Efficient Quantum Measurement Engines

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We propose quantum engines powered entirely by a position-resolving measurement performed on a quantum particle. These engines produce work by moving the quantum particle against a force. Unlike classical information-driven engines (e.g. Maxwell's demon), the energy is not extracted from a thermal hot source but directly from the observation process via partial wave-function collapse of the particle. We present results for the work done and the efficiency for different values of the engine parameters. Feedback is required for optimal performance. We find that unit efficiency can be approached when one measurement outcome prepares the initial state of the next engine cycle, while the other outcomes leave the original state nearly unchanged.

Traditional discussions on quantum measurements focus on the probability to find different possible results, and how a given result changes the quantum state of the system being interrogated. However, researchers have begun to study the measurement process with point of view of thermodynamics, bringing surprising new insights. On the one hand, it was shown that resources may be required to make measurements [1], and certain measurements are even forbiden from taking place as they violate conserved quantities, such as energy [2]. The Wigner, Araki, Yanase (WAY) theorem [3–5] and its generalizations [6, 7], implies that a source of energy is needed if one wants to measure repeatedly and accurately observables which does not commute with the system's Hamiltonian.

On the other hand, when a measurement is allowed, it can be seen as a *thermodynamic resource*, analogous to heat or work reservoirs, such as a battery, in classical thermodynamics. This resource is two-fold: First, just as for measurements on classical systems, the information gained during a quantum measurement can be used to design work extraction via an appropriate feedback. This is the principle of Maxwell's demon already implemented in various quantum and classical systems [8–13]. In these setups, energy is extracted from a hot thermal reservoir just as in a regular heat engine. Second, in contrast with the classical world where observation does not impact the measured system, quantum measurement induces a non-unitary evolution that may change (and in certain cases increase) the energy of the system [14]. Combining these two properties allows quantum engines to be designed that extract energy *from the observation process*, in the absence of any hot thermal reservoir. A recent work [15] proposed a quantum Maxwell demon being able to extract energy from measurement-induced coherences in a qubit, using Rabi oscillations to transfer work into a coherent optical tone. However, as the dispersive measurement of the qubit also involved an optical field, the net result is simply using the qubit as an energy transducer from one optical mode to another. Ref. [16] exploits measurement to let the system do work on a classical magnetic field or a time-varying external potential. Ref. [17] also proposes a measurement-induced work extraction relying on non-Markovian effects in a zero temperature thermal reservoir.

In the current paper, we propose quantum measurement-fueled engines able to drive a single-particle current against a potential barrier. These engines thus do useful work such as raising an elevator, or charging a battery. We stress that this form of work extraction does not involve a time-dependent Hamiltonian of the system: the work output is directly stored in the potential energy of the particle, described in a fully quantum-mechanical way. In addition, the energy comes entirely from the process of observation: Measuring the system in a basis that does not commute with its Hamiltonian allows energy to be taken away from the measurement apparatus and given to the system in such a way as to be turned into useful work. This energy transfer is stochastic in nature, so has some similarities to heat in a stochastic thermodynamics context [14]. However, this similarity is only superficial, in that we show the existing thermodynamic bounds do not apply, and we are able to design engines with this "quantum heat" that can approach unit efficiency [15]. As recently stated, such an engine would not operate if the measuring apparatus was isolated [18]: input power must be provided to the apparatus to perform such measurements, which is taken into account for the engine's efficiency.

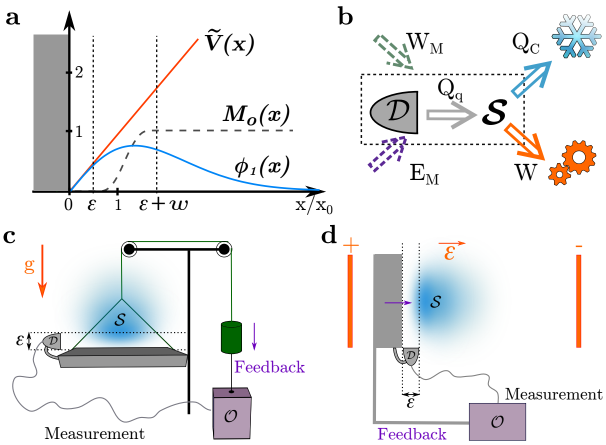

FIG. 1. **a**: Situation under investigation. The particle's initial wave function $\phi_1(x)$ (blue solid) is the ground state which is confined between the tilted scaled potential $\tilde{V}(x) = V(x)/(\hbar/(2mx_0^2) = x/x_0$ (depicted in solid red) and the wall (in gray), initially located at position $x_{\mathrm{wall}} = 0$ for the drawing. The generalized measurement is characterized by the function $M_o(x)$ (dotted line) able to tell that, for sure, the particle is outside the region $[x_{\mathrm{wall}}, x_{\mathrm{wall}} + \varepsilon x_0]$ when outcome o is found. **b**: Energy exchanges occurring during the engine cycle: the measurement provides the quantum heat Q_{q}, which is split between useful work W and the heat Q_{C} dissipated in the cold bath. The dotted arrows stand for the details of the energy exchanges during the measurement according to our measurement model (see SI [19]): the work W_{M} is needed to entangle $\mathcal{S}$ and the meter, and E_{M} is the average energy provided to the meter to reset it before the next cycle. **c,d**: Two possible implementations. **c**: The elevator. An atom $\mathcal{S}$ is on a platform and experiences gravitational acceleration g. The detector $\mathcal{D}$ checks every cycle if the atom is within a distance ε from the platform and sends the outcome to the elevator operator $\mathcal{O}$ (lift attendant) who shifts the elevator to the "next floor" of height εx_0 for free if the outcome is o. **d**: The single electron battery. The negatively charged particle experiences an electric field of intensity $\mathcal{E}$ between two electrodes. The wall is a piece of neutral insulator that can be moved depending on the outcomes of $\mathcal{D}$. The electron successfully moved distance L between the electrodes charges the battery with energy $e\mathcal{E}L$.

Setup.— We will now make a quantum measurement do useful work by having a particle climb a tilted linear potential. The setup is the following (see Fig. 1): a particle is described by a pure state $|\psi\rangle$ in a potential $\hat{V} = V(\hat{x}) = V_0\hat{x}/\xi + V_{\mathrm{wall}}(\hat{x})$. The term $V_{\mathrm{wall}}(x)$ corresponds to a barrier of infinite height preventing the particle from reaching the positions $x < x_{\mathrm{wall}}$. The time-independent Schrödinger equation for a particle with mass m and energy E, $-(\hbar^2/2m)\partial_x^2\phi(x) + V(x)\phi(x) = E\phi(x)$, can be rewritten for $x > x_{\mathrm{wall}}$ as an Airy differential equation $\phi''(z) - z\phi(z) = 0$ in term of the variable $z \equiv (x - \xi E/V_0)/x_0$. We have introduced the characteristic length $x_0 = (\hbar^2\xi/2mV_0)^{1/3}$.

Taking into account the boundary condition $\phi(x_{\mathrm{wall}}) = 0$, we can express the eigenstates of the Hamiltonian in terms of the Airy function $\mathrm{Ai}(x)$ [20] and its zeros $\{a_l\}_{l\geq 1}$ with $a_l < 0$ and $a_{l+1} < a_l$:

$$\phi_n(x) = \frac{1}{\sqrt{x_0}} \frac{\mathrm{Ai}[(x - x_{\mathrm{wall}})/x_0 + a_n]}{\mathrm{Ai}'(a_n)}, \quad (1)$$

for $x \geq x_{\mathrm{wall}}$, and 0 otherwise. The energy eigenvalues are

$$E_n = \frac{\hbar^2}{2mx_0^2}|a_n| + \frac{\hbar^2}{2mx_0^2}(x_{\mathrm{wall}}/x_0). \quad (2)$$

Let us start the system so the particle is in the ground state $\phi_1(x)$ and the wall is at position $x = 0$. An ideal position measurement of the particle is in fact impossible, because it would require an infinite amount of energy. Let us therefore consider another kind of position measurement, and simply determine whether the particle is within some distance εx_0 of the wall, or not. As shown in the analysis below, even this "yes-no" question introduces discontinuities in the wavefunction and is also too costly. We therefore adopt a minimal model, and consider two possible outcomes of a generalized measurement, each associated with Kraus operators M_o and M_i, where the labels i, o denote that particle is found inside or outside the region $[0, \varepsilon x_0]$ from the wall. We smooth the abrupt transition with an interpolating region from εx_0 to $(\varepsilon + w)x_0$. Let us choose M_o to be

$$M_o = \begin{cases} 0, & x/x_0 < \varepsilon, \\ \sin[\pi(x/x_0 - \varepsilon)/2w], & \varepsilon < x/x_0 < \varepsilon + w, \\ 1, & x/x_0 > \varepsilon + w. \end{cases} \quad (3)$$

$M_i^2 + M_o^2 = 1$ for all space (let us choose M_i also real) because M_o, M_i are Kraus operators [21, 22], so we must then have M_i decreasing from 1 at the wall as a cosine function down to 0. Regardless of the specific form for $M_{o,i}$, quantum mechanics dictates that the probability of finding result i, o is given by $P_{i,o} = \langle\phi_1|M_{i,o}^2|\phi_1\rangle = \int dx M_{i,o}^2(x)\phi_1^2(x)$, with a conditional post-measurement state given by $|\phi_\alpha\rangle = M_\alpha|\phi_1\rangle/\sqrt{P_\alpha}$, $\alpha = i, o$.

Engine cycle.— The three strokes of the engine cycle can now be described. The engine consists of the system (a single particle), a detector, and a controller to either move the wall's position or keep it in place. The object of the engine is to convert energy given by the measurement process into useful work.

1. Measurement: A measurement of the particle's position occurs, resulting in the stochastic result i or o with probabilities P_i, P_o. Generally, the new (disturbed) state of the particle is no longer in its ground state and therefore has a greater internal energy, regardless of which outcome occurs. The energy gained by the particle during this step must be provided by the measurement because total energy is conserved. We refer to the average energy gain over both outcomes, $Q_\mathrm{q} \geq 0$, as "quantum heat" because of its stochastic nature.

2. Feedback: If outcome i was found (particle is close to the wall), then the engine controller does nothing. If outcome o is found (the particle must be a distance larger than ε from the wall), then the controller suddenly moves [23] the wall to the right of a distance $x_\mathrm{M} = \varepsilon x_0$. This costs no work in principle because the wavefunction's value is 0. Further, it has been shown that motion of the wall through a region of zero wavefunction makes no change to the rest [24].

3. Reset: Whatever the outcome, we let the particle relax in contact with a bath of temperature T_R very low with respect to $T^* = (E_2 - E_1)/k_\mathrm{B} \simeq 1.75\hbar^2/2mx_0^2 k_\mathrm{B}$, where E_n is defined in Eq. (2). For a relaxation time long enough, the particle is in its ground state, possibly with an advanced wall. As the particle is in a known pure state after the measurement, this step can in principle be replaced with a coherent energy extraction step mapping the post-measurement states to the ground state(s), such that no thermal bath is required. The corresponding system unitaries may be included as part of the measurement operator, so that no energy is wasted on the cold bath. This unitary version of the reset operation has the great advantage to be achievable in a finite time, while a perfect thermalization is constrained by the third law of thermodynamics. However, this replacement may be impractical in certain cases. Note that if the wall has been advanced (outcome o) the new ground state has an energy increased by an amount $W = (\hbar^2/2mx_0^2)\varepsilon$: this corresponds to work extracted during the cycle used to increase the particle's potential energy.

We note that there is a conditional step in the above cycle, and the engine is instructed to do different things depending whether outcome i or o is found. If we consider this system to be an isolated thermodynamic system of the same type of Szilard, see e.g. [25] with the observer acting as a quantum Maxwell demon, the demon resets its memory in a bath of temperature T_D. Although we are not extracting work from a thermal bath, but rather from the quantum measurement process, the use of finite resources used cyclically still requires erasure of memory. The erasure cost is $W_\mathrm{er} = -k_\mathrm{B} T_\mathrm{D} \sum_{\alpha=i,o} P_\alpha \log P_\alpha \leq k_\mathrm{B} T_\mathrm{D} \log 2$. It can be set much smaller than W either by considering a sufficiently low T_D (in particular for $T_\mathrm{D} = T_\mathrm{R}$) or choosing $P_o \gg P_i$ or $P_i \gg P_o$. These latter two situations are reached in the Zeno regime and the gradual measurement limit, respectively, as detailed below.

Results.— We now analyze the engine's performance. The engine cycle is stochastic, so it is possible that from run to run, a large amount of work may be done (i.e. a long sequence of o results). However, we will consider the average performance of the engine. The engine cycle is constructed so that the system always begins in the ground state, and therefore the average work per cycle is given by the work in steps 2 and 3, times the probability of o,

$$W = \varepsilon \frac{\hbar^2}{2mx_0^2} \int dx M_o^2(x) \phi_1^2(x). \tag{4}$$

The average amount of energy given by the measurement apparatus to the system (per cycle) is given by $Q_\mathrm{q} = \sum_\alpha P_\alpha \langle \phi_\alpha | H | \phi_\alpha \rangle - \langle \phi_1 | H | \phi_1 \rangle$, or

$$Q_\mathrm{q} = -\frac{\hbar^2}{2m} \int dx \left(\sum_\alpha M_\alpha(x) M_\alpha''(x) \right) \phi_1^2(x), \tag{5}$$

where we have assumed $M_{o,i}$ are diagonal in the position basis, causing the potential energy term to drop out. The conversion efficiency is defined as

$$\eta = \frac{W - W_\mathrm{er}}{Q_\mathrm{q}}. \tag{6}$$

For the specific choice of the Kraus operators in Eq. (3), the quantum heat takes on the simple form, $\tilde{Q}_\mathrm{q} = \left(\frac{\pi}{2w}\right)^2 \int_{\varepsilon x_0}^{(\varepsilon+w)x_0} dx\, \phi_1^2(x)$, where the tilde symbol denotes the work or heat divided by $\hbar^2/(2mx_0^2)$.

It may naively be thought that this engine is most efficient when the variables ε, w are very small, such that the outcome o becomes much more frequent than the outcome i, a kind of "Zeno limit engine" as in [15]. In fact, this is false because the quantum heat diverges when $w \to 0$, reflecting the problematic nature of the strict "yes-no" question mentioned in the setup (See SI [19]). As shown in Fig. 2, the best work performance ($\tilde{W} = 0.80$) is given for

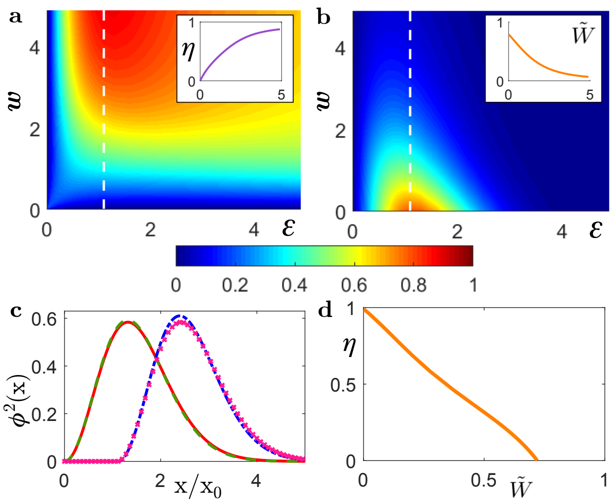

FIG. 2. **a,b**: Efficiency η (**a**) and average work extracted per cycle $\tilde{W}$ (**b**) as a function of ε and w. Insets: Section of the efficiency (**a**) and the work (**b**) along the dashed white line corresponding to $\varepsilon = \varepsilon^*$. **c**: Comparison of the post-measurement state corresponding to result o (blue dashed-dotted) [resp. i (green dashed)], with $\varepsilon = \varepsilon^*$ and $w = 10$ with the new ground state wavefunctions of the linear trap potential displaced by an amount $\varepsilon^* x_0$ (pink "x") [resp. original ground state wavefunction (red solid)]. **d**: Efficiency η as a function of the work $\tilde{W}$ when w is varied while ϵ is kept constant equal to ϵ^*. For plots **a,d**, $T_R = 10^{-2} T^*$.

$\varepsilon \approx 1.18$, and for $w = 0$, which corresponds to vanishing efficiency. Conversely, the efficiency nearly reaches unity for large w, i.e. in the case of "gradual" measurements that have a very slow turn-on outside the region $[0, x_0\varepsilon]$. Define the asymptotic efficiency, $\eta_{asyp} = \lim_{w\to\infty} \eta$ to find [26]

$$\eta_{asyp} = \varepsilon \frac{\int_{\varepsilon x_0}^{\infty} dx\, (x/x_0 - \varepsilon)^2 \phi_1^2(x)}{\int_{\varepsilon x_0}^{\infty} dx\, \phi_1^2(x)}. \qquad (7)$$

Incredibly, the efficiency of the measurement approaches 1 at $\varepsilon = \varepsilon^* \approx 1.100$, corresponding to a maximal efficiency of $\eta_{max} \approx 0.998$. We can understand more deeply why this optimal value of ε^* corresponds to maximal efficiency by plotting the (normalized) post-measurement state $\phi_o(x) \propto (x - \varepsilon^*)\,\mathrm{Ai}(x + a_1)$, and comparing it to the new ground state of the (displaced) trap potential ($\propto \mathrm{Ai}(x + a_1 - \varepsilon x_0)$), shown in Fig. 2**c**; they are nearly identical. Similarly, the comparison between $M_i \phi_1(x)/\sqrt{P_i}$ with the original ground state look nearly the same. Consequently, either moving the wall by εx_0 or leaving it in place almost perfectly realizes the next phase of the engine cycle [27]. Eventually, a trade-off between efficiency and work can be found for intermediate values of w (see Fig.2**d**), setting $\epsilon = \epsilon^*$ [28].

Implementations.— We implement two different variations of the engine shown in Fig. 1. The first is a single atom elevator: a gravitational potential of $V(x) = mgx$ acts on the atom, resulting in the characteristic length of $x_0 = (\hbar^2/2m^2g)^{1/3} \approx 6\mu\mathrm{m}/\tilde{m}^{2/3}$ [29] for an atom near the surface of the earth of relative atomic mass $\tilde{m}$. The temperature needed to cool to the ground state is $T^* = \tilde{m}^{1/3}12\mathrm{nK}$, so for e.g. a Rb atom, we have $x_{0,Rb} \approx 300\mathrm{nm}$ and require $T^*_{Rb} \approx 50\mathrm{nK}$, which is quite possible to realize in cold atom experiments. Alternatively, we can consider one ultra-cold neutron above a neutron mirror, which is the setup of recent gravity-resonance spectroscopy experiments [30, 31]. In the sketch of Fig. 1**a**, the elevator has a platform that has a counter balanced weight over the pulley. Since the net force is zero, the elevator can be raised to the "next floor" by the elevator operator with no work done, so long as the movement only occurs when the atom has no amplitude to be near the platform.

In our second example shown in Fig. 1**b**, we consider a parallel plate capacitor that is being charged, one electron at a time (a battery). We consider a potential difference of $1V$ across a $1cm$ gap. This gives a characteristic length scale of $x_0 = (\hbar^2/(2me^2\mathcal{E}))^{1/3} \approx 72nm$, where $\mathcal{E}$ is the electric field between the plates. The required reset temperature is only $T^* \approx 0.15K$ because the electron is so light. An insulating, uncharged plate with negligible susceptibility can be moved through the electric field without any work done. The plate stops the electron from accelerating back to the positively charged electrode. A measurement of the electron's position away from the plate allows the controller to advance the position of the plate to bring the electron to the other side of the capacitor, charging the battery.

In the SI [19], we present a model of the measurement, implemented by a spin-1/2 meter impulsively interacting with the particle, in order to track the energy exchange. Letting the spin begin with energy $E_o - E_i$ the difference of the energies of the states corresponding to outcomes o and i, during the interaction, an amount of work $W_\mathrm{M} = E_i - E_1 \approx 0$ is performed on the joint spin-particle system. The average energy given away by the spin is $E_\mathrm{M} = P_o(E_o - E_i)$. These energies provide the "fuel" for the quantum measurement engine, $Q_\mathrm{q} = E_\mathrm{M} + W_\mathrm{M}$, and must be replenished for the engine to continue working, see Fig. 1**b**: as dictated by the WAY theorem, the measurement is not repeatable if the meter is not externally powered.

Conclusions.— We have constructed an explicit quantum engine that converts energy from quantum measurement to do useful work on the system. This process requires feedback in general. We stress that a simple transfer of energy is not sufficient to make a working engine. The energy must be transferred in such a way that it can be efficiently extracted. To this end, our three stroke engine is near optimal because one outcome produces nearly the correct ground state of the system in the next cycle, while the other outcomes leaves the state nearly the same as before. The ability to advance our wall with no work expended allows efficient conversion of kinetic to potential energy to make the particle do work against an opposing force, provided by the measurement process. In spite of the stochastic nature of the measurement process, we are able to attain efficiencies approaching unity. This result clearly illustrates the differences with quantum thermal engines.

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