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Yunxiang Liao and Matthew S. Foster

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Dephasing catastrophe in $4-\epsilon$ dimensions: A possible instability of the ergodic (many-body-delocalized) phase

Yunxiang Liao[1] and Matthew S. Foster[1,2]

[1]*Department of Physics and Astronomy, Rice University, Houston, Texas 77005, USA*
[2]*Rice Center for Quantum Materials, Rice University, Houston, Texas 77005, USA*

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In two dimensions (2D), dephasing by a bath cuts off Anderson localization that would otherwise occur at any energy density for fermions with disorder. For an isolated system with short-range interactions, the system can be its own bath, exhibiting diffusive (non-Markovian) thermal density fluctuations. We recast the dephasing of weak localization due to a diffusive bath as a self-interacting polymer loop. We investigate the critical behavior of the loop in $d = 4 - \epsilon$ dimensions, and find a nontrivial fixed point corresponding to a temperature $T^* \sim \epsilon > 0$ where the dephasing time *diverges*. Assuming that this fixed point survives to $\epsilon = 2$, we associate it to a possible instability of the ergodic phase. Our approach may open a new line of attack against the problem of the ergodic to many-body-localized phase transition in $d > 1$ spatial dimensions.

Introduction.— The interplay between quantum interference and inelastic quasiparticle scattering in a disordered medium takes center stage in the problem of many-body localization (MBL) [1–4]. Although the MBL phase [5] and the ergodic metal-to-MBL insulator transition [6–8] have been explored extensively in $d = 1$ spatial dimension, their nature or even existence in higher dimensions remain open questions [9].

Instead of the many-body localized phase, in this work we reconsider the *standard theory of the ergodic phase* [10–12] in $d > 1$ spatial dimensions [13]. We identify a "hole" in this theory, when applied to a system that could transition to the MBL phase at low temperature. The hole concerns dephasing, which stabilizes the ergodic phase at finite temperature in 2D for an isolated system of fermions with weak disorder and short-range interactions. We show that calculating the dephasing rate due to short-ranged interactions for the first quantum correction to transport (weak localization) is tantamount to computing a certain correlation function in a strongly coupled, auxiliary quantum field theory (QFT). While there exists a standard result for this case (e.g., [14]), it is in fact a mean-field approximation (the self-consistent Born approximation SCBA), and mean field theory is expected to be unreliable for any field theory below its upper critical dimension [15, 16]. Within a controlled ϵ-expansion, we identify a nontrivial fixed point corresponding to a nonzero critical temperature at which the dephasing of weak localization appears to fail. This hints at the possibility of describing the ergodic-to-MBL phase transition by approaching from the ergodic side.

We emphasize that we consider *weak* disorder and "order one" strength interactions, similar to the standard literature [10–12] but different from Basko, Aleiner, and Altshuler [1] (who focused on strong disorder and weak interactions). The dephasing problem identified and treated in this work does not arise in the theory of diffusive electrons in solid state materials, owing to the long-ranged nature of the Coulomb interaction. In the latter case, the Markovian character of the (approximate) dephasing kernel admits an exact solution [10, 17], which happens to be the same as the SCBA [18].

The problem considered here is different from that of a system with well-localized single particle states interacting with an external bath. In that case there are arguments (see e.g. [1, 19, 20]) that any bath with a

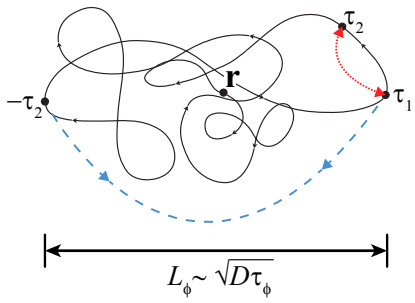

FIG. 1: Cooperon as a self-interacting polymer loop: The Cooperon $C^t_{\eta,-\eta}(\mathbf{r},\mathbf{r})$ entering the expression for the weak localization correction [Eq. (1)] is given by the return probability of a diffusing particle interacting with a fluctuating density field bath $\rho_{\rm cl}(\mathbf{r},t)$ [10–12, 17, 18, 21]. The diffusion process starts and ends at the same point $\mathbf{r}$ from time $-\eta$ to η. For short-range interactions, the density fluctuations are diffusive and, similar to the virtual diffusion process, controlled by the diffusion constant D [see Eq. (4)]. The fluctuation-averaged Cooperon (return probability) in Eq. (3) can be interpreted as the path integral of a self-interacting polymer loop. The action is given by Eqs. (3b) and (3c), where S_0 is the action for the unperturbed random walk, and S_I is the self-interaction term. The first term in S_I describes the repulsive (causal) interaction between $\mathbf{r}(\tau_1)$ and $\mathbf{r}(\tau_2)$ of range $\sqrt{D_c|\tau_1-\tau_2|}$, and is indicated by the red dotted line in the figure; the second term, denoted by the blue dashed line, represents the attractive (anticausal) interaction between $\mathbf{r}(\tau_1)$ and $\mathbf{r}(-\tau_2)$ of the same range. The intrinsic length scale of the polymer loop is determined by the dephasing length $L_\phi = \sqrt{D\tau_\phi}$, which should diverge as the system approaches the MBL transition point from the ergodic phase [1].

continuous spectrum leads to thermalization. We treat the weak localization correction and its dephasing on the same footing, within the hydrodynamic framework [10–12, 21] that describes how quantum interference corrections to transport are dephased *within the same system*.

For disordered, interacting fermions in the ergodic phase at finite energy density (temperature), the weak localization correction to the conductivity, which results from the quantum interference of wave functions scattered by impurities, is cut off in the infrared by dephasing [10–12]. The dephasing problem is equivalent to a virtual random walk (Cooperon return probability) that interacts with a stochastic bath. At low temperatures in a metal, the system serves as its own bath due to inelastic electron-electron collisions, mediated by screened Coulomb interactions. In this case the bath is Ohmic, i.e. the fluctuations in time are Markovian for the relevant frequency range $\omega \lesssim k_B T$. The Markovian case is exactly solvable and gives a finite dephasing rate $1/\tau_\phi > 0$ for any $T > 0$ [10]. In contrast to the long-range Coulomb interaction [10, 17], the self-generated bath of a fermion system with short-range interactions is *non-Markovian* (diffusive) [21], and does not admit an exact solution. Since in this case the thermal fluctuations of the diffusive bath are slow, these could prove ineffective at amputating quantum interference corrections for sufficiently small T and/or small diffusion constant D.

If the dephasing rate vanishes, the weak localization correction diverges logarithmically in the infrared for an infinite system in 2D. The system is therefore completely localized and unable to act as a heat bath for itself. This suggests the possibility to access the MBL-ergodic transition in 2D by approaching it from the metallic side. The transition could be explored as a "dephasing catastrophe" in a model with short-range interactions; the latter are believed to be a requirement for MBL [22]. In particular, if there exists a 2D system with a many-body mobility edge at a finite energy density corresponding to temperature T_{MBL}, the dephasing of quantum interference corrections would fail as T approaches T_{MBL} from above.

In this Letter, we consider the lowest order weak localization correction due to the virtual return probability of the Cooperon. We recast the dephasing of the Cooperon due to a diffusive bath as a geometric statistical-mechanics problem of a self-interacting polymer loop, analogous to the self-avoiding random walk (SAW)—see Fig. 1. The self-interactions are themselves diffusive, as mediated by the bath. As with the SAW, we construct a replica field theory [15] whose upper critical dimension d_c is 4, and investigate the critical behavior using the renormalization group (RG) approach.

In addition to the Gaussian fixed point (corresponding to decoupled Cooperon and bath), we identify a nontrivial fixed point controlled by an ϵ-expansion with $\epsilon \equiv d_c - d > 0$. The fixed point has only one relevant direction, and its location corresponds to a finite temperature $T^* \sim \epsilon > 0$ at which the dephasing rate ("mass of the Cooperon") vanishes. We also compute the correlation length critical exponent ν to one-loop level, and find in d spatial dimensions $\nu = 2/d$, saturating the Harris-Chayes bound [26, 27].

As with all studies of this type [15, 16, 23], we cannot be sure that the fixed point we find survives to finite $\epsilon = 2$, which would be the relevant dimension for demonstrating an instability of the ergodic phase. However, like all ϵ-expansions we *can* be sure that our result is not invalidated by the next order in perturbation theory, at least for sufficiently small $\epsilon \ll 1$. Moreover, we show that the dephasing problem represents a type of geometric criticality, and as such is governed by a nonunitary QFT. Critical points in nonunitary QFTs can arise even at the "lower critical dimension" ($\epsilon = 2$), as is well-known for the self-avoiding walk [23, 24] and weak antilocalization in the symplectic class [25]. In 3D ($\epsilon = 1$), our result predicts a kink in the temperature-dependence of the conductivity for an isolated, weakly disordered fermion system with short-ranged interactions, since the infrared part of the weak localization correction is analytic in the inverse dephasing length.

While our result suggests a possible instability of the ergodic phase in 2D, we cannot immediately identify this with the ergodic-MBL transition. It is possible that the MBL phase itself does not exist for $d > 1$ [9]. Moreover, at the ergodic-MBL transition, one must account for higher order quantum interference corrections; we return to this issue in the conclusion. We note however that our results should be testable via a classical lattice polymer simulation in two and three dimensions.

The problem.— The weak localization (WL) correction to the conductivity, which is caused by the quantum interference between pairs of time-reversed paths, can be written as

$$\delta\sigma_{\mathsf{WL}} = -\frac{4e^2}{\hbar\pi} D \int_\eta \left\langle C^t_{\eta,-\eta}(\mathbf{r},\mathbf{r}) \right\rangle_\rho , \tag{1}$$

where $C^t_{\eta,-\eta}(\mathbf{r},\mathbf{r})$ is the Cooperon satisfying the differential equation [10, 21]:

$$\left\{ \partial_\eta - \frac{D}{2}\nabla^2 + \frac{i}{2}\left[\rho_{\mathsf{cl}}\left(\mathbf{r}, t+\frac{\eta}{2}\right) - \rho_{\mathsf{cl}}\left(\mathbf{r}, t-\frac{\eta}{2}\right)\right]\right\} \times C^t_{\eta,\eta'}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\delta(\eta-\eta')\delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

Here D is the diffusion constant. The variables t and η represent the average and relative time of the time-reserved paths, respectively. ρ_{cl} denotes the "classical" (versus quantum [21]) component of the hydrodynamic density field, whose interaction with the Cooperon results in phase relaxation. The calculation of the WL correction requires averaging $C^t_{\eta,-\eta}(\mathbf{r},\mathbf{r})$ in Eq. (1) over

stochastic thermal fluctuations of ρ_{cl} (denoted by the angular brackets).

Following Refs. [10] and [17], the solution to Eq. (2) can be represented in the form of a Feynman path integral [28]. After averaging over the density fluctuations, we obtain

$$\left\langle C^{t}_{\eta,-\eta}(\mathbf{r},\mathbf{r})\right\rangle_{\rho} = \frac{1}{2}\int_{\mathbf{r}(-\eta)=\mathbf{r}}^{\mathbf{r}(\eta)=\mathbf{r}} \mathcal{D}\mathbf{r}(\tau) \times \exp\left\{-S_0[\mathbf{r}(\tau)] - S_I[\mathbf{r}(\tau)]\right\}, \quad (3a)$$

$$S_0[\mathbf{r}(\tau)] = \int_{-\eta}^{\eta} d\tau \frac{1}{2D}\dot{\mathbf{r}}^2(\tau), \quad (3b)$$

$$S_I[\mathbf{r}(\tau)] = \frac{1}{4}\int_{-\eta}^{\eta} d\tau_1 \int_{-\eta}^{\eta} d\tau_2 \times \begin{bmatrix} i\Delta_{\rho}^{(K)}\left(\mathbf{r}(\tau_1)-\mathbf{r}(\tau_2), \dfrac{\tau_1-\tau_2}{2}\right) \\ -i\Delta_{\rho}^{(K)}\left(\mathbf{r}(\tau_1)-\mathbf{r}(-\tau_2), \dfrac{\tau_1-\tau_2}{2}\right) \end{bmatrix}, \quad (3c)$$

where $i\Delta_{\rho}^{(K)}$ is the Keldysh correlation function of ρ_{cl}. Notice that the dependence of the Cooperon on the average time t is removed by the fluctuation average.

For long range Coulomb interactions [10, 17], the noise kernel $i\Delta_{\rho}^{(K)}$ in Eq. (3c) is instantaneous in time, and thus of Markovian type. By contrast, the kernel $\Delta_{\rho}^{(K)}$ for short-range interactions is diffusive and non-Markovian. At temperature T, it is given by [14, 21]

$$i\Delta_{\rho}^{(K)}(\mathbf{r},\tau) \simeq k_{\mathsf{B}}T\frac{\gamma^2}{\kappa}\left(\frac{1}{4\pi D_c|\tau|}\right)\exp\left(-\frac{\mathbf{r}^2}{4D_c|\tau|}\right), \quad (4)$$

where γ indicates the short-range interaction strength, κ is the charge compressibility, and $D_c = D/(1-\gamma)$ is the charge diffusion constant.

Eq. (3) can be interpreted as the path integral of a self-interacting polymer loop with boundary condition $\mathbf{r}(-\eta) = \mathbf{r}(\eta) = \mathbf{r}$. The Gaussian term S_0 in Eq. (3b) describes the unperturbed random walk. The interaction term S_I in Eq. (3c) consists of two parts: the repulsion between $\mathbf{r}(\tau_1)$ and $\mathbf{r}(\tau_2)$, and the attraction between $\mathbf{r}(\tau_1)$ and $\mathbf{r}(-\tau_2)$. The repulsive (attractive) interaction arises due to causal (anticausal) correlations within a path (between time-reversed paths). The ranges of both types of interactions are $\sqrt{D_c|\tau_1-\tau_2|}$. Fig. 1 shows a schematic illustration of this polymer loop, where the repulsive and attractive interactions are indicated by the red dotted and blue dashed lines, respectively. The characteristic length scale of the loop corresponds to the dephasing length $L_\phi = \sqrt{D\tau_\phi}$.

Another self-interacting polymer model described by Eq. (3) is the self-avoiding random walk (SAW), where S_I instead acquires the form

$$S_I[\mathbf{r}(\tau)] = g\int d\tau_1 \int d\tau_2\, \delta\left(\mathbf{r}(\tau_1)-\mathbf{r}(\tau_2)\right), \quad (5)$$

with g the interaction constant. While the SAW interaction in Eq. (5) is local in space (a contact interaction), it is entirely nonlocal in "time," while the interaction in Eq. (4) is nonlocal in space and time.

Replica approach.— It is well-known that the critical scaling of the SAW can be studied using a replica approach [15]. We apply a similar strategy for the dephasing problem by defining the replica field theory,

$$Z = \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}\rho_{\mathsf{cl}}\, e^{-S_{\Psi}[\bar{\Psi},\Psi]-S_{\rho}[\rho_{\mathsf{cl}}]-S_c[\bar{\Psi},\Psi,\rho_{\mathsf{cl}}]}, \quad (6a)$$

$$S_{\Psi}[\bar{\Psi},\Psi] = \int_{\mathbf{k},\omega} \bar{\Psi}^a(\mathbf{k},\omega)\left(\xi k^2 - ih\omega + r\right)\Psi^a(\mathbf{k},\omega), \quad (6b)$$

$$S_{\rho}[\rho_{\mathsf{cl}}] = \frac{1}{2}\int_{\mathbf{k},\omega} \rho_{\mathsf{cl}}(\mathbf{k},\omega)\rho_{\mathsf{cl}}(-\mathbf{k},-\omega)\,\frac{k^4+\omega^2}{2k^2}, \quad (6c)$$

$$S_c[\bar{\Psi},\Psi,\rho_{\mathsf{cl}}] = \frac{i}{2}\lambda\int_{\mathbf{q},\Omega}\int_{\mathbf{k},\omega} \rho_{\mathsf{cl}}(\mathbf{q},\Omega) \times \begin{bmatrix} \bar{\Psi}^a\left(\mathbf{k}+\mathbf{q},\omega+\dfrac{\Omega}{2}\right) \\ -\bar{\Psi}^a\left(\mathbf{k}+\mathbf{q},\omega-\dfrac{\Omega}{2}\right) \end{bmatrix}\Psi^a(\mathbf{k},\omega), \quad (6d)$$

where λ and ξ are (in $d=2$) defined by

$$\lambda \equiv \sqrt{k_B T\gamma^2/\kappa D_c}, \qquad \xi \equiv D/2D_c. \quad (7)$$

The Cooperon is encoded by the (bosonic or fermionic) field Ψ^a; the superscript $a \in \{1,2,...N\}$ indexes the replica space. The fluctuation-averaged Cooperon $\langle C\rangle_{\rho}$ can be obtained as the correlation function $\tilde{C}(\mathbf{k},\omega) \equiv \left\langle \Psi^a(\mathbf{k},\omega)\bar{\Psi}^a(\mathbf{k},\omega)\right\rangle$ in the replica limit $N \to 0$, $\langle C\rangle_{\rho}(\mathbf{k},\omega) = \frac{1}{2}\tilde{C}(\sqrt{D_c}\mathbf{k},\omega)$. In Eq. (6b), h denotes the scaling factor of the frequency (which will acquire anomalous corrections), while the "mass term" r is not present in the bare action, but will be generated by the RG transformation described below.

One can integrate out the density field ρ_{cl} in Eq. (6), and introduce a matrix field $\hat{Q}$ to decouple the generated quartic interaction terms. After integrating out Ψ, one arrives at an effective field theory for $\hat{Q}$, whose saddle point Q_{SP} gives the SCBA dephasing rate (in $d=2$) [13, 14]:

$$\tau^{-1}_{\mathsf{SCBA}} = 2\int_{\mathbf{k},\omega}\left[Dk^2 - i\omega + \tau^{-1}_{\mathsf{SCBA}}\right]^{-1}\left[i\Delta_{\rho}^{(K)}(\mathbf{k},\omega)\right] = \frac{1}{2\pi D_c\kappa}\frac{\gamma^2}{(2-\gamma)}\,k_{\mathsf{B}}T\ln\left(\frac{k_{\mathsf{B}}T}{\tau^{-1}_{\mathsf{SCBA}}}\right). \quad (8)$$

This result is identical to that obtained from the lowest order cumulant expansion of Eq. (3a), when the infrared limit of the integral is cut off at $\tau_{\mathsf{SCBA}}^{-1}$ "by hand" [18]; the result also obtains via self-consistent diagrammatic perturbation theory [12, 14, 21]. The SCBA is exact for long-range screened Coulomb interactions [10], but there can be corrections for a correlated (non-Markovian) bath.

We return to the replicated field integral in Eq. (6). In d spatial dimensions, the engineering dimensions of the fields and coupling constants are $[\Psi(\mathbf{k},\omega)] = [\rho_{\mathsf{cl}}(\mathbf{k},\omega)] = -2-d/2$, $[\lambda] = (4-d)/2$, $[r] = 2$, and $[\xi] = [h] = 0$. Here we have adopted the convention that momentum carries dimension 1, i.e. $[\mathbf{k}] = 1$, and frequency carries the engineering dimension $z = 2$, $[\omega] = 2$. The upper critical dimension at which coupling constant λ becomes dimensionless is $d_c = 4$. Below (above) d_c, the interaction term S_c in Eq. (6d) is relevant (irrelevant) in a renormalization group (RG) sense. This suggests that, below d_c spatial dimensions, there might exist a nontrivial fixed point that is perturbatively accessible in a controlled ϵ-expansion from d_c.

Besides its dephasing effect, the density fluctuation ρ_{cl} also contributes Altshuler-Aronov corrections to the conductivity [11, 21]; these are ignored here. Moreover, since the weak localization correction in Eq. (1) is itself the virtual shift of the diffusion constant D, we assume that D does not change under the RG flow *within the dephasing problem*. In particular, we employ the renormalization scheme where, contrary to frequency scaling factor h in Eq. (6b) that flows under the RG transformation, ξ defined in Eq. (7) is fixed by a wave function renormalization. A more sophisticated approach might impose a scale-dependent self-consistent condition on D, as it enters both the Cooperon and bath correlation functions that together determine the weak localization correction up to that scale.

Carrying out a Wilsonian RG analysis in $d = 4-\epsilon$ dimensions at one-loop level, we obtain the β-functions to leading order in ϵ [13]:

$$\begin{aligned}\frac{dg}{dl} &= \epsilon g - g^2\frac{4\xi}{(h+2\xi)^3},\\ \frac{dr}{dl} &= 2r - gr\frac{2\xi}{(h+2\xi)^3} + \frac{g}{2}\frac{1}{(h+2\xi)},\\ \frac{dh}{dl} &= -g\frac{2\xi}{(h+2\xi)^3}h,\end{aligned} \tag{9}$$

where l is the logarithm of the length scale, and g is defined by $g \equiv \lambda^2/(4\pi^2)$. These equations possess a nontrivial fixed point at $g^* = 2\xi^2\epsilon$, $r^* = -\epsilon\xi/4$, $h^* = 0$.

In Figs. 2(a) and (b), we show the RG flow described by Eq. (9) in the $g-r$ ($h=0$) and $g-h$ planes, respectively. We have set $\xi = 1$ and $\epsilon = 0.1$ in both plots. The red dashed line slightly tilted from the horizontal g-axis in Fig. 2(a) represents the critical surface in the $h = 0$ plane. Under the RG transformation, any point on the critical surface flows towards the nontrivial fixed point (denoted by the red dot) corresponding to $T^* > 0$ (since $g \propto T$). The dephasing rate $\tau_\phi^{-1} = 2(r - r^*)$ vanishes at this critical point. By contrast, for those points off of the critical surface and associated with $T > T^*$ [shaded region in Fig. 2(a)], the RG flow is directed away from the critical fixed point and towards the ergodic phase where a nonzero dephasing rate τ_ϕ^{-1} is generated, as in the SCBA [Eq. (8)]. We only consider the critical point as approached from the ergodic phase, so we do not consider parts of the flow outside the critical surface or shaded region. Fig. 2(b) shows that the frequency scale h flows to zero at the fixed point.

At the critical fixed point, the Cooperon has the scaling behavior $\langle C\rangle_\rho(\mathbf{k},\omega\to 0) = 1/Dk^2$, which is the bare (undephased) result [13]. Inserting this into Eq. (1) gives

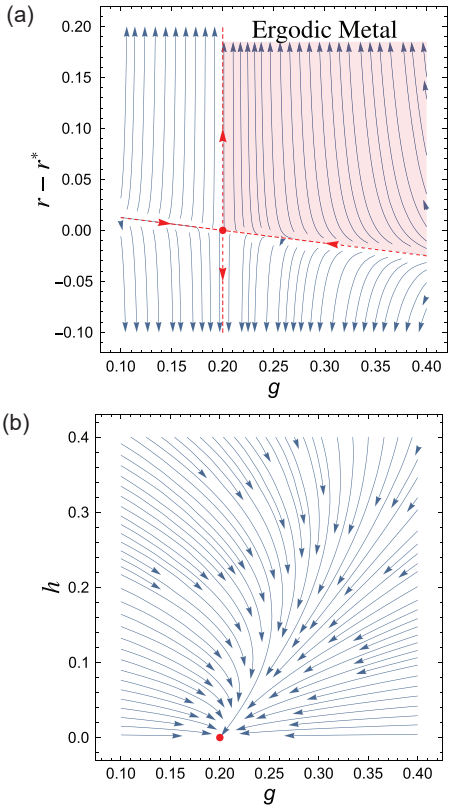

FIG. 2: One-loop RG flow in spatial dimensions $d = 4-\epsilon$. Panel (a) depicts the RG flow in the $g-r$ plane with $h = 0$, $\xi = 1$ and $\epsilon = 0.1$. Here the coupling constant g is proportional to k_BT/D (in $d = 2$). The nontrivial fixed point denoted by the red dot has two irrelevant and one relevant directions. (The relevant direction and one irrelevant direction lie in the $g-r$ plane and are indicated by the red dashed lines.) The dephasing rate $\tau_\phi^{-1} = 2(r - r^*)$ vanishes at the critical fixed point corresponding to temperature $T^* > 0$. Above T^*, the system flows towards the ergodic metal phase where the dephasing rate τ_ϕ^{-1} is nonzero. The RG analysis suggests that the system undergoes a temperature-tuned transition that can be considered as a possible instability of the ergodic phase (in $d = 2$). Panel (b) shows the RG flow in the $g-h$ plane.

a logarithmic divergence of the weak localization correction in the infrared in 2D. By linearizing the RG flow near the nontrivial fixed point [13], we find that in spatial dimensions $d = 4-\epsilon$, the coupling constant $(r-r^*)$ has the scaling dimension $y_r = 2 - \epsilon/2 + O(\epsilon^2)$, which leads to a correlation length exponent $\nu = 1/2 + \epsilon/8 + O(\epsilon^2)$. We note that the critical exponent $\nu = 2/d$, which saturates the Harris-Chayes bound ($\nu \geq 2/d$) [26, 27].

As emphasized above, our result gives only a hint of a possible instability of the ergodic phase in $d = 2$ dimensions. A challenging but worthwhile direction for future work is to impose a scale-dependent self-consistency condition between the RG controlling the dephasing due to real processes, and the running renormalization of the diffusion constant due to the virtual ones. This self-consistent [1, 20] ("RG-improved perturbation theory") approach would account for the fact that the diffusion constant itself changes with scale. At the transition in 2D, it may reach a critical (universal) average value or vanish altogether [1, 3, 6–8]. A further refinement would incorporate higher order quantum interference conductance and Altshuler-Aronov corrections.

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