Deterministic Remote Entanglement of Superconducting Circuits through Microwave Two-Photon Transitions
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Deterministic remote entanglement of superconducting circuits through microwave two-photon transitions

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Large-scale quantum information processing networks will most probably require the entanglement of distant systems that do not interact directly. This can be done by performing entangling gates between standing information carriers, used as memories or local computational resources, and flying ones, acting as quantum buses. We report the deterministic entanglement of two remote transmon qubits by Raman stimulated emission and absorption of a traveling photon wavepacket.

We achieve a Bell state fidelity of 73%, well explained by losses in the transmission line and decoherence of each qubit.

INTRODUCTION

Entanglement, which Schroedinger described as “the characteristic trait of quantum mechanics” [1], is instrumental for quantum information science applications such as quantum cryptography and all the known pure-state quantum algorithms [2]. Two distant systems Alice and Bob can be entangled if they interact locally with a third traveling system acting as a mediator. Since they can travel over long distances, photons are natural candidates for this role [3]. Remote entanglement was first demonstrated between two atomic clouds [4] traversed by a light beam measuring non-destructively a joint property. The difficulty of this scheme is to render the extracted information from the two systems indistinguishable. Superconducting circuit implementations [5, 6] also face this issue. Another protocol, widely used in trapped ions [7], solid-state spin qubits [8], quantum dots [9] and superconducting circuits [10] relies on the simultaneous emission of photons by both Alice and Bob, either through fluorescence or stimulated Raman emission. Entanglement is then heralded by detection of one of these photons, whose origin is erased by recombining them on a beam-splitter. This scheme is robust, in particular against photon losses, as long as the photons are indistinguishable to the detector. It should be possible to entangle in this way two arbitrary nodes of a network for modular quantum computing [11–13]. But can we build an even simpler remote entangler, which would not require a which-path eraser and detector?

As depicted in Fig. 1a, a minimal protocol consists of entangling Alice with a propagating electromagnetic field – for instance by concurrently exciting the standing system and a photon in this field – whose state is then swapped to Bob. Entanglement of atomic clouds using this method was reported in ref. [14], albeit with very low success probability. On the other hand, deterministic generation of entanglement requires an efficient absorption by one node of the field emitted by the other, which is also desirable to propagate information through a network. Efficient absorption by the receiving node requires to shape the “pitched” wavepacket by controlling the emission rate in time at the emitting node [15, 16]. In circuit-QED, many experiments [17–21] have focused on pitching a rising exponential wavepacket, which can be easily absorbed [22–25] by the receiver. Another approach [15, 26] consists of modulating both the emitter and receiver couplings to the transmission channel in time to pitch and catch a time-symmetric wavepacket. While efforts were made in that direction [27–29], the full protocol has not been demonstrated so far [30]. In this letter, we report deterministic entanglement generation between two distant transmon superconducting qubits using such a scheme. We employed microwave pumps to concurrently and coherently excite a transmon and a photon in a buffer resonator [10, 20]. The photon leaks out in a transmission line, and after traveling along ∼ 1 m cable and through microwave components, is captured by a second transmon qubit with a similar scheme. The entanglement purity is limited by photon losses in the line, which could be corrected for by purification [31, 32], and intrinsic decoherence of each qubit, which could also be improved.

DRIVING A TWO-PHOTON TRANSITION

The experimental setup is schematically depicted in Fig. 1b. Two superconducting transmon qubits [33], Alice and Bob, are embedded in two indium-plated copper cavities, anchored to the base stage of a dilution refrigerator (see [10, 34] for device fabrication and setup details). The photon damping rate \( \kappa = 2\pi \times 1 \text{ MHz} \) for the lowest energy mode of each cavity is set by relaxation through a well-coupled port into a common microwave transmission line, which dominates over both the internal losses and relaxation through the second port. This last port is used to apply resonant microwave
\[ H = \frac{\hbar}{2} (q^\dagger q + \tilde{c}^\dagger c - \frac{\alpha}{2} (q^\dagger q)^2 - \chi q^\dagger q c^\dagger c + e^{-i(\omega_1 + \omega_2)t} g_s(t) q^\dagger c^\dagger + h.c. + e^{-i(\omega_1 - \omega_2)t} g_c(t) q^\dagger c + h.c. \]  

where \( e \) and \( q \) are the annihilation operators for the cavity and qubit modes, \( \alpha \) is the anharmonicity of the transmon mode, \( \chi \) the dispersive shift \([38]\), and \( \tilde{\omega}_q(t) \) and \( \tilde{\omega}_c(t) \) are the Stark shifted frequencies of the transmon and cavity modes in presence of the pumps. These dressed frequencies and the squeezing and conversion strengths \( g_s(t) \) and \( g_c(t) \) are slow varying compared to \( \Delta \) and read

\[
\begin{align*}
\tilde{\omega}_q &= \omega_q - \chi |\xi_2|^2 - 2\alpha |\xi_1|^2 \\
\tilde{\omega}_c &= \omega_c - \chi |\xi_1|^2 \\
g_s &= \chi \xi_1 \xi_2 \\
g_c &= \chi \xi_1^* \xi_2^* 
\end{align*}
\]

FIG. 1. a) Minimal logical circuit for remote entanglement. Alice is entangled with the ancillary system \( C \) by a Hadamard and a CNOT gate. The information propagates to \( C' \) (green wave) where it is swapped to Bob. b) Setup schematics and c) energy level diagram. Two transmon qubits Alice (in dark blue, dressed frequency \( \tilde{\omega}_{qA} \), see text for details) and Bob (in red, dressed frequency \( \tilde{\omega}_{qB} \)) are dispersively coupled to two resonant cavities (in green, dispersive couplings \( \chi_{A,B} \)). The cavities lowest energy modes are frequency matched \( (\tilde{\omega}_{cA} - \chi_A \approx \tilde{\omega}_{cB}) \) and are strongly coupled to a directional transmission line routing photons from Alice to Bob. By simultaneously driving Alice (Bob) with the detuned purple microwave at \( \omega_{1A} \) (orange, at \( \omega_{1B} \) and her cavity with the detuned light blue microwave at \( \omega_{2A} \) (light pink, at \( \omega_{2B} \)), we drive a Raman-type two-photon transition. For Alice, we choose \( \omega_{1A} + \omega_{2A} = \tilde{\omega}_{qA} + \tilde{\omega}_{cA} = \chi_A \) to resonantly drive \( |g0\rangle \leftrightarrow |e1\rangle \) (see (c) left diagram). A photon can eventually be emitted in the line (green wave). The wavepacket is shaped by modulating the pump amplitude. This photon is absorbed by Bob by driving \( |g1\rangle \leftrightarrow |e0\rangle \) with \( \omega_{2B} - \omega_{1B} = \tilde{\omega}_{cB} - \tilde{\omega}_{qB} \) (right diagram). After a full photon pitch and catch, the system is in \( |e0\rangle_A |e0\rangle_B \) (in magenta). After a “half” pitch, the qubits are entangled.

Drives to perform control operations on a single mode, such as qubit rotations at \( \tilde{\omega}_{qA,B}/2\pi \sim 5 \text{ GHz} \), or cavity displacements at \( \tilde{\omega}_{cA,B}/2\pi \sim 7.5 \text{ GHz} \). Interestingly, we can also directly drive common two-excitation transitions of these modes such as \( |g0\rangle \leftrightarrow |e1\rangle \) or \( |g1\rangle \leftrightarrow |e0\rangle \) \([35]\). Here \( |0\rangle \) and \( |1\rangle \) designate Fock states of the cavity and \( |g\rangle \) and \( |e\rangle \) the ground and first excited states of the qubit. This is done by simultaneously applying a sideband pump at \( \omega_{1A,1B} \) detuned from the qubit frequency by \( \Delta/2\pi = 100 \text{ MHz} \) (purple and orange waves on Fig 1) and another at \( \omega_{2A,2B} \) detuned from the cavity frequency by \( \pm \Delta \) (light blue and pink waves).

Let us consider separately each system Alice or Bob. One can show \([34, 36, 37]\) that in a displaced frame and using a rotating wave approximation, the system Hamiltonian in presence of pumps at \( \omega_1 \) and \( \omega_2 \) reads

\[
\begin{align*}
\hat{H} &= \omega_q(t) q^\dagger q + \tilde{\omega}_c(t) c^\dagger c - \frac{\alpha}{2} (q^\dagger q)^2 - \chi q^\dagger q c^\dagger c + e^{-i(\omega_1 + \omega_2)t} g_s(t) q^\dagger c^\dagger + h.c. + e^{-i(\omega_1 - \omega_2)t} g_c(t) q^\dagger c + h.c. 
\end{align*}
\]

Here, \( \omega_q \) and \( \omega_c \) are the frequencies in the absence of the pumps. \( \xi_1 \) and \( \xi_2 \) are the effective pump amplitudes – which correspond to the frame displacements used to get to Eq. (1) – and are proportional to the amplitude of the pump tones. Note that since the cavity mode is only weakly anharmonic, we have neglected a frequency shift of the cavity mode proportional to \( |\xi_2|^2 \) \([34]\). The conversion or squeezing process (red or blue sideband) can be selected by setting either

\[
\begin{align*}
\tilde{\omega}_q + \tilde{\omega}_c - \chi &= \omega_1 + \omega_2 \quad \rightarrow \quad |g0\rangle \leftrightarrow |e1\rangle \\
\tilde{\omega}_q - \tilde{\omega}_c &= \omega_1 - \omega_2 \quad \rightarrow \quad |g1\rangle \leftrightarrow |e0\rangle 
\end{align*}
\]

in driving the two-photon transition. The resonance condition Eq. (3a) is used for Alice. As shown by the energy-level diagram of Fig. 1, this pumping, combined with the cavity dissipation, eventually brings the system to the state \( |e0\rangle \) (highlighted in magenta). If the qubit is initially in \( |g\rangle \), a photon is emitted in the line (green wave). Conversely, the resonance condition Eq. (3b) is used for Bob, and if the qubit is initially in \( |g\rangle \), it can absorb the incoming photon and excite to \( |e\rangle \) (level highlighted in magenta), provided that the photon is resonant with the cavity frequency. This is made possible by designing the two cavities so that their transition nearly match \( (\tilde{\omega}_c - \chi - \tilde{\omega}_{qB})/2\pi = 600 \text{ kHz} \), and by modulating the amplitude and frequency of the pumps in time (see Fig. 3a), in order to shape the pitched wavepacket and to catch it efficiently. Accurate control of the drive strengths while matching the resonance conditions (3) is the main difficulty of this experiment.

First, we must determine the unknown scaling factor linking the amplitude of the applied pumps to the effective amplitudes \( \xi_{1,A,2,A,1,B,2,B} \). This is done by measuring the shift of the qubit transition peaks in presence of the pumps and using Eq. (2a), or any other quantity predicted by Eqs. (2). Such spectroscopic measurements are presented in \([34]\). While the Stark
FIG. 2. **Top panels** Rabi oscillations when driving a two-photon transition for a varying duration $t_{\text{pulse}}$, are recorded in the qubit excited state populations (dots). Alice is initialized in $|g\rangle$ and Bob in $|e\rangle$. The amplitude values $\xi_1$ and $\xi_2$ are calibrated through Stark-shift measurements (see text and [34]). As for all population measurements presented in this letter, statistical error bars are smaller than the dots size. Lines are fits for the two-photon drive strengths $g_s$ and $g_c$. **Inset**: Pulse sequence schematics. Pump pulse edges are smoothed to 128 ns and the pump 1 pulse is 100 ns longer for accurate control of the drive ramp up and down. **Bottom panels** The extracted drive strengths are plotted when varying $\xi_2$ (dots, the green stars are from the top panel fits). For each point, the cavity pump frequency is tuned to match the resonance condition Eq. (3). Lines are linear fits of the non-saturated regions and their slopes are used as a calibration for the release and capture of a shaped photon. Dashed black lines are the drive strengths $g_{s,c} = |\chi| \xi_1 \xi_2$ predicted from Stark-shift calibration of $\xi_{1,2}$ [34].

shifts display a characteristic linear dependence in the pump powers, some of the predictions from Eqs. (2) do not agree quantitatively (a detailed analysis is presented in [34]). In practice, we use an empirical approach. The amplitude of the two-photon drives being determined by the product of the pump amplitudes, we set $\xi_1$ and $\omega_1$ at a constant value. The cavity frequency is then fixed (see Eq. (2b)), and so is the frequency of the released photon. To vary $g_s$ or $g_c$, we only vary $\xi_2$ and change accordingly the frequency $\omega_2$ to fulfill the resonance condition (3). Following this protocol, we record Rabi oscillations of these two-photon transitions, presented on Fig. 2. The qubits are first initialized in $|g\rangle$ (Alice) or $|e\rangle$ (Bob) by single-shot dispersive measurement using a near quantum-limited Josephson Parametric Converter [39, 40] (JPC) and fast feedback control [41, 42]. We then drive the two-photon transition for a varying time $t_{\text{pulse}}$. For Alice, we record an oscillation in the excited state population decaying to 1 at a rate $\kappa$, as $|e\rangle$ is a dark state in presence of cavity dissipation (see Fig. 1). The edges of the pulses are smoothed as depicted in the top right inset so that the oscillation does not start at $P_e = 0$. We can fit this oscillation with $g_s$ as the only free parameter by solving a quantum Langevin equation [34, 43] on the qubit and cavity modes. Inversely, for Bob (right panel), the excited state population decays to 0. Note that this feature can be used for efficient cooling of the qubits before the experiment [34, 44]. In both cases, we then repeat the measurement when varying $\xi_2$. The extracted values of $g_s$ and $g_c$ display the expected linear dependence at low pump power (lines are linear fits) and are in good agreement with predictions from Eqs. (2c,2d) with the values of $\xi_1$, $\xi_2$ and dispersive shifts $\chi_A/2\pi = 8.3$ MHz, $\chi_B/2\pi = 3.3$ MHz extracted from spectroscopic measurements [34] (dashed black lines). This provides an accurate calibration of the drive strengths at low pump amplitude. Saturation for stronger drives is mainly attributed to non-ideal behavior of the mixers used to generate the pulses. Our model also neglected some non-linear effects such as the anharmonicity inherited by the cavity mode [34] and the non confining nature of the transmon cosine potential. For the actual release and capture presented in next sections, we use smaller values of $\xi_1 = 0.11$ and $\xi_2 < 1$ (see [34] for the corresponding Rabi oscillations) as the qubit coherence times were degraded at larger drive amplitude. This unexpected effect may originate from the aforementioned non idealities, compounded by the small pump detuning $\Delta$ - limited by our pulse generation scheme (see Fig. S1 in [34]) – compared to the transmon anharmonicity ($\Delta < \alpha A_B \sim 2\pi \times 200$ MHz).

**EXCITATION TRANSFER**

After calibrating the drive strengths, we turn to the task of generating a photon with Alice and capturing it with Bob. We choose the traveling wavepacket to be time-symmetric [15], Gaussian-shaped for spectral resolution, and with as short a characteristic time $\sigma = 800$ ns as permitted by the aforementioned maximum pump amplitudes. We also scale the wavepacket to contain one photon. With these constraints, the value of $g_s$ and $g_c$ required for the transfer are computed using a method adapted from [26] and described in detail in [34]. Note that beyond the slowly varying envelopes represented on Fig. 3a, the pump 2 pulses are modulated at $\omega_2$ and chirped to match the resonance conditions Eq. (2) at all times.

Unlike the ideal case of two perfectly frequency-aligned cavities [15], Alice and Bob’s control are not time-symmetric of one another. Indeed, to compensate for the small cavity mismatch, we modify Alice’s resonance condition Eq. (3a), so that the pitched wavepacket does not rotate in Bob’s frame. The resulting control $g_s$ is slowly...
The amplitude of $g_e$ determined with the same constraints but scaling the sequence as for the excitation transfer. The controls are realized by holding $\xi_1$ constant and varying $\xi_2$ as represented on the right axis. b) Excited state populations of Alice and Bob during the transfer (dots), measured by interrupting the transfer control pulses after a duration $t$ and subsequent dispersive readout of the qubits. Lines are predictions from cascaded quantum system simulation including all imperfections.

The photon transfer is validated by measuring the qubit populations in time (Fig. 3b), which reveals a transfer efficiency of 70 %, when not correcting for any experimental imperfections. After calibrating those through independent measurements [34], we reproduce the results with 1 % accuracy by performing full cascaded quantum system simulations [43] (lines). The dominant error sources are decoherence of the qubits (11 % error) and photon loss in the line (15 % error) [34]. This last figure is obtained by measurement induced dephasing and confirmed by measuring the fraction of the traveling wave packet power actually absorbed by Bob during the transfer (see [34]).

FIG. 3. Release and capture of a shaped photon. a) Calculated complex amplitude of the two-photon drive strength for Alice (blue) and Bob (red) to transfer a photon in a Gaussian traveling mode centered at Bob’s cavity resonance frequency with deviation $\sigma = 800$ ns. These controls are realized by holding $\xi_1$ constant and varying $\xi_2$ as represented on the right axis. b) Excited state populations of Alice and Bob during the transfer (dots), measured by interrupting the transfer control pulses after a duration $t$ and subsequent dispersive readout of the qubits. Lines are predictions from cascaded quantum system simulation including all imperfections.

plot the measured populations of Alice and Bob during the transfer on Fig. 4a (red and blue dots), which agree with the simulation predictions (lines) performed with the same parameters. We also plot the measured correlator $\langle Z_A Z_B \rangle_{\text{meas}}$ (where $Z = 2|e\rangle\langle e| - 1$) between these measurements (green dots). When considering the correlations after correcting for readout errors (dashed lines), we find that at final time the actual occupation of the excited state is $P(|e\rangle_A) = 0.5$ and the actual correlator is $\langle Z_A Z_B \rangle = 2P(|e\rangle_B)$ (within 1 %), which implies that Bob is excited only if Alice is. In other words, as a photon detector, Bob’s false positive probability beyond dispersive readout imperfections is below our detection precision. This property is crucial in non-deterministic entangling schemes, where the catch protocol could be used to perform single microwave photon detection [10, 45, 46].

Finally, we perform full tomography of the final joint state of Alice and Bob by rotating the qubits to measure all Pauli operators $X,Y,Z$ and their correlators. After rotating the $(X_B,Y_B)$ basis to compensate for the a priori unknown but deterministic differential phase accumulated by control and pump pulses along the input lines, one can directly compute the density matrix following $\rho = \frac{1}{2}\sum_{\alpha,\beta\in\{X,Y,Z\}}\langle\alpha A\beta B\rangle_{\text{meas}}\alpha_A \otimes \beta_B$. The fidelity to the target Bell state $|\Phi_+\rangle = (|gg\rangle + |ee\rangle)/\sqrt{2}$ is found to be $F = \text{Tr}(\rho |\Phi_+\rangle \langle \Phi_+|) = 73 %$, well above the entanglement threshold $F = 1/2$. Once again, the measured density matrix (colorbars on Fig. 4b, see [34] for a full representation of the two-qubit state Pauli vector components) is in quantitative agreement with simulation predictions (black transparent bars). The contribution of each experimental imperfection to the

REMOTE ENTANGLEMENT

We now turn to the task of entangling Alice and Bob. This is done by first having Alice release “half” of a photon and thus getting entangled with the traveling mode in the state $(|g0\rangle + |e1\rangle)/\sqrt{2}$, which corresponds to the Hadamard and CNOT gates in Fig. 1a. This operation is followed by a swap gate between the traveling mode and Bob, which corresponds to the same capture sequence as for the excitation transfer. The controls are determined with the same constraints but scaling the pitched wave packet to contain 1/2 photon on average. The amplitude of $g_e$ in this case is smaller than for the full release, so that we can use a traveling wave packet with a reduced characteristic time $\sigma = 450$ ns. We

FIG. 4. With Alice and Bob initially in $|g\rangle$, a pump control signal is applied on Alice to release half a photon (see text) while the capture sequence of Fig. 3a is played for Bob. a) Measured excited state populations and correlator (with $Z = 2|e\rangle\langle e| - 1$) when interrupting the control pulses after a duration $t$ and then performing simultaneous dispersive readout on both qubits. Plain lines are simulations including all imperfections. Dashed lines are the same simulations assuming perfect final readouts. b) Real part of the density matrix of the final entangled state measured by tomography of the two-qubit state (colored bars) and reconstructed by simulation (black contours). Fidelity to the Bell state $(|gg\rangle + |ee\rangle)/\sqrt{2}$ is 73%.
infidelity $1 - F$ is detailed in [34].

In this experiment, we have implemented a simple protocol to perform reliable operations between standing qubits and arbitrarily shaped traveling photons. The method was used to generate fast (2.5 $\mu$s) remote entanglement of two qubits separated by $\sim$ 1 m microwave cables and a circulator. This protocol could be readily extended to entangle larger systems in order to detect photon loss in the transmission line [14, 31, 32]. Moreover, by controlling the traveling photon wavepacket shape in frequency, the signal from one cavity could be routed to another arbitrary one connected on the same line. All these features are important primitives on the path to a reliable modular quantum computing architecture [13] or quantum internet [11].

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[30] During this letter writing, similar results were reported by two other groups [47, 48].


[34] See Supplementary Material for system characterization, details of the experimental setup and control pulses generation algorithm, which includes Refs. [49–51].


[48] Philipp Kupfers, Paul Magnard, Theo Walter, Baptiste

