Puzzling Two-Proton Decay of ^{67}Kr

S. M. Wang (□□□) and W. Nazarewicz

Phys. Rev. Lett. **120**, 212502 — Published 23 May 2018

DOI: 10.1103/PhysRevLett.120.212502

The puzzling two-proton decay of ^{67}Kr

S.M. Wang (王思敏)[1] and W. Nazarewicz[2]
[1]*FRIB/NSCL Laboratory, Michigan State University, East Lansing, Michigan 48824, USA*
[2]*Department of Physics and Astronomy and FRIB Laboratory, Michigan State University, East Lansing, Michigan 48824, USA*
(Dated: April 20, 2018)

Ground state two-proton ($2p$) radioactivity is a rare decay mode found in a few very proton-rich isotopes. The $2p$-decay lifetime and properties of emitted protons carry invaluable information on nuclear structure in the presence of low-lying proton continuum. The recently measured $2p$ decay of ^{67}Kr [1] turned out to be unexpectedly fast. Since ^{67}Kr is expected to be a deformed system, we investigate the impact of deformation effects on the $2p$ radioactivity. We apply the recently developed Gamow coupled-channel framework, which allows for a precise description of three-body systems in the presence of rotational and vibrational couplings. This is the first application of a three-body approach to a two-nucleon decay from a deformed nucleus. We show that deformation couplings significantly increase the $2p$ decay width of ^{67}Kr; this finding explains the puzzling experimental data. The calculated angular proton-proton correlations reflect a competition between $1p$ and $2p$ decay modes in this nucleus.

Introduction.– There are very few even-Z nuclei beyond the two-proton dripline that can decay by emitting two protons from their ground states. In such cases, the emission of a single proton is energetically forbidden or strongly suppressed due to the odd-even binding energy effect originating from proton pairing [2–8]. The corresponding half-lives are long enough to characterize this phenomenon as $2p$ radioactivity. Experimentally, $2p$ emission from the nuclear ground state (g.s.) was observed for the first time in ^{45}Fe [9, 10], and, later on, in ^{19}Mg [11], ^{48}Ni [12–14], and ^{54}Zn [15, 16]. Interest in this exotic phenomenon has been envigorated by measurements of proton-proton correlations in the decay of ^{45}Fe [17], ^{19}Mg [18], and ^{48}Ni [14], which have demonstrated the unique three-body features of the process and – when it comes to theory – the sensitivity of predictions to the angular momentum decomposition of the $2p$ wave function. The high-quality $2p$ decay data have called for the development of comprehensive theoretical approaches, capable of simultaneous description of structural and reaction aspects of the problem [4, 5].

The main challenge for theoretical studies of $2p$ radioactivity lies in the model's ability to tackle simultaneously nuclear structure aspects in the internal region and the three-body behavior in the asymptotic region. This becomes especially challenging for $2p$ decay since the Coulomb barrier strongly suppresses the wave function at large distances, which also makes the $2p$ lifetime quite sensitive to the low-ℓ wave function components inside the nucleus. So far, most of the theoretical models of $2p$ radioactivity, such as the WKB approach [19–21], R-matrix theory [22, 23], and three-body reaction model [24, 25], treat internal and asymptotic regions separately. In our previous work [26], we introduced the Gamow coupled-channel (GCC) method, which describes structure and decays of three-body systems within one coherent theoretical framework by utilizing resonant and scattering states in eigenfunction expansion. Consequently, this tool is suitable for unraveling the intriguing features of $2p$ g.s. decay of ^{67}Kr.

Being the heaviest g.s. $2p$ emitter observed so far, ^{67}Kr is of particular interest, since it provides unique structural data on medium-mass unbound systems in the presence of collective excitations. The measured $2p$ decay energy is 1690 ± 17 keV and the partial $2p$ lifetime 20 ± 11 ms [1] is significantly lower than the original theoretical prediction [27]. As suggested in Ref. [1], this may be due to configuration mixing effects and/or deformation in the daughter nucleus ^{65}Se. An alternative explanation involves the competition between two-body and three-body decay channels [25]: the partial $2p$ lifetime can be reproduced only if the two valence protons primarily occupy the $2p_{3/2}$ shell that is supposed to be already filled by the core nucleons.

The objective of this work is to incorporate a deformed, or vibrational, core into the GCC model, and study the $2p$ decay as the quadrupole coupling evolves. To benchmark the GCC Hamiltonian, we first consider the simpler case of spherical ^{48}Ni. Thereafter, we investigate deformation and configuration mixing effects on the $2p$ decay of ^{67}Kr.

Theoretical framework– To describe $2p$ emission, we extend the previously introduced [26] three-body core+nucleon+nucleon Gamow coupled-channel (GCC) approach by allow the pair of nucleons to couple to the collective states of the core. To this end, the wave function of the parent nucleus is written as $\Psi^{J\pi} = \sum_{J_p\pi_p j_c\pi_c}\left[\Phi^{J_p\pi_p}\otimes\phi^{j_c\pi_c}\right]^{J\pi}$, where $\Phi^{J_p\pi_p}$ and $\phi^{j_c\pi_c}$ are the wave functions of the two valence protons and the core, respectively. $\Phi^{J_p\pi_p}$ is constructed in Jacobi coordinates with the hyperspherical harmonics $\mathcal{Y}^{J_pM}_{\gamma K}(\Omega)$ for the hyperangle part. And the hyperradial part $\psi_{\gamma K}(\rho)$ is expanded in the Berggren ensemble that defines a complete basis $\mathcal{B}^{J_p\pi}_{\gamma \mathrm{n}}(\rho)$ in the complex-momentum plane including bound, decaying, and scattering states [26, 28]. As a result, $\Phi^{J_p\pi_p} = \rho^{-5/2}\sum_{\gamma nK} C^{J_p\pi}_{\gamma nK}\mathcal{B}^{J_p\pi}_{\gamma n}(\rho)\mathcal{Y}^{J_pM}_{\gamma K}(\Omega)$, where K is the hyperspherical quantum number and $\gamma = \{s_1, s_2, S_{12}, S, \ell_x, \ell_y, L, J_p, j_c\}$. By using the Berggren basis, the inner and asymptotic regions of the Schrödinger equation can be treated on the same footing, and this provides the natural connection between nuclear shell structure and reaction aspects of the problem.

The core+p+p Hamiltonian of GCC is

$$\hat{H} = \sum_{i=c,p_1,p_2}^{3} \frac{\hat{\boldsymbol{p}}_i^2}{2m_i} + \sum_{i>j=1}^{3} V_{ij}(\boldsymbol{r}_{ij}) + \hat{H}_c - \hat{T}_{\text{c.m.}}, \quad (1)$$

where V_{ij} is the interaction between clusters i and j, $\hat{H}_c$ is the core Hamiltonian represented by excitation energies of the core $E^{j_c\pi_c}$, and $\hat{T}_{\text{c.m.}}$ stands for the center-of-mass term. In this work, the proton-core interaction V_{pc} is approximated by a Woods-Saxon (WS) average potential including central, spin-orbit and Coulomb terms. At small shape deformations, we applied the vibrational coupling as in Refs. [29, 30]. At large quadrupole deformations we consider rotational coupling, which was incorporated as in the non-adiabatic approach to deformed proton emitters [31, 32].

In order to deal with the antisymmetrization between core and valence protons, one needs to eliminate the Pauli-forbidden states occupied by the core nucleons. Due to the transformation between different coordinates, the standard projection technique [26] can introduce small numerical errors in the asymptotic region. Since the wave function needs to be treated very precisely at large distances, we have implemented the supersymmetric transformation method [33–35] which introduces an auxiliary repulsive "Pauli core" in the original core-p interaction to eliminate Pauli-forbidden states. For simplicity, in this work we only project out those spherical orbitals which correspond to the deformed levels occupied in the daughter nucleus.

By using the hyperspherical harmonics and Berggren basis, the Schrödinger equation can be written as coupled-channel equation including the couplings not only among the hyperspherical basis but also among the collective states of core. The resulting complex eigenvalues contain information about resonance's energies and decay widths. However, for medium-mass nuclei, proton decay widths are usually below the numerical precision of calculations. Still, one can estimate decay widths through the current expression [36] as demonstrated in previous work [26, 37, 38]. According to the R-matrix theory, if the contribution from the off-diagonal part of the Coulomb interaction in the asymptotic region is neglected, the hyperradial wave function of the resonance $\psi_{\gamma K}(\rho)$ is proportional to the outgoing Coulomb function $H^+_{K+3/2}(\eta_{\gamma K}, k_p\rho)$ [24, 39, 40]. By assuming a small decay width and adopting the expression $\psi'/\psi = k_p H^{+\prime}/H^+$ [31, 32], one can bypass the numerical derivative of the small wave function in the asymptotic region that appears in the original current expression and increase numerical precision dramatically [41].

According to Refs. [38, 42], the high-K space of hyperspherical quantum numbers also has some influence on the decay width. Since practical calculations must involve some K-space truncation, we adopt the so-called Feshbach reduction method proposed in Refs. [38, 42]. This is an adiabatic approximation that allows one to evaluate the contributions to the interaction matrix elements originating from the excluded model space.

Hamiltonian and model space – For the nuclear two-body interaction between valence protons we took the finite-range Minnesota force with the original parameters of Ref. [43]. The proton-proton interaction has been augmented by the two-body Coulomb force. The core-valence potential contains central, spin-orbit and Coulomb terms. The nuclear average potential has been taken in a WS form including the spherical spin-orbit term with the "universal" parameter set [44], which has been successfully applied to nuclei from the light Kr region [45]. The depth of the WS potential has always been readjusted to the experimental value of Q_{2p}. The Coulomb core-proton potential is assumed to be that of the charge Z_ce uniformly distributed inside the deformed nuclear surface [44].

Since ^{48}Ni is doubly-magic, to discuss its $2p$ decay we limited our calculations to the spherical case. For ^{67}Kr, we assumed a deformed core of ^{65}Se described by the quadrupole deformation β_2, with the unpaired neutron treated as a spectator. According to calculations [46–48], the ^{65}Se core has an oblate shape. Based on the data from the mirror nucleus ^{65}Ga [49], we assume the g.s. of ^{65}Se to have $J^\pi = 3/2^-$ [50] and its rotational (vibrational) excitation to be a $J^\pi = 7/2^-$ state at 1.0758 MeV. This estimate is consistent with excitation energies of 2^+_1 states in the neighboring nuclei ^{64}Zn and ^{66}Ge [49]. In our coupled channel calculations, we included collective states of ^{65}Se with $J \leq j_c^{\max} = 15/2^-$; such a choice guarantees stability of our results. In particular, we checked that the calculated half-life differs by less than 3% when varying $j_c^{\max}$ from 11/2 to 15/2.

The calculations have been carried out in the model space of $\max(\ell_x, \ell_y) \leq 7$ with the maximal hyperspherical quantum number $K_{\max} = 50$ and the Feshbach reduction quantum number $K_f = 20$, which is sufficient for all the observables studied [26, 38, 42]. For the hyperradial part, we used the Berggren basis for the $K \leq 6$ channels and the HO basis for the higher angular momentum channels. The complex-momentum contour of the Berggren basis is defined as: $k = 0 \to 0.3-0.1i \to 0.5 \to 4 \to 8$ (all in fm^{-1}), with each segment discretized with 50 points. For the HO basis we took the oscillator length $b = 1.75$ fm and $N_{\max} = 60$.

Results.– We first investigate the spherical $2p$ emitter ^{48}Ni, which has been the subject of numerous theoretical studies [12, 19, 21, 51–54]. By assuming the experimental value of $Q_{2p} = 1.28 \pm 0.06$ MeV [55] we obtain $T_{1/2} = 30^{+133}_{-24}$ ms, which is consistent with the current experimental estimates: $T_{1/2} = 8.4^{+12.8}_{-7}$ ms [12] and $3^{+2.2}_{-1.2}$ ms [14]. Moreover, we found that calculations with different sets of WS parameters result in fairly similar decay widths, which is in accord with the conclusion of Ref. [21] that – as long as the sequence of s.p. levels does not change – the $2p$ lifetime should rather weakly depend on the details of the core-proton potential as the tunneling motion of the $2p$ system is primarily governed by the Coulomb interaction.

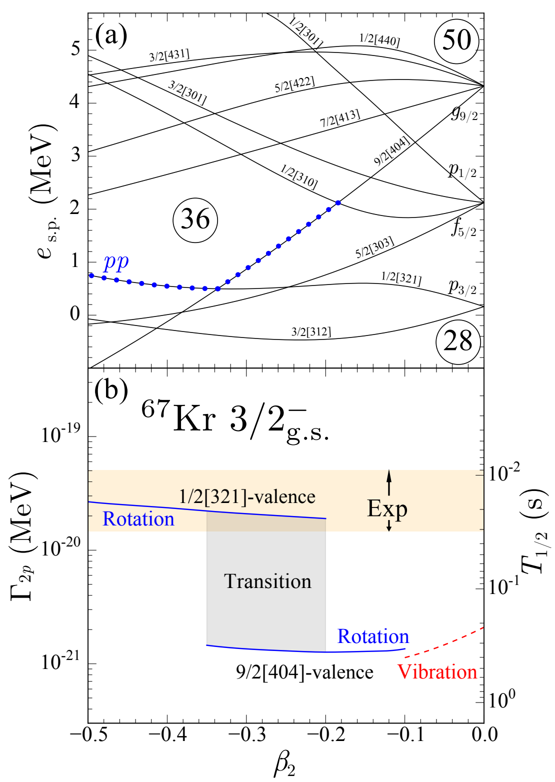

FIG. 1. Top: Nilsson levels $\Omega[{\rm N}n_z\Lambda]$ of the deformed core-p potential as functions of the oblate quadrupole deformation β_2 of the core. The dotted line indicates the valence level primarily occupied by the two valence protons. Bottom: Decay width (half-live) for the $2p$ g.s. radioactivity of ^{67}Kr. The solid and dashed lines mark, respectively, the results within the rotational and vibrational coupling. The rotational-coupling calculations were carried out by assuming that the 1/2[321] orbital is either occupied by the core (9/2[404]-valence) or valence (1/2[321]-valence) protons.

The lifetime of ^{67}Kr can be impacted by deformation effects [1]. Indeed, studies of one-proton ($1p$) emitters [30–32, 41, 56–60] have demonstrated the impact of rotational and vibrational couplings on $1p$ half-lives. Figure 1a shows the proton Nilsson levels (labeled by the asymptotic quantum numbers $\Omega[{\rm N}n_z\Lambda]$) of the WS core-$p$ potential. At small deformations, $|\beta_2| \leq 0.1$, the valence protons occupy the $f_{5/2}$ shell. The half-life predicted in the vibrational variant of calculations is $T_{1/2} > 218$ ms, which exceeds the experimental value by over an order of magnitude, see Fig. 1b. This result is consistent with previous theoretical estimates [19, 27].

As the deformation of the core increases, an appreciable oblate gap at $Z = 36$ opens up, due to the downsloping 9/2[404] Nilsson level originating from the $0g_{9/2}$ shell. This gap is responsible for oblate g.s. shapes of proton-deficient Kr isotopes [45, 61, 62]. The structure of the valence proton orbital changes from the 9/2[404] ($\ell = 4$) state at smaller oblate deformations to the 1/2[321] orbital, which has a large $\ell = 1$ component. While the exact crossing point of the 1/2[321] and 9/2[404] levels depends on details of the core-proton parametrization, the general pattern of Fig. 1a is robust: one expects a transition from the $2p$ wave function dominated by $\ell = 4$ components to $\ell = 1$ components as oblate deformation increases. Figure 1b shows the $2p$ decay width predicted in the two limits of the rotational model: (i) the 1/2[321] level belongs to the core, and the valence protons primarily occupy the 9/2[404] level; and (ii) the valence protons primarily occupy the 1/2[321] level. In reality, as the core is not rigid, proton pairing is expected to produce the diffused Fermi surface; hence the transition from (i) to (ii) is going to be gradual, as schematically indicated by the shaded area in Fig. 1b. The decreasing ℓ content of the $2p$ wave function results in a dramatic increase of the decay width. At the deformation $\beta_2 \approx -0.3$, which is consistent with estimates from mirror nuclei [63] and various calculations [45–48, 63] the calculated $2p$ g.s. half-live of ^{67}Kr is 24^{+10}_{-7} ms, which agrees with experiment [1].

Since the Minnesota force used here is an effective interaction that is likely to be affected by in-medium effects, one may ask how changes in the proton-proton interaction may affect the $2p$ decay process. Figure 2 displays the partial $2p$ width for the g.s. decay of ^{48}Ni and ^{67}Kr for two strengths of the pp interaction V^N_{pp}. The predicted Γ_{2p} of ^{48}Ni is quite sensitive to the strength of V^N_{pp}; namely, it increases by an order of magnitude when the interaction strength increases by 50%. For the original Minnesota interaction, the Q_p of ^{47}Co is 1.448 MeV, i.e., the $1p$ decay channel in ^{48}Ni is closed. Consequently, further increases in the valence proton interaction strength can only affect the pairing scattering from the $0f_{7/2}$ resonant shell into the low-ℓ proton continuum. The corresponding increase of low-ℓ strength in the $2p$ wave function results in the reduction of half-life seen in Fig. 2a.

The case of ^{67}Kr is presented in Fig. 2b. Here the trend is opposite: the decay width actually decreases with the strength of V^N_{pp}. To understand this we note that the $1p$ decay channel of the ^{67}Kr g.s. is open ($Q_p > 0$) for a large range of interaction strengths, see the insert in Fig. 2b. At the standard strength of $V^{\rm std}_{pp}$, the predicted Q_p of ^{66}Br is 1.363 MeV, i.e., one expects to see a competition between the sequential and three-body decay in this

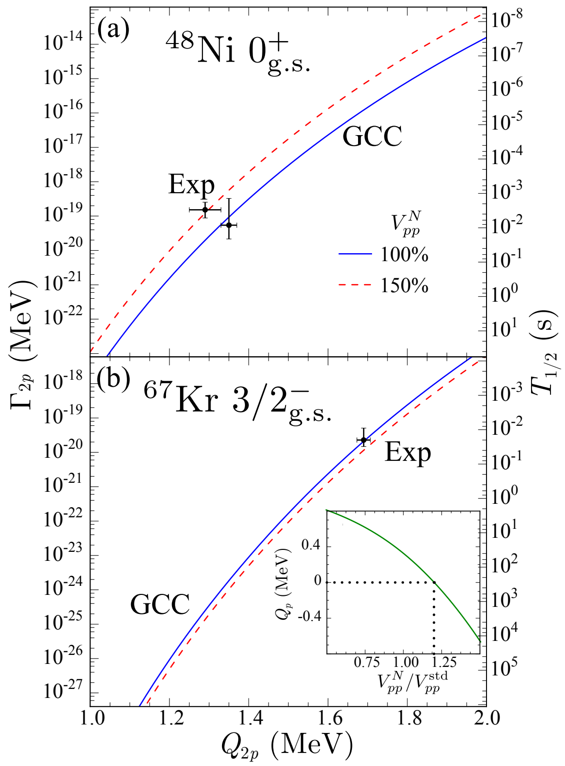

FIG. 2. Calculated $2p$ partial width (half-life) of the g.s. decay of (a) ^{48}Ni and (b) ^{67}Kr as a function of Q_{2p}. The results obtained with 100% (solid line) and 150% (dashed line) strength of the Minnesota force V^N_{pp} are marked. The experimental data are taken from Refs. [12, 14] (^{48}Ni) and [1] (^{67}Kr). The inset in (b) shows the $1p$ decay energy Q_p of ^{67}Kr at the experimental value of Q_{2p} obtained with different strengths of V^N_{pp} relative to the original value V^{std}_{pp}. The $Q_p = 0$ threshold is indicated by a dotted line.

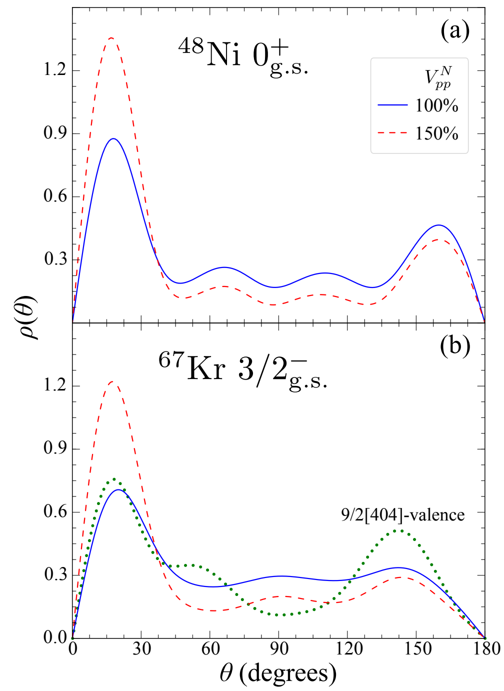

FIG. 3. Two-proton angular correlation for the g.s. of (a) ^{48}Ni and (b) ^{67}Kr obtained with the Minnesota force of standard strength (solid line) and 50%-increased strength (dashed line). The dotted line marks the angular correlation obtained with the standard strength assuming that the two valence protons occupy the 9/2[404] level.

case. With the increasing pairing strength, the odd-even binding energy difference grows, and the $1p$ channel gets closed around $V^N_{pp}/V^{\text{std}}_{pp} = 1.2$. The further increase of V^N_{pp} strength results in pairing scattering to higher-lying proton states originating from $0g_{9/2}$ and $0f_{5/2}$ shells with higher ℓ content, see Fig. 1. Both effects explain the reduction of Γ_{2p} seen in Fig. 2b.

Since the $1p$ channel is most likely open for ^{67}Kr [25], it is interesting to ask: How large is the diproton component in the ^{67}Kr decay? To this end, in Fig. 3 we study the $2p$ angular correlations $\rho(\theta)$ [26, 64] for the g.s. decays of ^{48}Ni and ^{67}Kr. In both cases, a diproton-like structure corresponding to a peak at small opening angles is very pronounced. Interestingly, while the shell-model structures of ^{48}Ni and ^{67}Kr are very different, the two valence protons are calculated to form very similar T-type Jacobi-coordinate configurations in these two nuclei. Namely, for ^{48}Ni the dominant (S_{12}, ℓ_x, ℓ_y) configurations in T-type Jacobi-coordinate are 58% (0, 0, 0) and 30% (1, 1, 1), while the corresponding amplitudes for ^{67}Kr are 59% and 27%. The diproton peak in ^{67}Kr is slightly lower than that in ^{48}Ni due to the fact that sequential decay is energetically allowed in ^{67}Kr. The $1p$ decay width of ^{67}Kr estimated by the core-proton model is 8.6×10^{-20} MeV, which has the same order of magnitude with the $2p$ decay width. Consequently, the $2p$ decay branch in ^{67}Kr is expected to compete with the sequential decay. With the pairing strength increased by 50% the diproton peak in $\rho(\theta)$ becomes strongly enhanced, see Fig. 3, as the $1p$ channel gets closed. The dotted line in Fig. 3 marks the $2p$ angular correlation of ^{67}Kr by assuming that the two valence protons occupy the 9/2[404] level. In this case, the correlation exhibits a minimum at 90° indicating that $\rho(\theta)$ is a good indicator of the valence proton structure.

Conclusions.– We extended the Gamow coupled-channel approach by introducing couplings to core excitations. We demonstrated that deformation effects are important for the $2p$ g.s. decay of ^{67}Kr. Due to the oblate-deformed $Z = 36$ subshell at $\beta_2 \approx -0.3$, the Nilsson orbit 1/2[321] with large $\ell = 1$ amplitude becomes available to valence protons. This results in a significant increase of the $2p$ width of ^{67}Kr, in accordance with experiment.

The sensitivity of $2p$ lifetime to the proton-proton interaction indicates that the pairing between the valence protons can strongly influence the decay process. Through the comparison of one-proton decay energies and angular correlations between ^{48}Ni and ^{67}Kr, we conclude that there is a competition between $2p$ and $1p$ decays in ^{67}Kr, while the decay of ^{48}Ni has a $2p$ character.

In summary, the puzzling $2p$ decay of ^{67}Kr has been naturally explained in terms of the shape deformation of the core. The explanation is fairly robust with respect to the details of the GCC Hamiltonian. We conclude that the Gamow coupled-channel framework provides a comprehensive description of structural and reaction aspects of three body decays of spherical and deformed nuclei. The future theoretical work will primarily focus on improving the quality of the underlying Hamiltonian. To this end, high-statistics angular correlation $2p$ data are needed to better constrain theoretical input and improve our understanding of $2p$ radioactivity.

Discussions with Kévin Fossez, Futoshi Minato, Nicolas Michel, Jimmy Rotureau, and Furong Xu are acknowledged. We appreciate helpful comments from Zach Matheson. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under award numbers DE-SC0013365 (Michigan State University), DE-SC0018083 (NUCLEI SciDAC-4 collaboration), and DE-SC0009971 (CUSTIPEN: China-U.S. Theory Institute for Physics with Exotic Nuclei).

[1] T. Goigoux *et al.*, Phys. Rev. Lett. **117**, 162501 (2016).
[2] V. Goldansky, Nucl. Phys. **19**, 482 (1960).
[3] M. Pfützner, Nucl. Phys. A **738**, 101 (2004), proceedings of the 8th International Conference on Clustering Aspects of Nuclear Structure and Dynamics.
[4] B. Blank and M. Płoszajczak, Rep. Prog. Phys. **71**, 046301 (2008).
[5] M. Pfützner, M. Karny, L. V. Grigorenko, and K. Riisager, Rev. Mod. Phys. **84**, 567 (2012).
[6] M. Pfützner, Phys. Scripta **2013**, 014014 (2013).
[7] J. Giovinazzo *et al.*, J. Phys. Conf. Ser **436**, 012057 (2013).
[8] E. Olsen, M. Pfützner, N. Birge, M. Brown, W. Nazarewicz, and A. Perhac, Phys. Rev. Lett. **111**, 139903(E) (2013).
[9] J. Giovinazzo *et al.*, Phys. Rev. Lett. **89**, 102501 (2002).
[10] M. Pfützner *et al.*, Eur. Phys. J. A **14**, 279 (2002).
[11] I. Mukha *et al.*, Phys. Rev. Lett. **99**, 182501 (2007).
[12] C. Dossat *et al.*, Phys. Rev. C **72**, 054315 (2005).
[13] M. Pomorski *et al.*, Phys. Rev. C **83**, 061303 (2011).
[14] M. Pomorski *et al.*, Phys. Rev. C **90**, 014311 (2014).
[15] B. Blank *et al.*, Phys. Rev. Lett. **94**, 232501 (2005).
[16] P. Ascher *et al.*, Phys. Rev. Lett. **107**, 102502 (2011).
[17] K. Miernik *et al.*, Phys. Rev. Lett. **99**, 192501 (2007).
[18] I. Mukha *et al.*, Phys. Rev. C **77**, 061303 (2008).
[19] M. Gonçalves, N. Teruya, O. Tavares, and S. Duarte, Phys. Lett. B **774**, 14 (2017).
[20] W. E. Ormand, Phys. Rev. C **55**, 2407 (1997).
[21] W. Nazarewicz *et al.*, Phys. Rev. C **53**, 740 (1996).
[22] F. C. Barker, Phys. Rev. C **63**, 047303 (2001).
[23] B. A. Brown and F. C. Barker, Phys. Rev. C **67**, 041304 (2003).
[24] L. V. Grigorenko, Phys. Part. Nuclei **40**, 674 (2009).
[25] L. V. Grigorenko, T. A. Golubkova, J. S. Vaagen, and M. V. Zhukov, Phys. Rev. C **95**, 021601 (2017).
[26] S. M. Wang, N. Michel, W. Nazarewicz, and F. R. Xu, Phys. Rev. C **96**, 044307 (2017).
[27] L. V. Grigorenko and M. V. Zhukov, Phys. Rev. C **68**, 054005 (2003).
[28] T. Berggren, Nucl. Phys. A **109**, 265 (1968).
[29] K. Hagino, N. Rowley, and A. Kruppa, Comput. Phys. Commun. **123**, 143 (1999).
[30] K. Hagino, Phys. Rev. C **64**, 041304 (2001).
[31] B. Barmore, A. T. Kruppa, W. Nazarewicz, and T. Vertse, Phys. Rev. C **62**, 054315 (2000).
[32] A. T. Kruppa and W. Nazarewicz, Phys. Rev. C **69**, 054311 (2004).
[33] I. J. Thompson, B. V. Danilin, V. D. Efros, J. S. Vaagen, J. M. Bang, and M. V. Zhukov, Phys. Rev. C **61**, 024318 (2000).
[34] I. Thompson, F. Nunes, and B. Danilin, Comput. Phys. Commun. **161**, 87 (2004).
[35] P. Descouvemont, C. Daniel, and D. Baye, Phys. Rev. C **67**, 044309 (2003).
[36] J. Humblet and L. Rosenfeld, Nucl. Phys. **26**, 529 (1961).
[37] L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Phys. Rev. Lett. **85**, 22 (2000).
[38] L. V. Grigorenko and M. V. Zhukov, Phys. Rev. C **76**, 014008 (2007).
[39] P. Descouvemont, E. Tursunov, and D. Baye, Nucl. Phys. A **765**, 370 (2006).
[40] V. Vasilevsky, A. V. Nesterov, F. Arickx, and J. Broeckhove, Phys. Rev. C **63**, 034606 (2001).
[41] H. Esbensen and C. N. Davids, Phys. Rev. C **63**, 014315 (2000).
[42] L. V. Grigorenko *et al.*, Phys. Rev. C **80**, 034602 (2009).
[43] D. Thompson, M. Lemere, and Y. Tang, Nucl. Phys. A **286**, 53 (1977).
[44] S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski, and T. Werner, Comput. Phys. Commun. **46**, 379 (1987).
[45] W. Nazarewicz, J. Dudek, R. Bengtsson, T. Bengtsson, and I. Ragnarsson, Nucl. Phys. A **435**, 397 (1985).
[46] Y. Aboussir, J. Pearson, A. Dutta, and F. Tondeur, At. Data Nucl. Data Tables **61**, 127 (1995).
[47] Mass Explorer, `http://massexplorer.frib.msu.edu/`.
[48] P. Möller, A. Sierk, T. Ichikawa, and H. Sagawa, At. Data Nucl. Data Tables **109-110**, 1 (2016).
[49] National Nuclear Data Center, `http://www.nndc.bnl.gov/`.
[50] E. Browne and J. Tuli, Nucl. Data Sheets **111**, 2425 (2010).
[51] L. Grigorenko, R. Johnson, I. Mukha, I. Thompson, and M. Zhukov, Nucl. Phys. A **689**, 567 (2001).
[52] J. Rotureau, J. Okołowicz, and M. Płoszajczak, Nucl. Phys. A **767**, 13 (2006).
[53] B. Blank, Int. J. Mod. Phys. E **18**, 2124 (2009).
[54] D. S. Delion, R. J. Liotta, and R. Wyss, Phys. Rev. C **87**, 034328 (2013).
[55] M. Pomorski *et al.*, Acta Phys. Pol. B **43**, 267 (2012).
[56] A. T. Kruppa, B. Barmore, W. Nazarewicz, and T. Vertse, Phys. Rev. Lett. **84**, 4549 (2000).
[57] C. N. Davids and H. Esbensen, Phys. Rev. C **64**, 034317 (2001).
[58] C. N. Davids and H. Esbensen, Phys. Rev. C **69**, 034314

(2004).
[59] G. Fiorin, E. Maglione, and L. S. Ferreira, Phys. Rev. C **67**, 054302 (2003).
[60] P. Arumugam, E. Maglione, and L. S. Ferreira, Phys. Rev. C **76**, 044311 (2007).
[61] M. Yamagami, K. Matsuyanagi, and M. Matsuo, Nucl. Phys. A **693**, 579 (2001).
[62] K. Kaneko, M. Hasegawa, and T. Mizusaki, Phys. Rev. C **70**, 051301 (2004).
[63] B. Pritychenko, M. Birch, B. Singh, and M. Horoi, At. Data Nucl. Data Tables **107**, 1 (2016).
[64] G. Papadimitriou, A. T. Kruppa, N. Michel, W. Nazarewicz, M. Płoszajczak, and J. Rotureau, Phys. Rev. C **84**, 051304 (2011).