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Ryohei Wakatsuki and Naoto Nagaosa

Phys. Rev. Lett. **121**, 026601 — Published 9 July 2018

DOI: 10.1103/PhysRevLett.121.026601

Nonreciprocal current in noncentrosymmetric Rashba superconductors

Ryohei Wakatsuki[1] and Naoto Nagaosa[1,2]

[1]*Department of Applied Physics, University of Tokyo, Hongo 7-3-1, 113-8656, Japan*
[2]*RIKEN Center for Emergent Matter Science (CEMS), Wako 351-0198, Japan*

We study theoretically the nonreciprocal charge transport in two-dimensional noncentrosymmetric superconductors with Rashba spin–orbit interaction. The resistivity R depends on the current I linearly under the external magnetic field B, i.e., $R = R_0\,(1+\gamma BI)$, which is called the magnetochiral anisotropy (MCA). It is found that the coefficient γ is gigantically enhanced by the superconducting fluctuation with the components of both spin singlet and triplet pairings, compared with that in the normal state. This finding offers a method to quantitatively estimate the ratio of the pairing interactions between the singlet and triplet channels including its sign.

The broken inversion symmetry P is expected to result in the directional nonreciprocal responses. For example, the propagation of light through the matter can be different depending on the direction. However, Onsager's reciprocal theorem [1], which originates from the time-reversal symmetry T of the microscopic dynamics, puts the constraint on the linear response function $K_{\alpha\beta}$ describing the response of α to the input β as

$$K_{\alpha\beta}(q,\omega,B) = \varepsilon_\alpha\varepsilon_\beta K_{\beta\alpha}(-q,\omega,-B), \tag{1}$$

where B is the magnetic field representing the T-breaking, $\varepsilon_\alpha(=\pm 1)$ (ε_β) is the even/odd nature of α (β) with respect to T, and q, and ω is the wavevector and frequency of the physical quantities α and β. An example is the case where α and β are the same component of the current, and $K_{\alpha\beta}$ describes the diagonal element of the dielectric function. Therefore, only with the broken T-symmetry propagation of light between q and $-q$, called directional dichroism, becomes possible [2].

On the other hand, the nonlinear nonreciprocal responses in the transport phenomena are characterized by the current-dependent resistivity R expressed as

$$R = R_0\,(1+\gamma BI)\,, \tag{2}$$

where I is the current, B is the magnetic field, and γ represents the nonreciprocity [3–8]. This means the nonlinear I-V characteristics are asymmetric between the positive and negative sign of I, i.e., the directional revistivity, called magnetochiral anisotropy (MCA) [3]. When the T-symmetry is intact, the electronic dispersion $\varepsilon_\sigma(k)$ (k: crystal momentum, σ: spin) has the symmetry between k and $-k$, i.e., $\varepsilon_\sigma(k) = \varepsilon_{-\sigma}(-k)$. Therefore, the magnetic field B, which breaks T-symmetry, is necessary in addition to P-breaking to induce the asymmetric energy dispersion and hence I-V characteristics as shown in Eq. (2). Microscopically, both the spin–orbit interaction λ and the Zeeman effect $\mu_B B$ are needed to make the energy dispersion asymmetric, which are usually small perturbations compared with the kinetic energy of electrons, i.e. the Fermi energy E_{F}. Therefore, the strength γ of MCA in Eq. (2) is usually very tiny because its expression contains the small factors of λ/E_{F} and $\mu_B B/E_{\mathrm{F}}$.

The superconductivity in noncentrosymmetric systems changes this situation. The superconductivity changes the transport phenomena within the narrow low energy window below the superconducting gap Δ or at low temperatures around and below the mean field transition temperature $T_{\mathrm{c}} \cong \Delta/k_{\mathrm{B}}$. The conductivity due to the fluctuating superconducting order parameter, i.e., paraconductivity, above T_{c} shows the enhanced nonreciprocal transport as shown experimentally in $\mathrm{MoS_2}$ [9]. It has been analyzed theoretically in terms of the time-dependent Ginzburg–Landau (GL) theory, and the enhancement of γ compared with that in the normal state γ_{N} is estimated as $\gamma/\gamma_{\mathrm{N}} \sim (E_{\mathrm{F}}/\Delta)^3$ [9]. This means that MCA provides a useful information about the electronic states and superconductivity of noncentrosymmetric materials. In the case of $\mathrm{MoS_2}$ with the out-of-plane magnetic field, the trigonal warping of the band structure leads to the third order terms in the wavenumber of order parameter, which is identified as the main mechanism of MCA.

In this paper, we study the MCA of two-dimensional superconductors with Rashba spin–orbit interaction in the temperature regime slightly above T_{c}, where the current is mainly carried by thermal fluctuation of the superconducting order parameter [10, 11]. The most essential aspect of the noncentrosymmetric superconductivity is the mixing of the spin-singlet even parity and spin-triplet odd parity pairings [12–16]. One of the consequence of this is the very large upper critical magnetic field $B_{\mathrm{c}2}$ beyond the Pauli limit. As we will show below, the noncentrosymmetric superconductors with Rashba spin–orbit interaction as given by Eq. (3) shows the nonreciprocal charge transport, which is very sensitive to the pairing interactions for singlet and triplet channels as shown in Eq. (19). Namely, once the Fermi energy E_{F}, the strength of the Rashba interaction and T_{c} are known, one can estimate the ratio of the pairing interactions between the singlet and triplet channels, i.e., r_{s} or r_{t}, including its sign. Note also here that the third order terms in the wavenumber of the order parameter, which was the main origin of γ for $\mathrm{MoS_2}$, is estimated to be much smaller in the present case as indicated by Eq. (22) below. We also show that the nonreciprocal current has a unique electric and magnetic fields angle dependence due to the symmetry constraints for the higher rank response tensor as shown in Figs. 1 and 2.

Before defining the Hamiltonian, we discuss the general form of the spin–orbit interaction in time-reversal preserved systems. If we express the spin–orbit interaction as $\boldsymbol{g_k}\cdot\boldsymbol{\sigma}$, time-reversal symmetry requires $\boldsymbol{g_k} = -\boldsymbol{g_{-k}}$. In this paper, we assume the simplest Rashba spin–orbit interaction, although the other forms will give the qualitatively same conclusions.

We start with the Rashba Hamiltonian which is given by [8]

$$H_{\boldsymbol{k}} = \xi_{\boldsymbol{k}} + \alpha\left(k_x\sigma_y - k_y\sigma_x\right) - \mu_{\mathrm{B}}\boldsymbol{B}\cdot\boldsymbol{\sigma}, \tag{3}$$

where $\xi_{\boldsymbol{k}} = \frac{\hbar^2\boldsymbol{k}^2}{2m} - E_{\mathrm{F}}$ is the dispersion without the spin–orbit interaction, α is the Rashba parameter, $\boldsymbol{B}$ is the in-plane magnetic field, and $\boldsymbol{\sigma}$ is the Pauli matrix. We have assumed that the g-factor is 2. Its eigenenergies are

$$\xi_{\pm\boldsymbol{k}} = \xi_{\boldsymbol{k}} \pm \sqrt{\left(\alpha k_y + B_x\right)^2 + \left(\alpha k_x - B_y\right)^2}. \tag{4}$$

Now we consider the superconductivity in the presence of the Rashba interaction [12, 16]. For even parity attractive interaction, we assume the standard BCS type onsite attractive interaction,

$$H_{\mathrm{int}} = -V^{\mathrm{g}}\sum_{\boldsymbol{k}\boldsymbol{k}'} c^\dagger_{\boldsymbol{k}\uparrow}c^\dagger_{-\boldsymbol{k}\downarrow}c_{-\boldsymbol{k}'\downarrow}c_{\boldsymbol{k}'\uparrow}, \tag{5}$$

with $c^\dagger_{\boldsymbol{k}\sigma}$ and $c_{\boldsymbol{k}\sigma}$ being the creation and annihilation operators of the electron with momentum $\boldsymbol{k}$ and spin σ. In general, the odd parity part is

$$-\sum_{\boldsymbol{k}\boldsymbol{k}'} V^{\mathrm{u}}_{ij}\left(\boldsymbol{k},\boldsymbol{k}'\right)\left(i\sigma_i\sigma_2\right)_{\alpha\beta}\left(i\sigma_j\sigma_2\right)_{\gamma\delta} c^\dagger_{\boldsymbol{k}\alpha}c^\dagger_{-\boldsymbol{k}\beta}c_{-\boldsymbol{k}'\gamma}c_{\boldsymbol{k}'\delta}, \tag{6}$$

with $V^{\mathrm{u}}_{ij}\left(\boldsymbol{k},\boldsymbol{k}'\right)$ being an odd function with respect to $\boldsymbol{k}$ and $\boldsymbol{k}'$, and invariant under the crystal symmetry transformations. For simplicity, we assume the simplest case $V^{\mathrm{u}}_{ij}\left(\boldsymbol{k},\boldsymbol{k}'\right) = V^{\mathrm{u}}\hat{\gamma}_i\left(\boldsymbol{k}\right)\hat{\gamma}_j\left(\boldsymbol{k}'\right)$ with $\hat{\gamma}\left(\boldsymbol{k}\right) = \frac{1}{k}\left(-k_y, k_y\right)$ in the Rashba system. We assume that the Rashba splitting is much larger than the critical temperature ($E_{\mathrm{R}} \gg T_{\mathrm{c}}$) and the inter-band pairings can be neglected. Then, the interaction Hamiltonian in the band basis reads to

$$H_{\mathrm{int}} = -\sum_{\boldsymbol{k}\boldsymbol{k}'\lambda\lambda'} t_{\boldsymbol{k}\lambda}t^*_{\boldsymbol{k}'\lambda'}\hat{g}_{\lambda\lambda'}\psi^\dagger_{\boldsymbol{k}\lambda}\psi^\dagger_{-\boldsymbol{k}\lambda}\psi_{-\boldsymbol{k}'\lambda'}\psi_{\boldsymbol{k}'\lambda'}, \tag{7}$$

where $\Psi^\dagger_{\boldsymbol{k}\lambda}$ and $\Psi_{\boldsymbol{k}\lambda}$ are the creation and annihilation operators with the band index $\lambda = \pm$, and $t_{\boldsymbol{k}\lambda} = \lambda i e^{i\phi_{\boldsymbol{k}}}$ with $\phi_{\boldsymbol{k}} = \arg\boldsymbol{k}$. The $\boldsymbol{k}$-independent matrix $\hat{g}$ is

$$\hat{g} = \begin{pmatrix} g_1 & g_2 \\ g_2 & g_1 \end{pmatrix}, \tag{8}$$

with $g_1 = \left(V^{\mathrm{g}} + V^{\mathrm{u}}\right)/4\ (>0)$ and $g_2 = \left(V^{\mathrm{g}} - V^{\mathrm{u}}\right)/4$. In this paper, we focus on two limiting cases. (1) $|V^{\mathrm{u}}| \ll |V^{\mathrm{g}}|$ case. We calculate the first order terms in the small parameter $r_{\mathrm{t}} = \frac{2V^u}{V^g+V^u}$, which is proportional to the triplet channel interaction V^u. (2) $|V^{\mathrm{u}}| \gg |V^{\mathrm{g}}|$ case. We are interested in the first order terms in the small parameter $r_{\mathrm{s}} = \frac{2V^g}{V^g+V^u}$, which is proportional to the singlet channel interaction. In most of the cases, we expect that the singlet interaction V^g is larger than the triplet interaction V^u, and hence we briefly mention the amplitude of r_{t}. If we assume that V^{g} and V^{u} correspond to the on-site and nearest-neighbor interactions, respectively, their amplitudes can be roughly estimated as e^2/a_0 and e^2/a with a_0 being the Bohr radius and a being the lattice constant. Therefore, $r_{\mathrm{t}} \sim 0.1$ is a reasonable value. Although we can not apply the same argument for r_{s}, we consider the other limit $r_{\mathrm{s}} \ll 1$ to see the global behavior of γ.

In order to calculate the superconducting fluctuation current slightly above the mean field critical temperature, it is convenient to employ the GL theory. The free energy quadratic with respect to the order parameters can be obtained by the equation [12]

$$F = \int\frac{d^2\boldsymbol{q}}{\left(2\pi\right)^2}\left[\sum_{\lambda\lambda'}\Psi^*_{\lambda\boldsymbol{q}}\left(\hat{g}^{-1}\right)_{\lambda\lambda'}\Psi_{\lambda'\boldsymbol{q}} - \sum_{\lambda}T\sum_{\omega_n}\int\frac{d^2\boldsymbol{k}}{\left(2\pi\right)^2}G_\lambda\left(\boldsymbol{k}, i\omega_n\right)G_\lambda\left(-\boldsymbol{k}+\boldsymbol{q}, -i\omega_n\right)\left|\Psi_{\lambda\boldsymbol{q}}\right|^2\right], \tag{9}$$

where $\Psi_{\lambda\boldsymbol{q}}$ is the order parameter and $G_\lambda\left(\boldsymbol{k}, i\omega_n\right) = \left(i\omega_n - \xi_{\lambda\boldsymbol{k}}\right)^{-1}$ is the non-interacting normal Green's function. We set the Boltzmann constant $k_{\mathrm{B}} = 1$.

Firstly, we assume $E_{\mathrm{F}} > 0$, and we will soon show that nonreciprocal current vanishes for $E_{\mathrm{F}} < 0$ in Eq. (18) below. After some calculations (see Supplementary Information), we obtain

$$F = \int\frac{d^2\boldsymbol{k}}{\left(2\pi\right)^2}\sum_{\lambda\lambda'}\Psi^*_\lambda\left[\left(\hat{g}^{-1}\right)_{\lambda\lambda'} + \delta_{\lambda\lambda'}N_\lambda\left(S_1 - L_{\lambda\boldsymbol{k}}\right)\right]\Psi_{\lambda'}, \tag{10}$$

$$L_{\lambda\boldsymbol{k}} = K_\lambda\boldsymbol{k}^2 - \lambda R_\lambda\left(B_yk_x - B_xk_y\right), \tag{11}$$

$$S_1 = \log\frac{2e^{\gamma_{\mathrm{E}}}E_{\mathrm{c}}}{\pi T}, \tag{12}$$

with $\delta_{\lambda\lambda'}$, γ_{E}, and E_{c} being the Kronecker delta, Euler constant, and cutoff energy respectively. The density of states N_λ and the other coefficients K_λ and R_λ are given in Supplementary Information. The critical temperatures are obtained by solving

$$\det\left(\hat{g}^{-1} - \hat{N}S_1\left(T_{\mathrm{c}}\right)\right) = 0, \tag{13}$$

with $\hat{N}_{\lambda\lambda'} = \delta_{\lambda\lambda'}N_\lambda$. It results in

$$\frac{1}{S_1\left(T_{\mathrm{c}}\right)} = \frac{g_1\left(N_- + N_+\right)}{2} \pm \sqrt{\left(\frac{g_1\left(N_- - N_+\right)}{2}\right)^2 + g_2^2N_-N_+}. \tag{14}$$

Due to the form of the interaction ($g_1 \approx g_2$ for the singlet dominant case and $g_1 \approx -g_2$ for the triplet dominant case), the solution with the plus sign has much higher critical temperature. Hence, we can ignore the order parameter with the lower critical temperature when we calculate the fluctuation current.

The fluctuation current can be obtained by evaluating the equation [9, 17]

$$\begin{aligned} \boldsymbol{j} = & -T \sum_{\boldsymbol{k}} C \left. \frac{\partial \eta\left(\boldsymbol{k}+2e\boldsymbol{A}\right)}{\partial \boldsymbol{A}} \right|_{\boldsymbol{A}=\boldsymbol{0}} \\ & \times \int_{-\infty}^{0} du \exp\left[-C\int_{u}^{0} dt \eta\left(\boldsymbol{k}-2e\boldsymbol{E}t\right)\right], \end{aligned} \tag{15}$$

where η is the eigenvalue of the matrix in Eq. (10) with the higher critical temperature, and $C = \frac{32T_{\mathrm{c}}}{\pi\hbar(N_- + N_+)} + O\left(r_{\mathrm{t,s}}\right)$. We expand the eigenvalue up to $O(r_{\mathrm{t,s}})$ because we can show that the coefficient of the second order term is half of the coefficient of the linear term, hence the higher order terms can be neglected. It is noted that the factor C should contain a $r_{\mathrm{t,s}}$-dependent correction from the relaxation time of order parameters in the time-dependent GL theory. However, we ignore it because it does not affect the γ-value in the lowest order of $r_{\mathrm{t,s}}$. As in the case of the normal state, we assume that the electric and magnetic fields are applied along the x and y directions respectively, and evaluate the current along the x direction up to $O\left(B_y E_x^2\right)$. We will discuss the case of general fields configurations later. After the integration in Eq. (15) is carried out (we employed Mathematica), the relation Eq. (14) is used to simplify the equation. The result is

$$j_x = \sigma^{(1)} E_x + \sigma^{(2)} E_x^2, \tag{16}$$

$$\sigma^{(1)} = \frac{e^2}{16\hbar\varepsilon}, \tag{17}$$

$$\begin{aligned} \sigma^{(2)} = & \frac{\pi e^3 B_y r_{\mathrm{t,s}}}{128\hbar\varepsilon^2} \\ & \times \frac{N_- N_+ \left(K_- N_- - K_+ N_+\right)\left(K_- R_+ + K_+ R_-\right)}{S_1\left(T_{\mathrm{c}}\right) T_{\mathrm{c}} \left(N_- + N_+\right)\left(K_- N_- + K_+ N_+\right)^2}, \end{aligned} \tag{18}$$

in the lowest order of $r_{\mathrm{t,s}}$. Here, we have defined the reduced temperature $\varepsilon = \frac{T-T_{\mathrm{c}}}{T_{\mathrm{c}}}$. The linear coefficient $\sigma^{(1)}$ is the conventional form of the fluctuation conductivity in two-dimensional superconductors. The nonlinear coefficient $\sigma^{(2)}$ grows faster than $\sigma^{(1)}$ toward the critical temperature as in the case of MoS_2 [9]. It is noted that the parity mixing is essential for the nonreciprocal current, which vanishes for $r_{\mathrm{t,s}} = 0$.

We mention the case when the Fermi energy is below the crossing point of the bands ($E_{\mathrm{F}} < 0$). In this case, because the density of states from the upper band is zero, the nonreciprocal current vanishes, whereas, the normal current contribution exists [8].

For $E_{\mathrm{F}} > 0$, the γ-value expressed with the microscopic parameters is

$$W\gamma_{\mathrm{S}} = \frac{\sigma^{(2)}}{B_y\left(\sigma^{(1)}\right)^2} = \frac{\pi\mu_{\mathrm{B}}\hbar S_3 E_{\mathrm{F}} \alpha r_{\mathrm{t,s}}}{eS_1 T_{\mathrm{c}}\left(2E_{\mathrm{F}} + E_{\mathrm{R}}\right)}, \tag{19}$$

with W being the sample width and $E_{\mathrm{R}} = \frac{m\alpha^2}{\hbar^2}$ being the energy splitting at the shifted momentum due to the Rashba spin–orbit interaction, and $S_3 = \frac{7\zeta(3)}{4\pi^2 T_{\mathrm{c}}^2}$. We have used the relation between $\sigma^{(1)}$, $\sigma^{(2)}$, and γ as shown in Ref. [9]. More precisely, $\sigma^{(1)}$ and $\sigma^{(2)}$ in Eq. (19) should contain normal state contributions. Therefore, the γ-value approaches the value in Eq. (19) when the fluctuation contribution excesses the normal contribution. Explicitly, according to the Drude formula for the normal conductivity, the γ-value develops below $\varepsilon^* = \frac{m}{16\hbar n\tau}$, with n being the electron density and τ being the relaxation time.

It should also be noted that the sign of Eq. (19) depends on the sign of $r_{\mathrm{t,s}}$, i.e., the sign of V^{u} for the singlet dominant case and V^{g} for the triplet dominant case. Therefore, we can determine whether the interaction is repulsive or attractive by MCA measurement.

Here, we consider the nonreciprocal current which relies on the third order term with respect to the wavenumber as transition metal dichalcogenides [9]. Although the Rashba Hamiltonian possesses rotational symmetry, the cubic term of the wavenumber of the order parameter appears in the GL free energy in the presence of the in-plane magnetic field. The detailed calculation of the γ-value is shown in Supplementary Information, and the result is

$$W\tilde{\gamma}_{\mathrm{S}}^{\mathrm{pos}} = \frac{3\pi\mu_{\mathrm{B}}\hbar\alpha}{2eT_{\mathrm{c}}\left(2E_{\mathrm{F}} + E_{\mathrm{R}}\right)^2} \qquad (E_{\mathrm{F}} > 0), \tag{20}$$

$$W\tilde{\gamma}_{\mathrm{S}}^{\mathrm{neg}} = \frac{15\pi\mu_{\mathrm{B}}\hbar\alpha}{4eT_{\mathrm{c}}E_{\mathrm{R}}\left(2E_{\mathrm{F}} + E_{\mathrm{R}}\right)} \qquad (E_{\mathrm{F}} < 0). \tag{21}$$

The ratio between the γ-values from the parity mixing and the cubic term mechanisms is

$$\frac{\gamma_{\mathrm{S}}}{\tilde{\gamma}_{\mathrm{S}}^{\mathrm{pos}}} \sim \frac{r_{\mathrm{t,s}} E_{\mathrm{F}}\left(2E_{\mathrm{F}} + E_{\mathrm{R}}\right)}{S_1 T_{\mathrm{c}}^2}. \tag{22}$$

Therefore, the MCA from the cubic term is negligible compared to that from the parity mixing.

Next, we compare the γ-values in the normal state and the superconducting fluctuating regime (Eq. (19)). In Ref. [8], it has been concluded that the MCA exists if the Fermi energy is below the crossing point of the bands ($E_{\mathrm{F}} < 0$). The amplitude of the MCA is

$$W\gamma_{\mathrm{N}} = \frac{3\pi\mu_{\mathrm{B}}\hbar\alpha}{2e\left[E_{\mathrm{R}}\left(E_{\mathrm{R}} - 2\left|E_{\mathrm{F}}\right|\right)\right]^{3/2}}. \tag{23}$$

We assume that the strength of the spin–orbit interaction is comparable with the Fermi energy ($E_{\mathrm{R}} \approx |E_{\mathrm{F}}|$) because it is difficult to realize $E_{\mathrm{F}} < 0$ with a small E_{R}. Then, we obtain

$$W\gamma_{\mathrm{N}} \sim \frac{\mu_{\mathrm{B}}\hbar^2}{e\sqrt{m}} \frac{1}{\left|E_{\mathrm{F}}\right|^{5/2}}. \tag{24}$$

In the superconducting fluctuation regime, the nonreciprocal fluctuation current exists in the case of $E_{\mathrm{F}} > 0$, which is

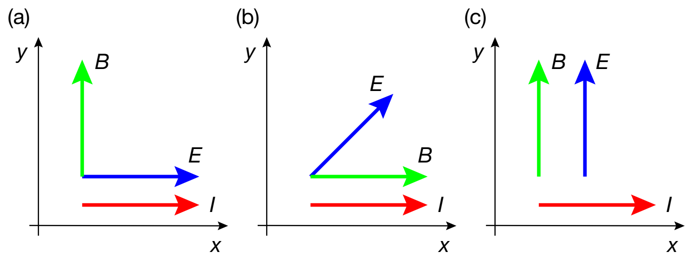

FIG. 1. (color online). The three fields configurations which correspond to (a) σ_{xyxx}, (b) σ_{xxxy}, and (c) σ_{xyyy}. B, E, and I in the figures represent the electric field, magnetic field, and nonreciprocal current respectively.

opposite to the normal state. With the same assumption for the normal state, we obtain

$$W\gamma_{\mathrm{S}} \sim \frac{\mu_{\mathrm{B}}\hbar^2}{e\sqrt{m}}\frac{r_{\mathrm{t,s}}E_{\mathrm{F}}^{1/2}}{S_1 T_{\mathrm{c}}^3}. \tag{25}$$

From Eqs. (24) and (25), we conclude that the MCA is drastically enhanced in the superconducting fluctuation regime because of the huge energy scale difference between the Fermi energy E_{F} and the critical temperature T_{c}. This is similar to the proceeding results for $\mathrm{MoS_2}$ [9].

We finally mention the electric and magnetic fields angle dependence of the nonreciprocal current. If we express the second order current as $j_i = \sigma_{ijkl}B_jE_kE_l$, the coefficient σ_{ijkl} is the pseudo tensor consistent with the crystal symmetry. Our model Eq. (3) possesses C_∞ symmetry and arbitrary in-plane mirror symmetries, which impose the restrictions that among σ_{xjkl}, only σ_{xxxy} $(= \sigma_{xxyx})$, σ_{xyxx}, and σ_{xyyy} can be finite (corresponding configurations are shown in Fig. 1), and $\sigma_{xyyy} = 2\sigma_{xxxy} + \sigma_{xyxx}$ and $\sigma_{yjkl} = -\sigma_{xjkl}$ are satisfied. According to calculations the same as that for σ_{xyxx} above, we obtain $\sigma_{xxxy} = -\frac{1}{3}\sigma_{xyxx}$ and $\sigma_{xyyy} = \frac{1}{3}\sigma_{xyxx}$, which satisfy the above conditions. If we define the angle between the current and magnetic (electric) field as $\theta_{\mathrm{B}}(\theta_{\mathrm{E}})$, the nonreciprocal current is

$$j^{(2)} = \sigma_{xyyy}\left(2\sin\theta_{\mathrm{B}} + \sin\left(\theta_{\mathrm{B}} - 2\theta_{\mathrm{E}}\right)\right)BE^2, \tag{26}$$

whose dependence in the $(\theta_{\mathrm{B}}, \theta_{\mathrm{E}})$ plane is shown in Fig. 2. It is noted that the normal state has the same angle dependence although it has not been discussed in the previous paper [8]. Realistic materials do not have such high symmetries, however, the above discussion should be applicable if the Fermi surface is almost circular.

We have investigated the MCA of the Rashba system in the superconducting fluctuation regime. The main result is the explicit form of the γ-value shown in Eq. (19). Here, we estimate the γ-value for the $\mathrm{LaAlO_3/SrTiO_3}$ interface, where the two-dimensional Rashba superconductivity is realized [18–21]. Its superconducting critical temperature is $T_{\mathrm{c}} \sim 100\mathrm{mK}$. We mentioned that the γ-value approaches the value in Eq. (19) when the fluctuation current exceeds the normal state contribution. The normal state sheet resistance is $R_{\mathrm{N}} \sim 1\mathrm{k}\Omega$

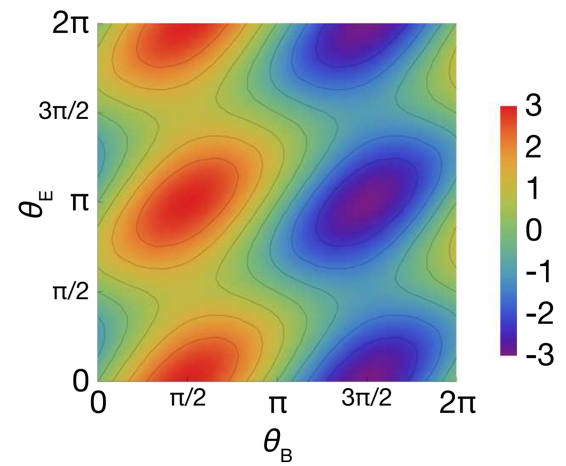

FIG. 2. (color online). The electric and magnetic fields angle dependence of $j^{(2)}$. θ_{B} (θ_{E}) represents the angle between the magnetic (electric) field and the nonreciprocal current. The amplitude is normalized by $\sigma_{xyyy}BE^2$.

[18–21] and the linear part of the fluctuation conductivity is shown in Eq. (17). By comparing them, we obtain $T - T_{\mathrm{c}} \sim 1.5\mathrm{mK}$, below which Eq. (19) is applicable. The carrier density is $n \sim 10^{13}\mathrm{cm}^{-2}$, spin–orbit field is $B_{\mathrm{SO}} \sim 1\mathrm{T}$, and the Debye temperature is $T_{\mathrm{D}} \sim 400\mathrm{K}$. If we assume $r_{\mathrm{t}} = 0.1$, we obtain $W\gamma_{\mathrm{S}} \sim 8\times10^{-2}\mathrm{T}^{-1}\mathrm{A}^{-1}\mathrm{m}$. With the typical sample width $W = 10^{-6}$, we obtain $\gamma_{\mathrm{S}} \sim 8\times10^{4}\mathrm{T}^{-1}\mathrm{A}^{-1}$, which is a very large value compared with the previously known systems [2–6].

Such a huge enhancement of the MCA originates from the energy scale difference between the Fermi energy E_{F} and the critical temperature T_{c} as indicated in Eqs. (24) and (25). This phenomenon is similar to the case of superconducting $\mathrm{MoS_2}$ [9], in which the large MCA stems from the trigonal warping term due to its three-fold rotational symmetry. However, the MCA originates from the parity mixing of the order parameter in the present case.

We have also shown the unique fields angle dependence of the nonreciprocal current, which is summarized in Fig. 1. It originates from the symmetry constraints of the higher rank response tensor. Especially, if the Fermi surface is almost circular and well approximated by our model, the fields angle dependence is given in Eq. (26) and shown in Fig. 2.

In addition to $\mathrm{LaAlO_3/SrTiO_3}$ as we have discussed, normal Rashba systems can be used with the aid of superconducting proximity effect. BiTeX (X=I, Br, Cl) [22, 23], the surface of Au(111) [24], or Bi/Ag(111) alloy [25] will work well.

Experimentally, the nonreciprocal current can be observed simply by measuring second order harmonic voltage drop under a fixed a.c. current. With such a simple method, we can observe the nontrivial second order response which reflects the crystal symmetry or the Hall response of the nonlinear current shown in Fig. 1(c). It is also possible to determine the sign of α from the sign of the γ-value. Moreover, we may estimate the amplitude of $r_{\mathrm{t,s}}$, which is the ratio between the even and odd parity attractive interactions by using the measured γ_{S}-value.

ACKNOWLEDGEMENTS

The authors thank Y. Saito, T. Ideue, and Y. Iwasa for valuable discussions. R.W. was supported by the Grants-in-Aid for Japan Society for the Promotion of Science No. JP15J09045. N.N. was supported by Ministry of Education, Culture, Sports, Science, and Technology Nos. JP24224009 and JP26103006, the Impulsing Paradigm Change through Disruptive Technologies Program of Council for Science, Technology and Innovation (Cabinet Office, Government of Japan), and Core Research for Evolutionary Science and Technology (CREST) No. JPMJCR16F1.

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