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Spin Transport and Accumulation in a 2D Weyl Fermion System

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In this work, we demonstrate a new route to realizing an experimentally controllable Spin Hall effect (SHE), on the surface of 3D topological insulators, which depends fundamentally on the spin-orbit coupled nature of the 2D Weyl fermions. Spin transport is solely due to the spin-torque current in this system, and the SHE arises from chiral spin-flip scattering off non-SOC scalar impurities, of potential strength V and size a . The resultant spin-Hall angle has a fixed sign, with $\theta^{SH} \sim O\left(\frac{v^2}{v_F^2/a^2}(k_F a)^4\right)$ being a strongly-dependent function of the Fermi wave-vector (k_F) and Fermi velocity (v_F). Hence, the SHE can be tuned by adjusting the Fermi energy or impurity size.

The spin Hall effect (SHE) has a long and rich history, starting with the initial proposal of asymmetric Mott scattering by Dyakonov and Perel [1, 2]. This extrinsic mechanism was re-introduced in 1999[3, 4], while an intrinsic SHE was first proposed in 2003[5, 6]. The proposal of a two-dimensional (2D) Z_2 -protected Quantum Spin Hall (QSH) state[7], and its successful prediction in HgTe/CdTe quantum well [8] quickly followed; thus giving rise to a new field of topological materials[9, 10], which now include 2D QSH states [11], 3D topological insulators (TI)[12, 13], topological Kondo insulators[14, 15] and Weyl semi-metals[16].

We propose a new route for realizing an experimentally controllable Spin Hall effect on the surface of 3D topological materials, by utilizing the spin-orbit coupled (SOC) nature of chiral Weyl fermions, on the surface of 3D TI materials, which are expected to provide highly efficient spin-charge conversion[17, 18], via the spin Hall effect or spin accumulation in the Rashba-Edelstein effect[19]. Hence, there is a strong interest in spintronic TI heterostructures, with many theoretical works[20–25], discussing a plethora of spin-charge phenomena, including magnetoresistance effects, inverse spin-galvanic effect, and spin-transfer torque, which have stimulated a flurry of experimental efforts[18, 26–30].

In heavy-metal/ferromagnet systems, e.g. FePt/Au, a giant spin Hall angle (SHA) of ~ 0.1 has been reported[31], which has been interpreted as resonant skew-scattering off the Fe impurities[32]. Furthermore, recent experiments on TI heterostructures[26, 29] have reported values of $\tan \theta^{SH} > 100\%$, with combined surface and bulk contributions. In order to disentangle the surface Weyl fermion contribution from the bulk bands, a Cu-layer inserted TI/Cu/ferromagnet heterostructure has recently been engineered, with $\tan \theta^{SH} \sim 50\%$ [30].

Similar to the anomalous Hall effect, there are both intrinsic Berry curvature and extrinsic scattering contributions to the SHE. For systems with weak SOC, it has been shown[33] that the extrinsic skew scattering mechanism dominates in the clean limit; hence, the spin Hall conductivity σ_{xy}^z scales with the longitudinal conductivity σ_{yy} ,

and the SHA, $\theta^{SH} = \frac{\sigma_{xy}^z}{\sigma_{yy}}$ is a well-defined measure of the SHE. The Rashba-Edelstein effect is a closely related transport-driven spin accumulation phenomenon, which also scales with σ_{yy} in the clean limit; the spin accumulation $\langle S^i \rangle = \sigma_\alpha^i E_\alpha$ is proportional to the applied electric field E_α (along α -direction) with a coefficient σ_α^i . In contrast to typical heavy-metal/ferromagnet systems, scattering off non-magnetic impurities is sufficient to drive an $O(\frac{1}{n_i})$ skew-scattering mechanism and an $O(\frac{1}{\gamma_t})$ Rashba-Edelstein effect, due to the chiral nature of the Weyl fermions; here, n_i is the impurity concentration and γ_t is the transport scattering rate.

In spin-orbit coupled systems, the proper definition of the spin current is more subtle as spin is not a conserved quantity. Reference [34] presented a bulk conserved spin current that satisfies a continuity equation, $\frac{dS^z}{dt} + \nabla \cdot (\vec{J}_s + \vec{P}_\tau) = 0$, with an additional spin-torque density term, $\nabla \cdot \vec{P}_\tau = \frac{i}{\hbar} [S^i, H^0]$, as well as the conventional spin current $\vec{j}_\tau^z = \psi^\dagger \frac{1}{2} \{\vec{v}, S^z\} \psi$. Hence, the transport spin current is the sum of a spin-polarized and a spin-torque current, $\vec{J}_s^i = \vec{j}_s^i + \vec{P}_\tau^i$, succinctly expressed as the time-derivative of a spin-dipole operator, $\hat{J}_s = \frac{d(\hat{\vec{r}}\hat{\vec{S}})}{dt}$.

As pointed out by several groups[35–37], there is no finite conventional spin current for Weyl systems; hence, spin transport for Weyl fermions is solely due to the spin-torque density P_τ coming from quantum-mechanical evolution of the electron spin. The spin torque current, P_x^z , arises from the intrinsic quantum-mechanical evolution of the electron spin, and the z -component of the spin-torque current along $\hat{x}$ is,

$$\begin{aligned} P_x^z(\vec{k}) &= \frac{i}{k_x} \frac{d\hat{S}^z(\vec{k})}{dt} \\ &= \frac{2v_F}{ik_x} \sum_{\vec{p}} c_{\vec{p},\sigma}^\dagger \left[\left(\vec{p} + \frac{\vec{k}}{2} \right)_y \sigma^x - \left(\vec{p} + \frac{\vec{k}}{2} \right)_x \sigma^y \right] c_{\vec{p}+\vec{k},\sigma'} \end{aligned} \quad (1)$$

We consider elastic scattering near the Fermi energy, E_F , of 2D Weyl fermions (Dresselhaus-type $v_F \vec{k} \cdot \vec{\sigma}$ system) from a dilute ($n_i \ll 1$) random distribution of non-magnetic impurities, with scattering off each impurity

FIG. 1: Figure (a) shows a colour density plot of the Fermi surface (FS) contribution to the Rashba-Edelstein effect $\langle \sigma_y^y \rangle$ (Eq. (4c)). Note that the spin direction is rotated by 90° compared to the usual Rashba direction. When the FS is shifted by $\delta k_y = eE_y \tau_t$ (shown as a grey dashed circle) due to an external electric field E_y , the non-equilibrium distribution gives rise to a net $\langle S^y \rangle$, and $\tau_t = \frac{1}{\gamma_t}$ is the transport relaxation time. The spin texture of the Weyl fermions on the FS is indicated by the red arrows. Figure (b) illustrates spin-dependent skew scattering, $\bar{T}_{\sigma,\sigma'}(\vec{k}, \vec{k}')$ and $\bar{T}_{\sigma',\sigma}(\vec{k}', \vec{k})$ having positive (ν) and negative ($-\nu$) chirality respectively, with the helical Weyl fermions defining positive (ν) chirality.

given by $H^{imp} = \sum_{\vec{r}} c_{\sigma}^{\dagger}(\vec{r}) V e^{-\frac{|\vec{r}|^2}{a^2}} c_{\sigma}(\vec{r})$, with impurity size a . Note that the results can be easily translated into the Rashba-type $v_F \hat{z} \times \vec{k} \cdot \vec{\sigma}$ case via rotation of the momentum by 90° . Choosing the chemical potential μ to lie in the upper helical band, we obtain the following Hamiltonian as,

$$H = H^0 + H^{imp} \quad (2)$$

$$H^0 = \sum_{\vec{k}, \alpha, \beta} c_{\vec{k}, \alpha}^{\dagger} v_F \vec{k} \cdot \vec{\sigma}_{\alpha, \beta} c_{\vec{k}, \beta} - \mu c_{\vec{k}, \alpha}^{\dagger} c_{\vec{k}, \alpha} \quad (3)$$

$$H^{imp} = \sum_{\vec{k}, \vec{k}'} c_{\vec{k}, \alpha}^{\dagger} V_{\vec{k}, \vec{k}'} c_{\vec{k}', \alpha}$$

Here, $V_{\vec{k}, \vec{k}'} = \sum_n V_n e^{in(\theta_k - \theta_{k'})}$, and $V_n \approx \frac{V a^2}{2^n \Gamma(\frac{n+1}{2})} (k_F a)^n$, while v_F and $\sigma^i \in [\mathbb{1}, \vec{\sigma}]$ are the Fermi velocity and spin Pauli matrices, and $k_F a$ determines V_n , which will be shown to control the skew scattering strength. Since the impurity is non-magnetic, the system is invariant under time-reversal symmetry, $H = \mathcal{T} H \mathcal{T}^{-1}$; with $\mathcal{T} = \mathcal{K} i \sigma_2$ being the time-reversal symmetry operator and $\mathcal{K}$ is complex conjugation.

We utilize the Kubo formalism for calculating the spin and charge conductivities, σ_{xy}^z and σ_{yy} , and the Rashba-

Edelstein coefficient,

$$\sigma_{xy}^z = -\hbar e v_F^2 \frac{N_0(E_F)}{2} \frac{1}{\gamma_t} \frac{\gamma_s}{\gamma_0 + \gamma_a} \quad (4a)$$

$$\sigma_{yy} = (e v_F)^2 \frac{N_0(E_F)}{2} \frac{1}{\gamma_t} \quad (4b)$$

$$\sigma_y^y = \hbar e v_F \frac{N_0(E_F)}{2} \frac{1}{\gamma_t} \quad (4c)$$

Our key result is Eq. (4a), which shows that when spin-momentum locked Weyl fermions scatter off a non-magnetic impurity in the p -wave channel, conservation of angular momentum necessarily leads to spin-flips, and the chiral spin-flip scattering rate, γ_s , drives an $O(\frac{1}{n_i})$ skew scattering mechanism; the strength of which can be measured via the spin-Hall angle,

$$\theta^{SH} = \frac{\hbar}{|e|} \frac{\gamma_s}{\gamma_0 + \gamma_a} \quad (5)$$

Here, $e < 0$ is the electron charge and $N_0(E_F)$ is the density of states, whilst γ_t is the charge transport scattering rate, and γ_0 and γ_a are the s and p -wave quasi-particle scattering rates. Power counting of $\gamma_t \sim \gamma_0 \sim n_i V_0^2 N_0(E_F)$ and $\gamma_s \sim \gamma_a \sim n_i V_0^2 V_1^2 N_0(E_F)^3$, gives $\theta^{SH} \sim O\left(\frac{V^2}{v_F^2/a^2} (k_F a)^4\right)$. This is our key result: θ^{SH} has a positive sign, and is a strongly-dependent function of $k_F a$; hence, the SHE can be tuned by E_F .

FIG. 2: The first line shows the Feynman diagram for the $\bar{T}(\vec{k}, \vec{k}', i\omega_n)$ matrix that sums up the infinite set of scattering events from a single impurity, which we represent by a diamond symbol. The Feynman diagram for the impurity-averaged $\bar{G}_{eff}(\vec{k}, i\omega_n)$ is shown on the second line.

Below, we describe how we obtained the results in Eq. (4a) - (4c). Since Weyl fermions are spin-momentum locked, scattering even from a non-magnetic impurity will necessarily involve spin-flip events, and the spin-dependent skew scattering is captured by the $\sigma^\pm$ terms as illustrated in Fig. 1. The following Dyson equations, in operator formalism, give the effective Green's function and $\bar{T}$ -matrix, and Fig. 2 shows the corresponding Feynman diagrams.

$$\hat{\bar{G}}_{eff} = \hat{\bar{G}}_0 + \hat{\bar{G}}_0 \hat{\bar{T}} \hat{\bar{G}}_0 \quad (6a)$$

$$\hat{\bar{T}} = \hat{\bar{V}} + \hat{\bar{V}} \hat{\bar{G}}_0 \hat{\bar{T}} \quad (6b)$$

In the Kubo formalism, the DC longitudinal charge conductivity (σ_{yy}), spin-Hall conductivity (σ_{xy}^z) and spin accumulation (σ_y^i) are given by analytic continuation of the corresponding Matsubara correlation functions $\pi_{yy}(\vec{k}, \omega)$, $\pi_{xy}^z(\vec{k}, \omega)$, and $\pi_y^i(\vec{k}, \omega)$, i.e. $\sigma = -\lim_{\omega \rightarrow 0} \lim_{\vec{k} \rightarrow 0} \text{Im} \left[\frac{\pi(\vec{k}, \omega)}{\omega} \right]$, and detailed expressions are given in the SOM. Note that π_{yy} and π_y^i are equal up to a

factor of $\frac{ev_F}{\hbar}$ for Weyl fermions due to spin-momentum locking, i.e. $\hat{j}_y = ev_F \hat{\sigma}^y$. The infinite subset of Feynman ladder diagrams for these correlation functions are shown in Fig. 3, with chiral spin-flip scattering starting to contribute at third-order. The Bethe Salpeter equation for the scattering vertex, with $\vec{k}$ and Ω_m being the external momentum and frequency, is,

$$\begin{aligned} \bar{\bar{\Gamma}}^y(\vec{k} + \vec{p}, \vec{p}, i\Omega_m + i\omega_n, i\omega_n) &= \sigma^y + \sum_{\vec{q}} \bar{\bar{T}}(\vec{k} + \vec{p}, \vec{k} + \vec{q}, i\Omega_m + i\omega_n) \bar{\bar{G}}_{eff}(\vec{k} + \vec{q}, i\Omega_m + i\omega_n) \\ &\quad \times \bar{\bar{\Gamma}}^y(\vec{k} + \vec{q}, \vec{q}, i\Omega_m + i\omega_n, i\omega_n) \bar{\bar{G}}_{eff}(\vec{q}, i\omega_n) \bar{\bar{T}}(\vec{q}, \vec{p}, i\omega_n) \end{aligned} \quad (7)$$

FIG. 3: Feynman diagram for the effective scattering vertex, $\bar{\bar{\Gamma}}^y(\vec{p}, \omega)$, which includes an infinite subset of scattering events from multiple impurities in the dilute limit. Each diamond represents all the scattering events from a single impurity, and skew scattering occurs at 3rd and higher orders in perturbation.

Utilizing rotational symmetry, we obtain a physically transparent picture that clarifies the contribution of the different scattering channels to spin and charge transport from a multipole expansion of $\bar{\bar{G}}_0(\vec{k}, i\omega_n)$, $\bar{\bar{G}}_{eff}(\vec{k}, i\omega_n)$, the $\bar{\bar{T}}$ -matrix, and the transport scattering vertex $\bar{\bar{\Gamma}}$. This allows us identify s -wave spin-independent scattering as the main contribution to charge transport, while the SHE is driven by p -wave chiral spin-flip skew scattering between the s and p -wave channels of the Weyl fermions.

In the dilute limit, the $\bar{\bar{T}}$ -matrix and $\bar{\bar{G}}_{eff}(\vec{k}, \omega)$ can be calculated in the non-crossing approximation[38], and scattering from multiple impurities is included by $\bar{\bar{G}}_{eff}^{(R)}(\vec{k}, \omega)$ in the Dyson equation for the retarded self-energy,

$$\bar{\bar{G}}_{eff}^{(R)}(\vec{k}, \omega) = \left[\omega - v_F \vec{k} \cdot \vec{\sigma} - \bar{\bar{\Sigma}}^{(R)}(\vec{k}, \omega) \right]^{-1} \quad (8a)$$

$$\begin{aligned} &= g_{eff}^0(k, \omega) \mathbb{1} + g_{eff}^a(k, \omega) (\cos \theta \sigma^x + \sin \theta \sigma^y) \\ \bar{\bar{\Sigma}}^{(R)}(\vec{k}, \omega) &= n_i \sum_{\vec{k}_1} \bar{\bar{V}}(\vec{k}, \vec{k}_1) \bar{\bar{G}}_{eff}^{(R)}(\vec{k}_1, \omega) \bar{\bar{T}}^{(R)}(\vec{k}_1, \vec{k}, \omega) \end{aligned} \quad (8b)$$

The s and p -wave components of $\bar{\bar{G}}_{eff}^{(R)}(\vec{k}, \omega)$ are $g_{eff}^0(\vec{k}, \omega)$ and $g_{eff}^a(\vec{k}, \omega)$ respectively, and detailed expressions as well as $O(\frac{\gamma}{E_F})$ terms have been relegated to the SOM[39]. We take v_F and E_F to be experimentally determined parameters, thereby dropping

$\text{Re}[\bar{\bar{\Sigma}}^{(R)}(k_F, E_F)]$, and assume an average quasi-particle scattering rate near the Fermi surface, i.e. $\bar{\bar{\gamma}} \equiv \text{Im}[\bar{\bar{\Sigma}}^{(R)}(k_F, E_F)]$.

$$\begin{aligned} \bar{\bar{\gamma}} &= \gamma_0 \mathbb{1} - \gamma_a (\cos \theta \sigma^x + \sin \theta \sigma^y) \\ &\quad - \gamma_b (\sin \theta \sigma^x - \cos \theta \sigma^y) + i \gamma_3 \sigma^z \end{aligned} \quad (9a)$$

$$\begin{aligned} \gamma_0 &= n_i N^{(0)}(E_F) \left[|T^0|^2 + |T_0^3|^2 \right. \\ &\quad \left. - 2(|T_1^3|^2 + |T^A|^2 - |T^S|^2) \right] \end{aligned} \quad (9b)$$

$$\gamma_a = 4n_i N^{(1)}(E_F) [|T^S|^2 - |T^A|^2] \quad (9c)$$

The main quasi-particle scattering rates are the s -wave γ_0 and p -wave γ_a terms, which determine γ_t (Eq. (12b)), and Eq. (4a) demonstrates that γ_0 and γ_a are important for both charge and spin transport. Refer to SOM[39] for the sub-leading γ_b and γ_3 terms. Since scattering events that result in a change of angular momentum, i.e involving the $l = 1$ DOS component $N^{(1)}(E_F)$, also cause a spin-flip due to SOC, it is clear that $\gamma_0 \propto V_0^2 N^{(0)}(E_F)$ and $\gamma_a \propto V_0^2 V_1^2 N^{(1)}(E_F)^3$ are due to spin-independent and dependent scatterings respectively. Here, the angular momentum resolved density of states (DOS) is defined as $N^{(i)}(\omega) = \int \frac{k dk}{2\pi} \text{Im} [g^i(k, \omega)]$, with $N^{(0)}(E_F)$ and $N^{(1)}(E_F)$ being the s and p -wave components respectively.

The scattering channels that drive γ_0 and γ_a can be clarified by a multipole analysis of $\bar{\bar{T}}$ -matrix, which is solved assuming that it varies slowly near E_F (see SOM for details). Keeping only the s and p -wave terms, $\bar{\bar{T}}(|\vec{k}| = |\vec{k}'| = k_F, \theta_k, \theta_{k'}, i\omega_n = E_F)$ is,

$$\begin{aligned} \bar{\bar{T}}(\theta_k, \theta_{k'}) &= T^0 \mathbb{1} + T_0^3 \sigma^z + T_1^3 (e^{i(\theta_k - \theta_{k'})} - e^{-i(\theta_k - \theta_{k'})}) \sigma^z \\ &\quad + \frac{T^S + T^A}{2} e^{i\theta_k} \sigma^- + \frac{T^S - T^A}{2} e^{-i\theta_k} \sigma^+ \\ &\quad + \frac{T^S + T^A}{2} e^{-i\theta_{k'}} \sigma^+ + \frac{T^S - T^A}{2} e^{i\theta_{k'}} \sigma^- \end{aligned} \quad (10)$$

Charge transport is dominated by the largest term, $T^0 \propto V_0$, while $T^{S/A}$ are the symmetric and asymmetric parts

of the p -wave spin-flip scattering channels, and T_0^3 and T_1^3 are the s and p -wave σ^z channels. It is now clear that γ_0 is mainly driven by the s -wave spin-independent T^0 channel, while γ_a arises solely from the spin-flip $T^{S/A}$ channels. Furthermore, the spin-flip T^S channel shall be shown to drive the skew scattering mechanism, Eq. (12b). These $\bar{T}$ -matrix coefficients are determined by V_n and $N^{(0/1)}(E_F)$, with detailed expressions in the SOM.

In the uniform DC limit, the on-shell component of the scattering vertex $\bar{\Gamma}^y(\vec{p}, \omega) = \bar{\Gamma}^y(\vec{p}, \omega - i\eta, \omega + i\eta)$ is solved self-consistently using a multipole expansion of $\bar{\Gamma}^y(\vec{p}, \omega) = \sum_n \Gamma_n^i e^{in\theta} \sigma^i$ (see SOM[39] for details), and keeping only the s and p -wave channels gives,

$$\bar{\Gamma}^y(k_F, E_F) = (\Gamma_{p_x}^0 \cos \theta + i\Gamma_{p_y}^0 \sin \theta) \mathbb{1} + \Gamma_0^1 \sigma^x + \Gamma_0^2 \sigma^y + (\Gamma_{p_x}^3 \cos \theta + i\Gamma_{p_y}^3 \sin \theta) \sigma^z \quad (11)$$

where,

$$\Gamma_0^2 = \frac{\gamma_0}{\gamma_t}, \quad \Gamma_{p_x}^3 = -\frac{\gamma_s}{\gamma_t}, \quad (12a)$$

$$\gamma_t = (\frac{1}{2}\gamma_0 + \gamma_a), \quad \gamma_s = \frac{n_i \pi N_0(E_F)}{2} |T^0| |T^S| \quad (12b)$$

are the main charge and spin transport scattering vertex channels respectively (refer to SOM[39] for all the $\bar{\Gamma}$ -coefficients). Since the Weyl fermions are spin-momentum locked, charge transport along $\hat{y}$ -direction necessarily depends on the σ^y component of $\bar{\Gamma}$, i.e. Γ_0^2 . For the SHE, the spin torque current, P_x^z , will clearly couple to the σ^z component of $\bar{\Gamma}$ along $\hat{x}$, i.e. $\Gamma_{p_x}^3$.

We define a skew scattering rate γ_s in Eq. (12b), which arises from the spin-flip scattering T^S terms. Upon projection into the upper helical band, this gives a $T^S \sin(\theta_k - \theta_{k'})$ term, which elucidates the skew scattering nature of the spin-flip scattering. This term arises at 3^{rd} and higher orders in perturbation; i.e. $T^S \propto V_0 V_1^2 N_0(E_F)^2$, in agreement with previous work [37]. Most importantly, this demonstrates that the skew scattering rate, and hence, the SHE can be tuned by varying $k_F a$, i.e. the Fermi energy or the impurity size.

In summary, the main results of this paper are the spin and charge conductivities, and the Rashba-Edelstein coefficient, and our key finding is Eq. (4a), which shows an $O(\frac{1}{n_i})$ skew scattering contribution to the SHE. For strongly SOC-systems, such as Weyl fermions, exchange of angular momentum upon scattering off non-magnetic impurities will lead to spin-flip processes, and p -wave chiral spin-flip scattering between the s and p -wave electrons will drive a skew-scattering contribution to the SHE. This is made explicit by writing out the spin and angular-momentum scattering channels for $\sigma_{xy}^z = \hbar e v_F^2 \text{Re}[\Gamma_{p_x}^3(\eta^{a0}(E_F) - \eta^{0a}(E_F))]$, where $\eta^{ij}(\omega) = \int \frac{dp}{2\pi} p^2 \partial_p g_{eff}^{i(R)}(p, \omega) g_{eff}^{j(A)}(p, \omega)$, and $\eta^{0a/a0}(E_F)$ evinces

the scattering between the s and p -wave electrons due to γ_s (see SOM for calculation details).

In addition, we briefly discuss the effects of band bending in Weyl systems. The leading $O(\frac{1}{m})$ correction comes from including a conventional spin current, $\vec{j}_s^z = \psi^\dagger \frac{1}{2} \{\vec{v}, S^z\} \psi$, with $\vec{v} = \frac{\hbar \vec{k}}{m}$. However, it has been pointed out[35–37] that $\vec{j}_s^z \propto \sigma_y$ for Rashba-type systems; hence, up to $O(\frac{1}{m})$, band bending does not give rise to a spin current for Weyl fermion systems.

To conclude, we have demonstrated an experimentally controllable SHE on the surface of 3D topological materials, which relies fundamentally on the chiral nature of the Weyl fermions. Utilizing a multipole expansion of $\bar{G}_{eff}$, $\bar{T}$ and $\bar{\Gamma}$, we identify the different scattering processes that dominate charge transport and the SHE. The results show that strong spin-orbit coupling in the band-structure is sufficient to cause chiral spin-flip scattering of the helical electrons off non-SOC scalar impurities, resulting in a skew-scattering contribution to the SHE. The strength of this mechanism is measured by the SHA, $\theta^{SH} = \frac{\hbar}{|e|} \frac{\gamma_s}{\gamma_0 + \gamma_a} \sim O\left(\frac{V^2}{v_F^2/a^2} (k_F a)^4\right)$, with a being the range of the impurity potential, and we highlight the fact that the skew scattering strength can be tuned by varying $k_F a$; thereby providing an experimentally-accessible parameter for controlling the SHE. We gratefully acknowledge I. Mertig, K. Kondou and Y. Tokura for helpful discussions, and this work was supported by CREST, Japan Science and Technology Agency (JST).

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