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Searching for near-horizon quantum structures in the binary black-hole stochastic gravitational-wave background

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Quantum gravity corrections have been speculated to lead to modifications to space-time geometry near black hole horizons. Such structures may reflect gravitational waves, causing echoes that follow the main gravitational waves from binary black hole coalescence. By studying two phenomenological models of the near-horizon structures under Schwarzschild approximation, we show that such echoes, if exist, will give rise to a stochastic gravitational-wave background, which is very substantial if the near-horizon structure has a near unity reflectivity for gravitational waves, readily detectable by Advanced LIGO. In case the reflectivity is much less than unity, the background will mainly be arising from the first echo, with a level proportional to the power reflectivity of the near-horizon structure, but robust against uncertainties in the location and the shape of the structure — as long as it is localized and close to the horizon. Sensitivity of third-generation detectors allows the detection of a background that corresponds to power reflectivity $\sim 3 \times 10^{-3}$, if uncertainties in the binary black-hole merger rate can be removed. We note that the echoes do alter the energy spectrum is given by

$$\sum_{l,m} X_{lm}(r_s, \omega_s) = S_{lm}(\omega, \rho),$$

where $r_s$ is the tortoise coordinate with $dr/dr_s = 1 - 2M/r$ with effective potential given by

$$V(r) = \left(1 - \frac{2M}{r}\right) \left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right).$$

with $M$ the mass of the BH. The source term is given by $S_{lm}(\omega, r) = W_{lm}(\omega) e^{-\mu r}$, where $W_{lm}$ is a functional of the trajectory of the test particle and its explicit expression can be found in Eqs. (19)–(21) of [18]. The wave function $X_{lm}$ is related to GW in the $r \to +\infty$ limit via $h_+ + i h_\times = 8\pi^{-1} \sum_{lm} \frac{1}{2} Y_{lm}(t)$, where $Y_{lm}$ are spin-$s$ weighted spherical harmonics and $X_{lm}(t) = \int_{-\infty}^{\infty} e^{-i\omega t} X_{lm}(\omega)$. The GW energy spectrum is given by

$$dE/d\omega = \sum_{lm} 16 \pi c^2 |X_{lm}(\omega, r_s \to \infty)|^2.$$
For BHs, imposing in-going boundary condition near the horizon and out-going condition near null infinity, solution to Eq. (1) is expressed as $X_{lm}^{(0)}(\omega, r_s \to \infty) = e^{i\omega t} Z_{lm}^{(0)}(\omega)$, with

$$
Z_{lm}^{(0)}(\omega) = \int_{-\infty}^{\infty} dr'_s \left[ S_{lm}(\omega, r'_s) X_{lm}^{(0)}(\omega, r'_s) \right] / W^{(0)}(\omega),
$$

(4)

with $W^{(0)} = X_{lm}^{(0)} \partial_r X_{lm}^{(0)} - X_{lm}^{(0)} \partial_r X_{lm}^{(0)}$ the Wronskian between the two homogenous solutions, with $X_{lm}^{(0)} \sim e^{-i\omega r}$ for $r_s \to -\infty$ and $X_{lm}^{(0)} \sim e^{i\omega r}$ for $r_s \to +\infty$, respectively.

Echoes from near-horizon structure.— Let us now modify the Schwarzschild geometry near the horizon by creating a Planck-scale potential barrier $V_p$: $V_1 \to V_1 + V_p$, with $V_p$ centered at $r^2 = 2M + \epsilon$, with $\epsilon \ll M$. In tortoise coordinate, $\epsilon = l_p$ corresponds to $r_p^2 \approx -182M$. As discussed by [14] the effect of $V_p$ is the same as replacing the horizon ($r_s \to -\infty$) boundary condition for Eq. (1) by

$$
X_{lm}^R \sim e^{-i\omega r}, \quad R^e^{i\omega r}, \quad \text{for} \quad r_s \to r_p^0,
$$

(5)

while keeping the $r_s \to +\infty$ boundary condition unchanged. Here, $R(\omega)$ can be viewed as a complex reflectivity of the potential barrier [33], the location of reflection is implicitly contained in its frequency dependence; for example, a Dirichlet boundary condition corresponds to $R(\omega) = -e^{-2i\omega t}$ [34].

Defining $X_{lm}^{(R)} = Z_{lm}^{(R)} e^{i\omega t}$, $Z_{lm}^{(R)}$ can be written as a sum the main wave (for BH) and a series of echoes [14]:

$$
Z_{lm}^{(R)} = Z_{lm}^{(0)} + R \sum_{\infty}^{(1)} (R R_{\text{BH}})^n,
$$

(6)

with $R_{\text{BH}}$ the complex reflectivity of the Regge-Wheeler potential $V_1$ [see Eq. (2.14) of [14]] and

$$
Z_{lm}^{(1)}(\omega) = \int_{-\infty}^{\infty} dr'_s S_{lm}(\omega, r'_s) X_{lm}^{(0)}(\omega, r'_s) / W^{(0)}(\omega) + R R_{\text{BH}} Z_{lm}^{(0)},
$$

(7)

with $X_{lm}^{(0)}$ the complex conjugate of $X_{lm}^{(0)}$.

Note that each echo delayed from the previous one by $2|\omega|^2$ in the time domain. For small $R$, we write $Z_{lm}^{(R)} \approx Z_{lm}^{(0)} + R Z_{lm}^{(1)}$ and

$$
\frac{dE}{d\omega} \approx 16\pi \omega^2 \sum_{lm} \left[ |Z_{lm}^{(0)}|^2 + |R Z_{lm}^{(1)}|^2 + 2\text{Re}(R Z_{lm}^{(1)} Z_{lm}^{(0)}) \right].
$$

(8)

This is the sum of energies from main wave, the first echo, and the beat between the main wave and the first echo. While the beat is linear in $R$, it is highly oscillatory in $\omega$, since the main wave and the echo are well separated in the time domain.

Models of Reflectivity and Energy Spectra of Echoes.— Without prior knowledge about details of near-horizon structures, we only assume it is short-ranged and localized at $r_p^0$. The simplest would be to introduce a $\delta$-potential $V_p = \mathcal{A} \delta(r_s - r_p^0)/M$, with parameter $\mathcal{A}$ defined as the area under the Planck potential: $\mathcal{A} = M \int_{-\infty}^{\infty} V_i dr_s$. Note that $\mathcal{A}$ is a dimensionless quantity. As a comparison, the area under the Regge-Wheeler potential is $[19] M \int_{-\infty}^{\infty} V_i dr_s = (l-1)(l+2)/2 + 1/4$.

Such a model corresponds to a reflectivity

$$
\mathcal{R}(\omega) = e^{-2i\omega t} \mathcal{A}/(2i\omega M - \mathcal{A}).
$$

(9)

This is more physical than the Dirichlet case, by reducing $|R|$ at larger $\omega$. Since $|\mathcal{R}(0)| = 1$ and $\mathcal{R}(+\infty) = 0$ are general properties of all physical potentials, we expect Eq. (9) to describe a large class of near-horizon quantum structures. To further explore the shape of $V_p$, we also study the Pöschl-Teller potential [20] $V_p = \alpha^2 \lambda (1 - \lambda)/M^2 \cosh^{-2}[\alpha(r_s - r_p^0)/M]$. Dimensionless parameters $\alpha$ and $\lambda$ are related to the area under $V_p$ via $\mathcal{A} = 2\alpha^2 \lambda (1 - \lambda)$. The corresponding reflectivity is [21]

$$
\mathcal{R}(\omega) = e^{-2i\omega t} \frac{\Gamma(i \frac{\lambda M}{\alpha}) \Gamma(\lambda - i \frac{\lambda M}{\alpha}) \Gamma(1 - \lambda - i \frac{\lambda M}{\alpha})}{\Gamma(-i \frac{\lambda M}{\alpha}) \Gamma(1 - \lambda) \Gamma(l)},
$$

(10)

where $\Gamma(\cdot)$ is the Gamma function. In the following, we will keep $\mathcal{A}$ fixed and vary $\alpha$ and $\lambda$ to explore shapes of $V_p$.

To estimate the echoes’ energy spectrum, we adopt the EOB approach [16, 17]; for BHs with $m_1$ and $m_2$, we consider a point particle with reduced mass $\mu = m_1 m_2/(m_1 + m_2)^2$ falling down a Schwarzschild BH with total mass $M = m_1 + m_2$; the symmetric mass ratio is defined as $\nu = \mu/M$. For motion in the equatorial plane, we have a Hamiltonian for $(r, p_r, \phi, p_\phi)$, with radiation reaction incorporated as a generalized force $F_\phi$ [Eqs. (3.41)–(3.44) of [17]]. Upon obtaining the trajectory (see Fig. 1 for $\nu = 0.25$), we obtain source term $S_{lm}$, and compute $Z_{lm}^{(0)}$ and $Z_{lm}^{(1)}$ using Eqs. (4) and (7), which will then lead to the GW energy spectrum.

We will focus on the $(l, m) = (2, 2)$ mode, which carries most of the GW energy.

FIG. 1: Trajectory of the EOB effective particle moving in a coalescing quasi-circular orbit. The symmetric mass ratio $\nu = 0.25$. The inner black sphere with radius $2M$ represents the horizon of a Schwarzschild BH. The outer translucent sphere with radius $3M$ represents the photon sphere.

FIG. 2: The main wave $Z_{22}^{(0)}$ and the wave $Z_{22}^{(1)}$ that generates echoes via Eq. (6).
As seen in Fig. 2, the main wave $|Z_{22}^{(0)}|$ recovers the $f^{-7/6}$ power law at low frequencies, as predicted by post-Newtonian approximation, also qualitatively mimics a BBH waveform at intermediate (merger) to high frequencies (ringdown). Note that the ringdown makes the the $|Z_{22}^{(0)}|$ curve turn up slightly near the leading $(2, 2)$ Quasi-Normal Mode (QNM) frequency of the Schwarzschild BH before sharply decreasing, similar to Fig. 3 of Ref. [23]. The wave $|Z_{22}^{(1)}|$ peaks roughly at the QNM frequency.

Horizon structures with $\mathcal{A}$ of order unity lead to significant modifications in GW energy spectrum $dE/d\omega$. In the upper panel of Fig. 3, we choose the reflectivity (9) with $\epsilon = l_p$ and $\mathcal{A} = 0.25, 0.5, 0.75$ and 1. At low frequencies, near-horizon structures add peaks separated by $\Delta \omega \sim 0.017 M^{-1} - \pi/r_\text{p}^2$ to the post-Newtonian $dE/df \propto f^{-1/3}$. These resonant peaks are related to the poles of $1/(1 - \mathcal{R} R_{\text{BH}})$ in the series sum of Eq. (6). Near the QNM frequency, there is substantial additional radiation, which is due to the large value of $|Z_{22}^{(1)}|$. In the left panel, we choose several different values of $\epsilon$ which lead to different peak separation at low frequencies. In the right panel, we consider reflectivity (10) and find that the shape of the Planck potential, as characterized by $\alpha$, has negligible influence to $dE/d\omega$ as long as the area keeps fixed.

**Stochastic Gravitational-Wave Background (SGWB).**— The SGWB is usually expressed as $\Omega(f) = \rho_c^{-1}d\rho_{GW}/df\Delta f$, where $\rho_c$ represents the critical density to close the universe and $\rho_{GW}$ the GW energy density; it is related the $dE/df$ of a single GW source via [24]

$$
\Omega(f) = \frac{f}{\rho_c} \int_0^{f_{\text{c}}} dz \frac{R_m(z)[dE/df]}{(1 + z)H(z)},
$$

where $f_{\text{c}} = f(1 + z)$ is the frequency at emission. Here we adopt the $\Lambda$CDM cosmological model with $H(z) = H_0[\Omega_M(1 + z)^3 + \Omega_{\Lambda}]/c^2$, where the Hubble constant $H_0 = 70 \text{ km/s Mpc}$, $\Omega_M = 0.3$ and $\Omega_{\Lambda} = 0.7$. $R_m(z)$ is the BBH merger rate per comoving volume at redshift $z$. We use the *fiducial model* described in [22], where $R_m(z)$ is proportional to the star formation rate with metallicity $Z < Z_{\odot}/2$ and...
The optimal signal-to-noise ratio (SNR) for a signal is given by

\[ \text{SNR} = \frac{\gamma(f)\Omega(f)\Omega_{\text{GW}}(f)}{\int_0^{\infty} dP_1(f)P_2(f)}, \]

(12)

where \(\gamma(f)\) is the normalized overlap reduction function between the detectors, and \(P_1(f)\) and \(P_2(f)\) are the detectors’ noise spectral densities. We consider advanced LIGO at design sensitivity [23], LIGO Voyager [26] and Einstein Telescope (ET) [27] at planned sensitivities. Advanced LIGO and LIGO Voyager have the same \(\gamma\) and we take the constant \(\gamma = -3/8\) for co-located ET detectors [28]. The 1-year SNRs are listed in Table I for values of \(\mathcal{A}\) at order unity, in which case the echoes contribute significantly to the SNRs.

For lower values of \(\mathcal{A}\), we apply the model-selection method of Ref. [25] to distinguish the SGWB with and without echo contributions. The log-likelihood ratio (LR) between two models is given by \(\ln \Lambda = \langle \Delta \Omega | \Delta \Omega \rangle / 4\) and two models considered discernible when \(\ln \Lambda > c > 1\). Here we choose \(c = 12\), which corresponds to a false alarm rate of \(10^{-6}\) [29]. Minimum distinguishable \(\mathcal{A}\) to reach this LR threshold is shown in Tab. II; with 5-year integration, Voyager can detect \(\mathcal{A} \approx 0.21\), while ET can detect \(\mathcal{A} \approx 0.042\).

Conclusions and Discussions.—As we have seen in this paper, the \(\Delta \Omega\) due to the echoes is largely independent from uncertainties in \(\epsilon\). For strong near-horizon structures, with \(\mathcal{A}\) the order of unity, SGWB from the echoes will be clearly visible. For weak near-horizon structures, \(\Delta \Omega\) is mainly given by the first echo, and is simply proportional to the power reflectivity \(|\mathcal{R}|^2\). The level detectable by ET corresponds to \(\mathcal{A} \sim 0.042\), which corresponds to \(|\mathcal{R}|^2 \approx 3 \times 10^{-7}\) near the peak of the echo energy spectrum. Further details of the background not only depends on details in the Planck potential barrier \(V_\rho\), we will also need to generalize the analysis to a Kerr BH.

Uncertainties also exist in the SGWB of the main, inspirial-merger-ringdown wave, e.g., arising from different star formation rates, different metallicity thresholds to form BHs, details in the evolution of binary stars and the distributions in the time delay between BBH formation and merger — all of these lead to uncertainties in the local BBH merger rate and the local distribution of mass \(M\) and symmetric mass ratio \(\nu\) [22]. It is believed these uncertainties will be well quantified and narrowed down by future BBH detections. For example, the range of BBH local merger rate has been narrowed down to \(12 - 213 \text{ Gpc}^{-3} \text{ yr}^{-1}\) using GW170104 [30]. On the other hand, as demonstrated by Zhu et al., these uncertainties only scale the background spectra linearly at low frequencies and hence keep the power law \(\Omega(f) \propto f^{2/3}\) for \(f < 100 \text{ Hz}\) unchanged [24]. Our result shows the appearance of the near-horizon structures changes the slope of \(\Omega(f)\), making it deviate from the \(f^{2/3}\) power law even at low frequencies. This may be used to alleviate the influence from uncertainties.

In addition to BBH, binary neutron star (BNS) mergers also contribute to the background with a comparable magnitude [31]. Within the bandwidth of ground-based GW detectors, this background arises solely from inspiral, which gives an \(f^{2/3}\) power law and is not influenced by the presence of the near-horizon structure. As a result, the echo SGWB \(\Delta \Omega\) remains unchanged and our analysis on detectability still holds.
Echoes may also be detectable from individual events. Our calculations indicate for an event similar to GW150914, to reach an echo SNR of 10 the value of $\mathcal{A}$ should be at least $0.24$ (LIGO), $0.050$ (Voyager) and $0.011$ (ET), respectively. However, in the matched filtering search of individual signal, the exact waveform is required, which in our model depends not only on $\mathcal{A}$, but also on $e$ and $\alpha$, but may depend further on other unknown details of the Planck-scale potential — making it less robust. An analysis combined both background and individual signals will be presented in a separate publication[32].

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[33] Here we will obtain $R(\omega)$ from $V_p$, while the problem of obtaining $V_p$ once $R(\omega)$ is measured is the so-called inverse scattering problem, see e.g., K. Chadan, Inverse problems in quantum scattering theory, Springer Science & Business Media, 2012.
[34] In general, if $R(\omega) = \rho(\omega)e^{i\phi(\omega)}$ with $\rho(\omega)$ a slowly varying complex amplitude and $\phi$ a fast-varying phase, then the effective location of reflection for a wavepacket with central frequency $\omega_0$ is around $[\partial\Psi/\partial\omega]_{\omega_0}/2$. 