and quantum optical communications [28].

Model.—We consider a spinning optical Kerr resonator as shown in Fig. 1. As a generic PB model [30, 32, 53], Kerr interactions can also be experimentally achieved in cavity-atom systems [33, 70], or magnon devices [71], and theoretically in optomechanical systems [42, 43]. For a resonator spinning at an angular velocity Ω , the light circulating in the resonator experiences a Fizeau shift, i.e., $\omega_0 \rightarrow \omega_0 + \Delta_F$, with [72]:

$$\Delta_F = \pm \frac{nr\Omega\omega_0}{c} \left(1 - \frac{1}{n^2} - \frac{\lambda}{n} \frac{dn}{d\lambda} \right), \quad (1)$$

where ω_0 is the resonance frequency of a non-spinning resonator, n is the refractive index, r is the resonator radius, and c (λ) is the speed (wavelength) of light in

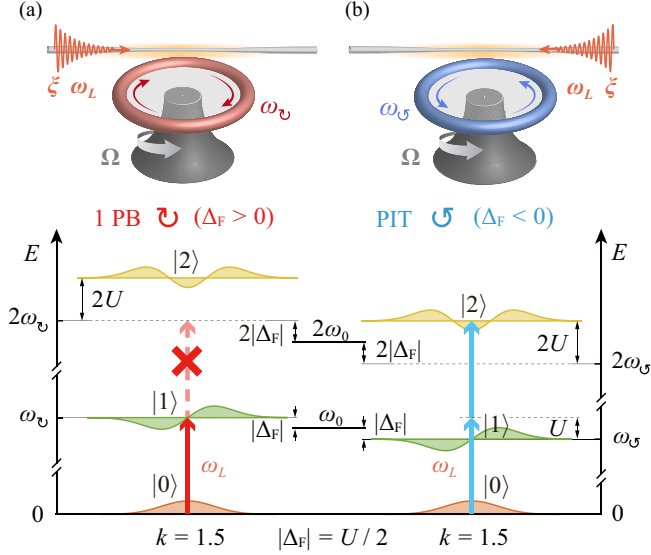


FIG. 1. Nonreciprocal 1PB in a spinning Kerr resonator. 1PB arises due to the anharmonic spacing of the energy levels $|n\rangle$. Here we take $n = 0, 1, 2$, and $\hbar = 1$, for simplicity. By fixing the CCW rotation of the resonator (the angular speed Ω fulfills the condition $\Delta_F = \pm U/2$), under the same driving power $P_{\text{in}} = 2 \text{ fW}$ and the same detuning $\Delta_L = -U/2$, i.e., $k = 1 - \Delta_L/U = 1.5$, (a) 1PB emerges by driving the device from its left side ($\Delta_F > 0$), while (b) PIT caused by two-photon resonance occurs by driving from the right side ($\Delta_F < 0$). This PIT exhibits $g^{(\mu)}(0) > 1$ ($\mu = 2, 3, 4$) [73].

vacuum. Usually, the dispersion term $dn/d\lambda$, characterizing the relativistic origin of the Sagnac effect, is relatively small (up to $\sim 1\%$) [68, 72]. We fix the CCW rotation of the resonator, hence $\Delta_F > 0$ ($\Delta_F < 0$) corresponds to the situation of driving the resonator from its left (right) side, i.e., the CW and CCW mode frequencies are $\omega_{\circ, \ominus} \equiv \omega_0 \pm |\Delta_F|$, respectively.

In a frame rotating at driving frequency ω_L , the effective Hamiltonian of the system can be written at the simplest level as [73]:

$$\hat{H} = \hbar(\Delta_k + \Delta_F)\hat{a}^\dagger\hat{a} + \hbar U\hat{a}^\dagger\hat{a}(\hat{a}^\dagger\hat{a} - k) + \hbar\xi(\hat{a}^\dagger + \hat{a}), \quad (2)$$

where $\Delta_k = \Delta_L + U(k - 1)$, $\Delta_L = \omega_0 - \omega_L$, the tuning parameter k is simply $k = 1 - \Delta_L/U$ for $\Delta_k = 0$, \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator of the cavity field, and $\xi = \sqrt{\gamma P_{\text{in}}}/(\hbar\omega_L)$, with the cavity loss rate γ and the driving power P_{in} . The Kerr parameter is [74]: $U = \hbar\omega_0^2 cn_2/(n_0^2 V_{\text{eff}})$, where n_0 (n_2) is the linear (nonlinear) refraction index, and V_{eff} is the effective mode volume. The Kerr coupling is also attainable by using other kinds of devices [33, 42, 43, 70, 71]. Note that the term Δ_F makes Eq. (2) fundamentally different from that used for studying conventional PB [53].

The energy eigenstates of this system are the Fock states $|n\rangle$ ($n = 0, 1, 2, \dots$) with eigenenergies

$$E_n = n\hbar\Delta_L + (n^2 - n)\hbar U \pm n\hbar|\Delta_F|, \quad (3)$$

where n is the cavity photon number. The second term, with U , leads to an anharmonic energy-level structure. The last term, with $\pm|\Delta_F|$, describing upper or lower shifts of energy levels with an amount being proportional to Ω , is the origin of nonreciprocal implementations of PB. When $|\Delta_F| = U/2$ and the probe with frequency $\omega_0 + |\Delta_F|$ ($k = 1.5$) comes from the left side, the light is resonantly coupled to the transition $|0\rangle \rightarrow |1\rangle$. As shown in Fig. 1(a), the transition $|1\rangle \rightarrow |2\rangle$ is detuned by $2\hbar U$ and, thus, suppressed for $U > \gamma$, i.e., once, a photon is coupled into the resonator, it suppresses the probability of the second photon with the same frequency going into the resonator. In contrast, by driving from the right side, there is a two-photon resonance with the transition $|0\rangle \rightarrow |2\rangle$, hence the absorption of the first photon favors also that of the second or subsequent photons, i.e., resulting in photon-induced tunneling (PIT), as defined below and shown in Fig. 1(b). This is a clear signature of nonreciprocal 1PB, i.e., *sub-Poissonian* light emerges by driving the system from one side, while *super-Poissonian* light emerges by driving from the other side.

Analytical results.—To confirm this intuitive picture, we study the μ th-order ($\mu = 2, 3$) correlation function with zero-time delay, i.e., $g^{(\mu)}(0) \equiv \langle \hat{a}^{\dagger\mu}\hat{a}^\mu \rangle / \langle \hat{n} \rangle^\mu$, with $\hat{n} = \hat{a}^\dagger\hat{a}$. The condition $g^{(2)}(0) > 1$ [$g^{(2)}(0) < 1$] characterizes PIT [34, 75] (1PB) via super-Poissonian (sub-Poissonian) photon-number statistics or photon bunching (antibunching) [76, 77]. The latter terms can also refer to different (i.e., two-time) optical correlation effects [77, 78], which are, however, not studied here. We stress that, although PIT has a classical-like property of super-Poissonian photon-number statistics [75, 79, 80], it is a purely quantum effect [34]. The analysis of higher-order correlation functions $g^{(\mu)}(0) > 1$ with $\mu > 2$ can reveal the relation of a particular PIT and multi-PB [73]. Thus, more refined criteria for PIT are sometimes applied [50, 79, 81], and we refer here to PIT if the conditions $g^{(\mu)}(0) > 1$ for $\mu = 2, 3, 4$ are satisfied [73]. We also note that partially coherent mixtures of the vacuum, and single- and multi-photon states, as generated here, can be described by μ th-order super-Poissonian correlations, i.e., $g^{(\mu)}(0) > 1$, for specific values of μ [82]. Particularly, $g^{(3)}(0) < 1$ [$g^{(3)}(0) > 1$] is a signature of third-order sub-Poissonian (super-Poissonian) statistics, which is also interpreted as three-photon antibunching (bunching) in recent experiments on multi-PB [51] and PIT [81]. Thus, $g^{(3)}(0)$, which is usually measured with extended Hanbury Brown-Twiss interferometers, provides a more refined test and classification of the nonclassical character of light, including 2PB (as studied below) or unconventional PB [83].

According to the quantum-trajectory method [84], the optical decay can be included in the effective Hamiltonian $\hat{H}_s = \hat{H} - (i\hbar\gamma/2)\hat{a}^\dagger\hat{a}$, where $\gamma = \omega_0/Q$ is the cavity dissipation rate and Q is the quality factor. In the weak-driving regime ($\xi \ll \gamma$), by truncating the Hilbert space

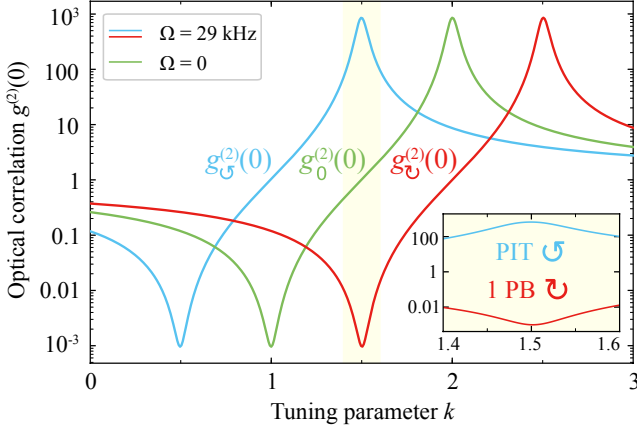


FIG. 2. The second-order correlation function $g^{(2)}(0)$ versus the tuning parameter k for different input directions. At $k = 1.5$, 1PB (red curve) or PIT (blue curve) occurs by driving the device from the left or right side, with the same strength. Here $P_{\text{in}} = 2 \text{ fW}$, $\Omega = 29 \text{ kHz}$ for the spinning resonator, and $g_0^{(2)}(0)$ corresponds to a non-spinning resonator (green). Note that Ω is related to Δ_F by Eq. (1). For the other parameter values, see the main text. On the scale of this figure, there are no differences between our numerical and (approximate) analytical results [73].

to $n = 2$, the state of this system is written as $|\varphi(t)\rangle = \sum_{n=0}^2 C_n(t)|n\rangle$, with probability amplitudes C_n . Then we have the following equations of motion

$$\begin{aligned} \dot{C}_0(t) &= -i\nu_0 C_0(t) - i\xi C_1(t), \\ \dot{C}_1(t) &= -i(\nu_1 - i\frac{\gamma}{2})C_1(t) - i\xi C_0(t) - i\xi\sqrt{2}C_2(t), \\ \dot{C}_2(t) &= -i(\nu_2 - i\gamma)C_2(t) - i\xi\sqrt{2}C_1(t), \end{aligned} \quad (4)$$

with $\hbar\nu_n = E_n$, $C_0(0) = 1$, $C_1(0) = C_2(0) = 0$. Solving these equations (and dropping higher-order terms) leads to the steady-state solutions

$$C_1(\infty) = \frac{-\xi}{(\nu_1 - \nu_0 - i\frac{\gamma}{2})}, \quad C_2(\infty) = \frac{-\sqrt{2}\xi C_1(\infty)}{(\nu_2 - \nu_0 - i\gamma)}. \quad (5)$$

Denoting the probability of finding m photons in the resonator by $P(m) = |C_m|^2$, we have

$$g^{(2)}(0) = \frac{2P_2}{(P_1 + 2P_2)^2} \simeq \frac{(\Delta_L + \Delta_F)^2 + \gamma^2/4}{(\Delta_L + \Delta_F + U)^2 + \gamma^2/4}. \quad (6)$$

1PB and PIT correspond to the minimum and the maximum of $g^{(2)}(0)$, respectively, i.e., when $U > \gamma$, $g_{\text{min}}^{(2)}(0) = 1/[4(U/\gamma)^2 + 1] < 1$ for $\Delta_L = -\Delta_F$, and $g_{\text{max}}^{(2)}(0) = 4(U/\gamma)^2 + 1 > 1$ for $\Delta_L = -\Delta_F - U$.

Numerical results.—In order to confirm our analytical results, now we numerically study the full quantum dynamics of the system. We introduce the density operator $\hat{\rho}(t)$ and then solve the master equation [85, 86]:

$$\dot{\hat{\rho}} = \frac{i}{\hbar}[\hat{\rho}, \hat{H}] + \frac{\gamma}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}). \quad (7)$$

The photon-number probability $P(n) = \langle n|\hat{\rho}_{\text{ss}}|n\rangle$ can be obtained for the steady-state solutions $\hat{\rho}_{\text{ss}}$ of the master equation. The experimentally accessible parameters are chosen as [87–91]: $V_{\text{eff}} = 150 \mu\text{m}^3$, $Q = 5 \times 10^9$, $n_2 = 3 \times 10^{-14} \text{ m}^2/\text{W}$, $n_0 = 1.4$, $P_{\text{in}} = 2 \text{ fW}$, $r = 30 \mu\text{m}$, and $\lambda = 1550 \text{ nm}$. V_{eff} is typically 10^2 – $10^4 \mu\text{m}^3$ [87, 88], Q is typically 10^9 – 10^{12} [89, 90], and $g^{(2)}(0)$ as low as ~ 0.13 was achieved experimentally [33]. Moreover, in Fig. 2, we set $\Omega = 29 \text{ kHz}$; a similar property of quantum nonreciprocity is also confirmed for $\Omega = 6.6 \text{ kHz}$ (see the Supplemental Material [73]). These values of Ω are experimentally feasible [68]. Very recently, spinning objects have reached much higher velocities, reaching the GHz regime [92, 93]; such systems could also be applied to study the nonreciprocal PB via Kerr-like optomechanical interactions [94, 95]. We note that the Kerr coefficient can be $n_2 \sim 10^{-14} \text{ m}^2/\text{W}$ for materials with potassium titanyl phosphate [91], and n_2 can be further enhanced with various techniques [96–101], e.g., feedback control [100, 101] or quadrature squeezing [98, 99].

An excellent agreement between our analytical results and the exact numerical results is seen in Fig. 2. Here we use $g_0^{(2)}(0)$, $g_{\text{C}}^{(2)}(0)$, and $g_{\text{C}}^{(2)}(0)$ to denote the cases with $\Delta_F = 0$, $\Delta_F > 0$, and $\Delta_F < 0$, respectively. For a non-spinning resonator, regardless of the driving direction, $g_0^{(2)}(0)$ always has a dip at $k = 1$ (i.e., $\Delta_L = 0$) or a peak at $k = 2$ (i.e., $\Delta_L = -U$), corresponding to 1PB or PIT, respectively. In contrast, for a spinning device, by driving from the left (right) side, we have $\Delta_F > 0$ ($\Delta_F < 0$) and, thus, a red (blue) shift for $g^{(2)}(0)$, leads to 1PB (PIT) at $k = 1.5$, i.e., $g_{\text{C}}^{(2)}(0) \sim 0.001$, $g_{\text{C}}^{(2)}(0) \sim 673$. This quantum nonreciprocity, with up to *six orders of magnitude* difference of $g^{(2)}(0)$ for opposite directions, is fundamentally different from the classical transmission-rate nonreciprocity.

Nonreciprocal 2PB.—The absorption of 2 photons can also suppress the absorption of additional photons [53]. This 2PB effect, featuring three-photon antibunching, but with two-photon bunching, satisfies [51, 73]:

$$g^{(3)}(0) < f \equiv e^{-\langle \hat{n} \rangle}, \quad g^{(2)}(0) \geq f^{(2)} \equiv e^{-\langle \hat{n} \rangle} + \langle \hat{n} \rangle \cdot g^{(3)}(0). \quad (8)$$

The third-order correlation function can be obtained analytically as [73]:

$$g^{(3)}(0) = \frac{6P_3}{(P_1 + 2P_2 + 3P_3)^3} \simeq \frac{(\Delta^2 + \gamma^2/4)g^{(2)}(0)}{(\Delta + 2U)^2 + \gamma^2/4}, \quad (9)$$

with $\Delta = \Delta_L + \Delta_F$, also agreeing well with the numerical results. Figures 3(a) and 3(b) show that 2PB emerges around $k = 2.5$ by driving from the left side, while we have PIT by driving from the right side, i.e., $g_{\text{C}}^{(2)}(0) \sim 36$, $g_{\text{C}}^{(3)}(0) \sim 1003$. By tuning the driving frequency to the three-photon resonance [see Fig. 3(d)], it is indeed possible to observe that $g^{(3)}(0)/g^{(2)}(0) \sim 100$, as shown in Fig. 3(a) for $\max\langle n \rangle = 0.0185$. This means that the prob-

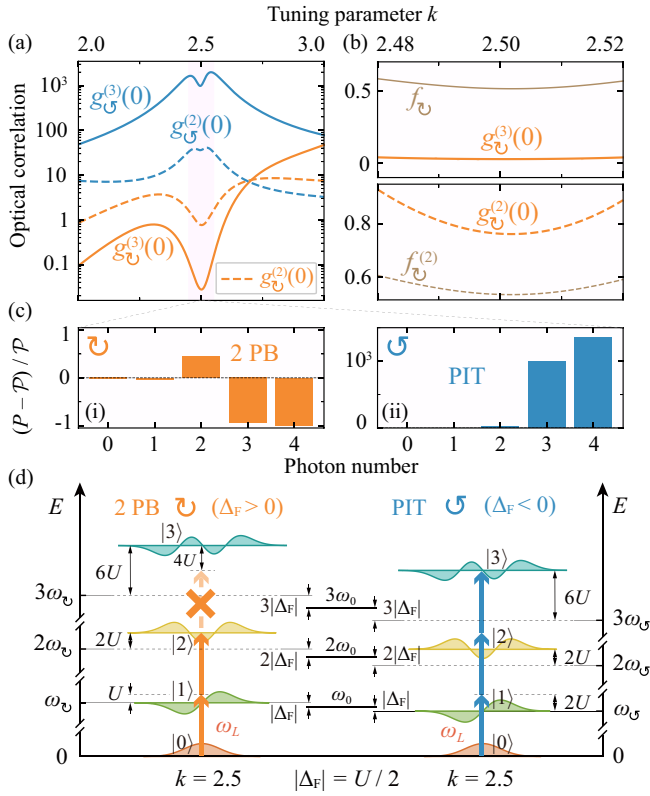


FIG. 3. (a) The correlation functions $g^{(3)}(0)$ (solid curves) and $g^{(2)}(0)$ (dashed curves) versus the tuning parameter k for different driving directions. Note that at $k = 2.5$, 2PB can emerge by driving the system from the left side (orange), while PIT occurs by driving from the right side (blue). In (b), 2PB is confirmed by the criteria given in Eq. (8) for the CW mode. (c) This nonreciprocal 2PB can also be recognized from the deviations of the photon distribution to the standard Poisson distribution with the same mean photon number. (d) The energy-level diagram shows the origin of this unidirectional 2PB: with enhanced driving power $P_{\text{in}} = 0.3$ pW, by choosing $\Delta_L = -3U/2$ (i.e., $k = 2.5$), 2PB emerges by driving the device from the left ($\Delta_F > 0$), while three-photon resonance-induced PIT emerges by driving from the right side ($\Delta_F < 0$). The other parameters are the same as those in Fig. 2.

ability of simultaneously measuring three photons can be much larger than that of two photons in this situation. Similar values of $g^{(3)}(0) \sim 10^3$, $g^{(2)}(0) \sim 10$ were also predicted in the PIT analysis in Ref. [81].

Our results can be further confirmed by comparing the photon-number distribution $P(n)$ with the Poisson distribution $\mathcal{P}(n)$. Figure 3(c) shows that $P(2)$ is enhanced while $P(n > 2)$ are suppressed by driving from the left side, which is in *sharp contrast* to the case when driving from the right side. This unidirectional 2PB effect can be intuitively understood by considering the energy-level structure of the system, as shown in Fig. 3(d). By choosing $\Delta_L = -3U/2$ or $k = 2.5$, the transition $|0\rangle \rightarrow |2\rangle$ is resonantly driven by the left input laser, but the transition $|2\rangle \rightarrow |3\rangle$ is detuned by $4\hbar U$, which features the 2PB

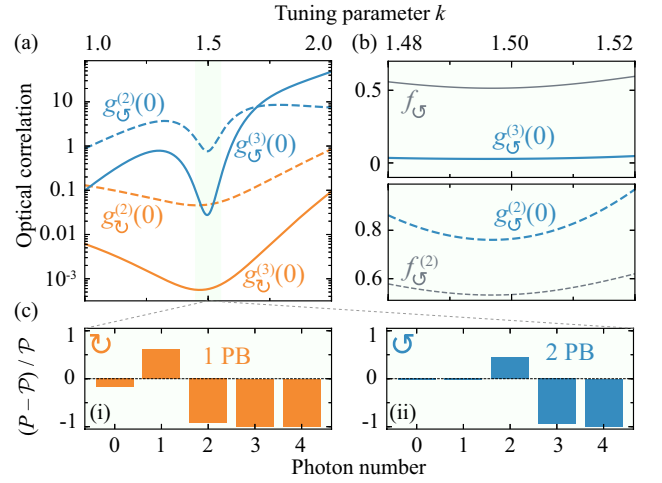


FIG. 4. (a) The correlation functions $g^{(3)}(0)$ (solid curves) and $g^{(2)}(0)$ (dashed curves) versus the tuning parameter k for different driving directions. 1PB can emerge around $k = 1.5$ by driving from the left side (orange), while 2PB occurs by driving from the right side (blue). In (b), 2PB is confirmed by the criteria given in Eq. (8) for the CCW mode. (c) This 1PB-2PB nonreciprocity can also be recognized from the relative photon population numbers in the resonator. For all plots, the parameters are the same as those in Fig. 3.

effect; in contrast, by driving from the right side, three-photon resonance happens for the transition $|0\rangle \rightarrow |3\rangle$, leading to PIT. Hence with such a device, *sub-Poissonian* light can be achieved by driving it from the left side, while *super-Poissonian* light is observed by driving it from the right side.

Nonreciprocity of 1PB and 2PB.—Fig. 4 shows that at $k = 1.5$, 1PB emerges by driving from the left side, due to $g_{\text{C}}^{(2)}(0) \sim 0.045$, while 2PB occurs by driving from the right side since the criteria given in Eq. (8) are fulfilled for $\Delta_F < 0$. This indicates a purely quantum device with *direction-dependent counting statistics*, a new nonreciprocal feature, which has not been revealed previously. This 1PB-2PB nonreciprocity, as also clearly seen in Fig. 4(c) for the populations of different Fock states, provides a route for creating or processing different quantum states in a single node of quantum networks [66, 67]. Figures 3–4 present our solutions of the standard master equation, given in Eq. (7), which describes both a slow continuous non-unitary evolution and quantum jumps occurring with a small probability [102]. By contrast, our approximate analytical solutions, based on the complex Hamiltonian H_s and the Schrödinger equation, were obtained by ignoring these quantum jumps following the standard approach of [103].

Conclusions.—We have studied nonreciprocal PB effects in a spinning Kerr resonator. By fixing the CCW rotation of the resonator, we find that, (i) for $P_{\text{in}} = 2$ fW, $\Delta_{\text{sag}} = \pm U/2$ and $k = 1.5$, we have 1PB and PIT for the CW and CCW modes, respectively; (ii) for $P_{\text{in}} = 0.3$ pW,

$\Delta_{\text{sag}} = \pm U/2$ and $k = 2.5$, we have 2PB and PIT for the CW and CCW modes, respectively; more interestingly, (iii) for $P_{\text{in}} = 0.3$ pW, $\Delta_{\text{sag}} = \pm U/2$ and $k = 1.5$, we have 1PB and 2PB for the CW and CCW modes, respectively (for more examples, see the Supplementary Material [73]). These results can be useful in achieving e.g., nonreciprocal few-photon sources and quantum one-way devices.

The basic mechanism of this work can be generalized to a wide range of systems, such as acoustic and electronic devices [12–17], to achieve e.g., nonreciprocal phonon blockade [46–48], as a test of the quantumness of mechanical devices [77]. Our work can also be extended to study, e.g., nonreciprocal photon turnstiles [104], nonreciprocal photon routers [105–107], and nonreciprocal extraction of a single photon from a laser pulse [108], by considering a hybrid device with atoms [109, 110], quantum dots [111], or nitrogen-vacancy centers [112].

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