Plasma electron holes are self-sustaining soliton-like structures, in which electron phase-space deficit on trapped orbits causes a local electric potential maximum that confines those trapped electrons[1]. Spacecraft observations now routinely see these localized electrostatic potential structures in a variety of plasma regions[2–12]. Specific instruments and algorithms are now implemented to detect them, for example on the current MMS mission[13]. Holes are most easily analysed as vortices in one space and one velocity dimension, but it is known that in three-dimensions even holes that start one-dimensional often break up quickly, by what is called the transverse instability. This effect was observed in the earliest computer simulations of unmagnetized holes[14], and has been confirmed since by many simulation studies[15–22]. It is also known that a strong enough parallel magnetic field can suppress the transverse instability. Despite the importance of the phenomenon, which decides the ultimate structure, persistence, and decay of holes, the transverse instability remains essentially unexplained. It is the purpose of this letter to identify its underlying mechanism.

Computational simulations[19, 23, 24] have established approximate quantitative stability criteria sufficient for many purposes, but their interpretation has left open many questions about the mechanism. The present study gives new and more comprehensive simulation results that provide strong evidence against the previously proposed mechanism, but in favor of a new understanding of the instability mechanism, explained here. It is based on electron hole kinematics: the overall conservation of momentum influenced by “jetting” (a form of energization) of particles by accelerating holes. A full-scale mathematical treatment of the new mechanism is beyond the scope of this paper.

To avoid confusing the transverse instability caused by holes with the instabilities driven by non-thermal electron distributions, we form a hole (artificially) initially as a one-dimensional slab-like structure (the y and z coordinates being ignorable). We then observe the growth in two space x and y (and three velocity) dimensions of transverse perturbations. For linear stability purposes there is no loss of generality in supposing the unstable wave-vector to be chosen along y, with z-dependence remaining absent.

The PIC code COPTIC[25], a 3D electrostatic code, is used here as 2D3V, pushing only electrons, the ions being taken as a uniform background. The simulation is initialized with a one-dimensional Schamel-type hole[26] having an initial potential shape (approximately) \( \phi = \psi \text{sech}^4(x/4) \) (measuring lengths in units of the Debye length \( \lambda_D \)) for chosen peak potential \( \psi \) (in units of electron temperature \( T_e/e \)). Time-steps have length 0.2 (times the inverse of the plasma frequency \( \omega_p^{-1} \)). The typical mesh is 64 \times 128. The domain \(-32 < x < 32\) resolves the hole and prevents the open x-boundaries from influencing the instability. Using up to \(-128 < y < 128\) is sufficient to resolve the \( y \)-variation of perturbations, as convergence tests with different domain lengths have shown, (although not long enough to prove the periodic \( y \)-boundaries completely negligible for the longest wavelengths). The transverse velocity distribution and the passing particle distribution are Maxwellian of equal temperature. Typically 200 million particles are used.

On the basis of their pioneering simulations, Muschietti et al[19, 23] proposed a criterion for the parallel magnetic field strength required to suppress the transverse instability: that the electron cyclotron frequency \( \Omega \) exceed the bounce frequency of deeply trapped electrons \( \omega_b \). For a Schamel hole \( \omega_h \simeq \omega_p \sqrt{\psi}/2 \). Systematic exploration of the parameter space using COPTIC approximately confirms this criterion.

Fig. 1 shows the time evolution of the peak potential in a series of two dimensional simulations. The initial amplitude is given by the value at the left end of the traces \((t = 0, \psi = 0.6, 0.4, 0.2, 0.1, 0.05)\) where the hole is 1-D. Various different magnetic field values (expressed \( \Omega/\omega_p \)) are used, as shown by the line labels. Stable cases preserve the initial value of the peak potential (with some small decay attributable to noise). Unstable cases kink and then decrease in \( \psi \) as a nonlinear result. However, unstable cases with magnetic field values not too much below the threshold of instability do not evolve to zero \( \psi \). Instead, they decrease to a finite value and then continue stably. Values of \( \psi \) below about 0.01 are at approximately the noise level.
FIG. 1. Evolution with time of the peak potential of electron holes for different magnetic fields.

The domain of initial stability and instability is shown in Fig. 2(a). On it is plotted the line \( \Omega = \omega_b = \omega_{pe} \sqrt{\psi/2} \).

It can be seen that this estimate of the stability threshold predicts reasonably well the observed stability. However, there is some ambiguity in identifying instability at the marginal level, which appears somewhat to the right of the scaling line.

FIG. 2. Domain of stability/instability in respect of (a) initial hole potential, (b) final hole potential.

In view of the apparent self-stabilization of the holes, it is perhaps more interesting to plot the final peak potential versus cyclotron frequency. That is shown in Fig. 2(b), strongly compressing the observed values closer to a single line, thus lending support to the hypothesis that self-stabilization is a matter of reducing the linear growth rate to the threshold. This stabilization threshold line is now more obviously to the right of the \( \Omega = \omega_b \) scaling: by a factor of approximately 1.5.

FIG. 3. An example of density (particles per Debye cell) contours in the \((y,x)\) plane during the early development of the transverse instability. The initial hole peak potential is \( \psi = 0.1T_e/e \) and no magnetic field is present. The electron density enhancement (lighter shading) in the outer regions adjacent to the hole peak is greater on the convex side.

In simulations with finite magnetic field, up to at least \( \Omega/\omega_p = 0.3 \). And although at higher magnetic fields it merges into the noise, no case examined shows significant concave-side enhancement.

Why the present results are different has not been established. But these observations argue against the electron focusing mechanism Muschietti et al hypothesized to
be the general cause of the instability, since the charge
supposed to cause it does not occur in the present simu-
lations.

I now explain what I call the *kinematic* mechanism of
transverse instability. We have shown[30] that for 1-D
holes when the frequency of a perturbation is small com-
pared with the inverse transit time of particles across the
hole, there is a ‘jetting’ effect whereby x-acceleration of a
hole (preserving its shape, so that $\phi(x, t) = \phi_0(x - x_h(t))$
and $x_h$ is non-zero) gives rise to net x-momentum rate
of change, $\dot{P}_x$, of the particles. Because particle momen-
tum is much larger than hole field momentum, the sum
of the jetting of electrons and ions must be effectively
zero. When the ion response is ignorable, which is the
case in most of the simulations studying transverse hole
instability, the 1-D momentum conservation (kinemat-
ics) requires $\dot{P}_x$ to be zero. This is satisfied by a non-
accelerating hole $\dot{x}_h = 0$.

Now consider a 2-D situation where the hole center
position is given by $x_0(y, t) = x_0 \exp(i(ky - \omega t))$, rep-
resenting a perturbative kink of displacement $x_0$. In the
absence of any magnetic field, the unperturbed trans-
verse motion of particles (in the direction $y$) is simply
a constant velocity $v_y$. A growing unstable perturba-
tion has positive imaginary part of the frequency $\omega_i$. If
the real part ($\omega_r$) of $\omega$ is negligible, as is observed
to be in the simulations, a transverse-moving particle
still feels an oscillating hole position because its orbit
is $y = v_y t$ and on it $x_h(y, t) = x_0 \exp(ikv_y + \omega_i)t$. We
have also recently[31] calculated the effects of hole ve-
locity oscillations at finite frequency on the jetting of a
single particle stream (of ions but electrons are simi-
lar). In summary, there is a coefficient $K(\omega')$, arrived
at by integrating over the 1-D $v_x$-distribution function
and the hole x-extent, such that a 1-D hole that oscil-
lates in position at frequency $\omega'$ gives $\dot{P}_e(\omega') = K(\omega')x_h$. This
approach is transferable to particles moving at spe-
cific transverse velocity in a kinked 2-D hole by identi-
fying $kv_y - i\omega_i = \omega'$. In the limit $\omega' \to 0$, $K(\omega')$ tends
in a constant value $K_0$, and if long enough transverse wave-
length is considered (small enough thermal $kv_y$), $K_0$ will
apply (to lowest order) to all relevant transverse velocities.
Then $\dot{x}_h(y, t) = -(kv_y - i\omega_i)^2 x_h$. When this effect is
integrated over a Maxwellian $v_y$-distribution (symmetric
in $v_y$) the imaginary cross term $kv_y\omega$ cancels and we find
$\langle \dot{x}_h(y, t) \rangle_{v_y} = -(kv_y)^2 x_0 = -(k^2T_y/m_e + \omega_i^2)x_0$,
so $\dot{P}_e = (\omega_i^2 - k^2T_y/m_e)K_0x_0$.

Since momentum balance is $\dot{P}_e = 0$, it does not ac-
tually matter what the magnitude of $K_0$ is (so long as
it has its low-frequency sign). We deduce that the kink
growth rate is

$$\gamma = \omega_i = \pm k\sqrt{\frac{T_y}{m_e}}.$$  \hspace{1cm} (1)

This, I propose, is the Transverse Instability at low $k$. It
has nothing to do with transverse “focusing”.

The heuristic explanation of the transverse instabil-
ity is simple. For a single $v_y$ it is that the combination of
the apparent x-acceleration of the hole in the parti-
cle’s transverse frame of reference, arising because of the
kinked hole’s curvature (the $(kv_y)^2$ term) is exactly can-
celled by the actual acceleration of the hole in the kink’s
$y$-position rest-frame, represented by the growth of the
kink: $(\omega_i^2)$. So the electron of velocity $v_y$ sees zero total
hole acceleration and experiences no jetting.

If the transverse wave-number is increased, then even-
tually $kv_y$ and hence $\omega'$ becomes comparable to the par-
allel electron transit time and the coefficient $K(\omega')$ de-
creases in real part (and acquires an imaginary part[31, Fig. 4]). Heuristically, when reversal of $R(K)$ occurs for the
majority of the particles responsible for jetting, so
that the sign of $P$ is reversed, the instability is sup-
pressed. Our prior kinematic analysis[30, section III.D]
showed that for $\psi < 1$ the parallel velocity extent of the
particles responsible for jetting is approximately $\sqrt{\psi}$.
Particles up to that velocity have a transit time approx-
imately $L/\sqrt{\psi}$, where $L$ is the hole x-length: equal to
roughly 4 (Debye-lengths). We may therefore estimate
the wavenumber at which $R(K(\omega')) \sim 0$ will occur as
being where $\omega'L/\sqrt{\psi} \sim 1$. Substituting a thermal trans-
verse velocity $v_y = 1$ we get an estimate for the cut-off
wavenumber for full stabilization $k_c \sim \sqrt{\psi}/L$.

Since for small $k$ the growth rate $\omega_i$ is proportional to
$k$, there must be a maximum growth rate somewhere be-
low $k_c$, perhaps at approximately half $k_c$, and having a rate
perhaps half the linear extrapolation. Thus we esti-
mate the maximum growth rate as $\gamma \simeq k_c/4 \simeq \sqrt{\psi}/4L \sim
\sqrt{\psi}/16$ in units of $\omega_p$.

In order to test the scaling expected from the kinematic
analysis of the transverse instability a series of runs was
carried out over a systematic range of hole depths from
$\psi = 0.05$ to $2T_i/e$. Fig. 4 shows the results. The growth
rate is found by fitting an exponential to the systemati-
cally rising part of the mode amplitude measured in two
ways whose difference indicates approximately the un-
certainty (error bars). The $k$ values are determined by
finding the mean mode number of the dominant mode,
treated either by finding the centroid of (one side of) the
absolute value of the Fourier Transform of $x(y)$, or by in-
terpolating only at the largest mode and those adjacent
to it (peaked). Again the two values are indicative of the
uncertainties.

The results agree with expectations. Both $\gamma$ and $k$
scale approximately proportional to $\sqrt{\psi}$. The observed
absolute values of growth rate $\gamma$ are of the same order
of magnitude as $k$, but not exactly equal to it. Equality
would be expected only for $k \ll k_c$. But the simu-
lation is presumably dominated by the peak growth rate
at which we have estimated $\gamma \simeq k/2$ which agrees with
observations. The absolute value of $\gamma$ is quite close to the
estimate $\sqrt{\psi}/16$.

The kinematic mechanism worked out here takes the
magnetic field to be negligibly small, although it will also apply at non-zero parallel magnetic fields of low magnitude. The higher fields that lead to stabilization of the transverse instability require accounting for the electrons’ helical orbits. Heuristically one might suppose that when the cyclotron period becomes short enough compared to the transit time (and hence the Larmor radius small compared with the Debye length) the destabilizing effects of transverse motion will be suppressed. But a rigorous mathematical treatment well beyond the present analysis is essential both to show how full stabilization occurs and to determine the precise value of the criterion. (Note added in proof: this analysis has now been undertaken by the author and will be published elsewhere. It confirms the results of the present paper.) Previous mathematical analyses[32, 33] have been misled by adopting a symmetric potential eigenmode as a first approximation and expanding in inverse powers of frequency. In all simulations the observed eigenmode is approximately a symmetric potential eigenmode as a first approximation and expanding in inverse powers of frequency. In all mathematical analyses there is b) a demonstration that trapped particles are responsible for the instability, because the significant passing particles’ transit frequency approximates the minimum bounce frequency.

Despite exploring a wide parameter range, the present simulations with isotropic Maxwellian background distributions never gave rise to the “whistler” or “streaked” waves. However, such phenomena do occur in COPTIC simulations with anisotropic temperature $T_\perp > T_\parallel$, and can readily cause break up of holes in those situations. I interpret these facts tentatively as an indication that these different phenomena are most probably (in COPTIC simulations certainly) instabilities provoked by the background plasma distribution. They might be important for holes in nature, but depend upon the details of the plasma through which they are passing, and are not the intrinsic transverse instability.

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