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Topological Floquet-Thouless energy pump

Michael H. Kolodrubetz[1,2,3], Frederik Nathan[4], Snir Gazit[2], Takahiro Morimoto[2], Joel E. Moore[1,2]
[1]*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA, USA*
[2]*Department of Physics, University of California, Berkeley, CA, USA*
[3]*Department of Physics, University of Texas at Dallas, Richardson, TX, USA*
[4]*Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark*

We explore adiabatic pumping in the presence of periodic drive, finding a new phase in which the topologically quantized pumped quantity is energy rather than charge. The topological invariant is given by the winding number of the micromotion with respect to time within each cycle, momentum, and adiabatic tuning parameter. We show numerically that this pump is highly robust against both disorder and interactions, breaking down at large values of either in a manner identical to the Thouless charge pump. Finally, we suggest experimental protocols for measuring this phenomenon.

The Thouless charge pump serves as a simple yet fundamental example of topology in quantum systems [1]. The hallmark of this effect is the transport of a precisely quantized amount of charge during an adiabatic cycle in parameter space. This remarkable phenomenon has been demonstrated experimentally in various physical systems such as few-body semiconductor quantum dots [2–5] and more recently in a one-dimensional chains of ultra-cold atoms trapped in an optical lattice [6–8].

Recently, the classification of topological phases of matter has been extended to periodically driven (Floquet) systems far from equilibrium [9–13]. In particular, periodic driving can lead to new topological phases that have no analogy in undriven systems [14–28], an idea which has been confirmed experimentally [29, 30]. A natural question to ask is whether these far-from equilibrium systems can also exhibit new topological pumping effects?

In this paper, we answer this question in the affirmative by explicitly constructing a generalized adiabatic pump in a Floquet system. We find a novel phase in which energy, rather than charge, undergoes quantized pumping. Specifically, upon adiabatic cycling of a particular parameter, partially filled systems in this phase transport energy from one side of the filled region to the other, as illustrated in Fig. 1. The energy transported per cycle is quantized in units of the drive frequency $\hbar\Omega$.

Using numerical and analytical arguments, we show that this phenomenon is stabilized by disorder and, via many-body localization, remains robust in the presence of interactions. In this way, we demonstrate the existence of a stable topological pump that can only be realized in the presence of periodic driving.

Model. Let us begin by introducing a simple model that exhibits topological energy pumping, which we will later demonstrate is topologically robust to perturbations. The model consists of a five-step driving protocol,

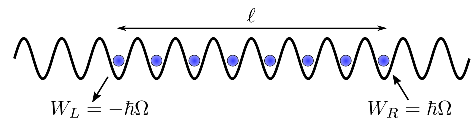

Figure 1. Illustration of the topological energy pump. Upon ramping the pump adiabatically around a cycle, the filled region of length $\ell \gg 1$ remains localized, but nevertheless quantized work is performed at the edges of the filled region in quanta of the drive energy $\hbar\Omega$.

with Hamiltonians $H_j = h_j + h.c.$, where

$$h_1 = -J \sum_{x=1}^{L/2} c^\dagger_{A,x} c_{B,x}, \qquad h_2 = -J \sum_{x=1}^{L/2} e^{i\lambda} c^\dagger_{B,x} c_{A,x+1}$$
$$h_3 = -J \sum_{x=1}^{L/2} c^\dagger_{B,x} c_{A,x+1}, \qquad h_4 = -J \sum_{x=1}^{L/2} e^{i\lambda} c^\dagger_{A,x} c_{B,x},$$
$$h_5 = \frac{\Delta}{2} \sum_{x=1}^{L/2} \left(c^\dagger_{A,x} c_{A,x} - c^\dagger_{B,x} c_{B,x} \right) \tag{1}$$

acting on L sites with open boundary conditions. The protocol is chosen to be time periodic with $H(t) = H(t+T)$ such that $H(0 < t < T/5) = H_1$, $H(T/5 < t < 2T/5) = H_2$, etc. This model is particularly simple if the tunneling strength J takes the value $J_{\rm tuned} \equiv 5\hbar\Omega/4$, where $\Omega = 2\pi/T$. At this fine-tuned point, the fermions hop exactly one site at each step, such that a fermion initialized at any site returns to the same site after one driving cycle, as illustrated in Fig. 2a.

Using the Floquet formalism, we write the single-particle time evolution U in the form $U(t) = \mathcal{P}(t) e^{-iH_F t}$, where the micromotion $\mathcal{P}(t) = \mathcal{P}(t+T)$ describes the dynamics within each cycle and H_F is the effective Hamiltonian that describes stroboscopic behavior at multiples of the period T [31]. For $J = J_{\rm tuned}$, the Floquet eigenstates are localized states $|x, \alpha\rangle \equiv c^\dagger_{\alpha,x}|\mathrm{vac}\rangle$. The eigenvalues of H_F, known as quasienergies, are only well-defined modulo $\hbar\Omega$. For a particle initially located on a site in the bulk, the phase $e^{i\lambda}$ acquired during step 2 is cancelled by

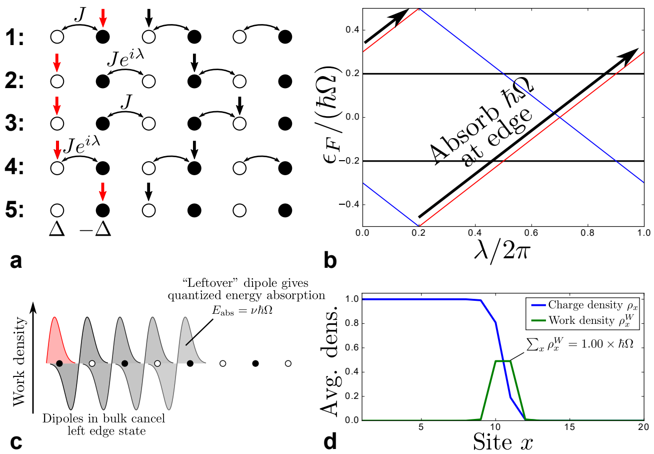

Figure 2. (a) Illustration of the anomalous Floquet pump (Eq. 1), which involves five steps of period $T/5$ with fined-tuned hopping $J_{\rm tuned} = 5\hbar\Omega/4$. Red and black arrows trace the positions of edge and bulk states respectively. (b) Quasienergy spectrum as a function of the tuning parameter λ show bulk bands (black), left edge state (red), and right edge state (blue). (c) Illustration of response measured in numerics, for which only the left half of the system is filled. (d) Numerical results for the local work and charge density for the model in Eq. 1 averaged over a single ramp from $\lambda = 0$ to 2π with $L = 20$, $N_c = 12$, and $N_\lambda = 1$. Data for ρ_x^W is in units of $\hbar\Omega$.

the phase $e^{-i\lambda}$ during step 4, yielding flat quasienergy bands at $\epsilon^F_{\rm bulk} = \pm\Delta/5$. However, a particle initially located at site $|1, B\rangle$ or $|L, A\rangle$ is unable to hop during steps 2 and 3, causing it pick up a λ-dependent phase during the driving cycle, which translates into a λ-dependence of these edge state quasienergies (Fig. 2b). While the bulk bands are trivial and can be shown to have vanishing Chern number with respect to λ and quasimomentum k [32], the edge states (red and blue) clearly exhibit topologically nontrivial winding. The question, then, is how to characterize and measure the topological properties of this model?

Topology and measurement. The main insight for understanding our model comes from noting that the band structure in Fig. 2b is identical to that found in the two-dimensional anomalous Floquet insulator (cf. Fig. 1 in Ref. [11]) with the the pump parameter λ playing the role of momentum k_y. In this way, our model is a dimensionally reduced version of the anomalous Floquet insulator [11, 17], in the same way that the Thouless pump may be thought of as the dimensional reduction of a Chern insulator. This immediately implies the existence of a topological invariant characterizing our pump, namely the winding number of the micromotion,

$$\nu = \frac{1}{8\pi^2}\int dt d\lambda dk \mathrm{Tr}\left(\left[\mathcal{P}^\dagger\partial_\lambda\mathcal{P}, \mathcal{P}^\dagger\partial_t\mathcal{P}\right]\mathcal{P}^\dagger\partial_k\mathcal{P}\right), \quad (2)$$

defined on the compact three-dimensional parameter space (t, λ, k). While the micromotion and thus the winding number in principle depend on the branch cut defining H_F, the fact that Chern numbers of the bulk bands vanish implies that the winding number is independent of this choice [11]. In particular, the winding number for a branch cut at quasienergy $\epsilon_{\rm cut}$ in some gap gives precisely the number of edge states crossing that gap. For the model we consider here, $\nu = 1$.

One hint for the observable consequences of this topological index comes from examining the quasienergy spectrum in the presence of open boundary conditions (Fig. 2b). Upon adiabatically ramping λ from 0 to 2π, the bulk remains unchanged while the left (right) edge state wraps around the Floquet Brillouin zone, absorbing (emitting) a quantum of energy. Upon completing the cycle, the system returns to its initial electronic state. Therefore the nontrivial topology does not lead to any direct pumping of the fermions. Instead, as we will show, ramping λ performs quantized work on the external driving fields.

Specifically, we now show that the quantized observable is the λ-averaged "force polarization" $P_F \equiv \sum_x x\rho_x^F$, where

$$\rho_x^F = \frac{1}{2}\left\langle\left\{\sum_\alpha c^\dagger_{\alpha,x}c_{\alpha,x}, \partial_\lambda H\right\}\right\rangle \quad (3)$$

is the local generalized force required to change λ by a small amount. Here curly braces denote the anti-commutator, $\alpha = \{A, B\}$ sums over sublattices, and the expectation value is taken with respect to an arbitrary quantum state [33]. Changing λ by a finite amount thus requires a local work

$$\rho_x^W = \int \rho_x^F\left[\lambda(t), t\right]\dot{\lambda}(t)dt.$$

While the above expressions hold for arbitrary non-equilibrium situations, the work becomes independent of speed in the limit of slow ramps, for which the wave function is given by (Floquet) adiabatic transport. Thus a finite work polarization $P_W = \int P_F d\lambda = \sum_x x\rho_x^W$ implies that work is done on one half of the system and done by the other half. We will see that quantization of P_W thus implies that this differential work is quantized, as illustrated in Fig. 1.

Quantization of P_W follows immediately from dimensionally reducing the anomalous Floquet insulator, as the topologically quantized magnetization [34] immediately reduces to P_W. In practice, the work polarization may be directly measured by filling part of the system and measuring the time-dependence local force ρ_x^F near the edges of the filled region, as illustrated in Fig. 1. Within the fully filled or fully empty regions nothing is able to move, hence no work is done: $\rho_x^W = 0$. Furthermore, as the net work on the entire system vanishes, the work done near the left edge of the filled region, W_L, must exactly cancel that done near the right edge: $W_R = -W_L$.

For a filled region of length ℓ lattice sites which is much larger than the localization length ξ, the total work polarization is then given by $P_W^{\text{tot}} \approx (W_R - W_L)\ell/2$. As the average work polarization per filled unit cell is quantized to be $\overline{P}_W = \nu\hbar\Omega$, we also have $P_W^{\text{tot}} = \nu\hbar\Omega\ell$. Equating these expressions, we find that

$$W_R = -W_L = \nu\hbar\Omega. \tag{4}$$

Further details on this derivation may be found in the supplement [35].

To confirm these predictions, we consider a slightly different setup in which we fill only the left half of the system, i.e., sites 1 through $L/2$. Then the only contribution to the force comes from the density step at $L/2$, such that the entire system absorbs/emits an integer number of photon quanta. Fig. 2c illustrates how this emerges from adding the quantized polarization in each localized state. Numerically, we start from this initial state and ramp λ from 0 to $2\pi N_\lambda$ at a constant rate $\dot{\lambda} = 2\pi/(N_c T)$. While slow time-dependence of λ formally breaks the T-periodicity, it has been shown than an appropriate extension of adiabaticity may be defined [36–38], which is nevertheless subtle due to the presence of resonances which must be crossed diabatically. In practice, we find that an appropriate adiabatic limit is reached for $N_c \gg 1$ and ramping over many adiabatic cycles ($N_\lambda \gg 1$) to remove initial transients [39]. We then expect the total energy absorbed by the system,

$$E_{\text{abs}} \equiv \int \langle \partial_\lambda H \rangle \dot{\lambda} dt, \tag{5}$$

to be quantized in units of $\hbar\Omega$. In the supplement [35] we show this analytically for our simple model, and we verify this numerically in Fig. 2d.

Disorder and interactions. Having determined the basic properties of our topological energy pump in an analytically tractable limit, we now demonstrate its robustness to disorder and interactions. One might naively expect this robustness to be trivial, as topological states are often argued to be protected against weak perturbations. However, in the presence of disorder, the ability to adiabatically track a given localized eigenstate is known to be ill-defined, as the eigenstate will undergo weakly avoided crossings on arbitrary length scales [40]. We will address this issue analytically in a follow up work [41], but for now we provide numerical support regarding its stability.

Specifically, we add static chemical potential disorder to our Floquet system,

$$H_{\text{dis}} = \sum_{\alpha,x} w_{\alpha,x} c^\dagger_{\alpha,x} c_{\alpha,x} \ , \tag{6}$$

where the disorder is drawn from a box distribution $w_{\alpha,x}/\Omega \in [-W, W]$. We also consider deviating from the fine-tuned limit by a "detuning" α [42]. such that

$$\Delta = \alpha\Omega \ , \ J = J_{\text{tuned}}(1-\alpha). \tag{7}$$

We then carry out the same procedure as in Fig. 2c to measure topological energy absorption.

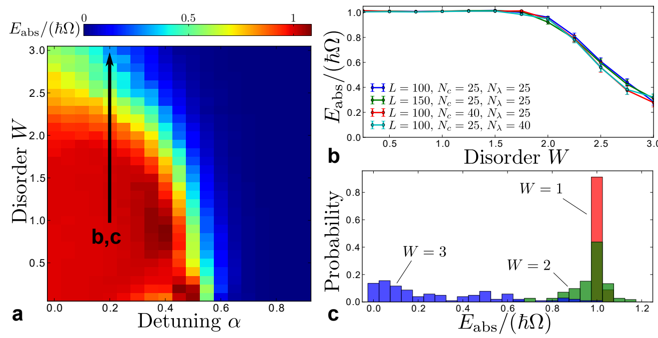

Figure 3. (a) Phase diagram of energy pump as a function of disorder W and detuning α (see Eq. 7) at fixed $L = 100$ and $N_c = N_\lambda = 25$. In the absence of disorder a phase transition occurs at $\alpha = 1/2$. In the presence of disorder, the topological plateau appears stable over a wide region. Note that some data, particularly at small W, is not fully converged to the $L, N_c, N_\lambda \to \infty$ limit due to large localization length. We analyze this limit further in the supplement [35]. (b) Cut at fixed $\alpha = 0.2$ showing a slow crossover to topologically trivial independent of L, N_c, and N_λ. (c) Histogram over disorder configurations of E_{abs} at three points along the crossover showing the breakdown of topological quantization.

The disorder-averaged phase diagram for a wide range of disorder strengths and detunings is shown in Fig. 3a. There is clearly a wide region with well-quantized energy pumping (red), up to disorder strengths and detuning of order $\hbar\Omega$. In fact, for the majority of the phase diagram, disorder is actually necessary to see quantization of the energy transport. The simplest reason for this is that, in the absence of disorder, any generic model will not be localized and our measurement of E_{abs} at the localized density edge is not meaningful. This is seen in our phase diagram for $\alpha \neq 0$, where a small amount of disorder clearly improves the quantization for the system size shown. Furthermore, we will show in a follow up work [41] that even the appropriately defined clean limit of P_F has a non-topological contribution which is suppressed by localization. In either case, the phase diagram clearly shows a large nearly quantized plateau at weak disorder below the topological transition at $\alpha = 1/2$. For instance, the data in Fig. 3b is quantized to within 0.4% and 0.8% at $W = 1$ and 3/2 respectively for $L = 150$, $N_c = N_\lambda = 40$. We also note that the quantized work polarization is robust to choice of initial conditions, as demonstrated numerically in the supplement [35].

At large disorder strengths, we expect a topological transition to a trivial state while maintaining Anderson localization throughout [43]. Surprisingly, we instead find a slow crossover for which energy is still pumped, but not quantized. This is unlike the sharp transition found in the anomalous Floquet Anderson insulator [17], and illustrates a fundamental difference regarding the role of disorder in one dimensional pumps compared to their higher-dimensional counterparts. For the energy pump, one of the tuning parameters, λ, couples strongly to the quasienergies, even when the system is localized. For the anomalous Floquet Anderson insulator, the winding number is defined as in Eq. (2) with angles θ_x and θ_y defining twisted boundary conditions in place of the parameters λ and k. For that model, the localization of Floquet eigenstates implies that the change of quasienergy due to either twist angles is exponentially suppressed. In contrast, the "dimensional extension" of the energy pump features Floquet states that are delocalized in the y-direction. Hence the quasienergy spectrum is sensitive to changes of θ_y, i.e., λ.

The breakdown of topological energy pumping may be traced to this increased sensitivity to λ. As the disorder strength W is increased, the L individual quasienergy mini-bands $\varepsilon_n(\theta_x, \lambda)$ may undergo topological gap closings and reopenings, potentially introducing non-trivial Chern numbers. This yields a Floquet branch cut dependence of the winding number $\nu(\epsilon^F_{\text{gap}})$ [11], where in the disordered case the winding number is defined as in Eq. (2) with θ_x in place of k_x. As our measurement populates quasienergy states at random (the "infinite temperature" ensemble), we stochastically sample over these winding numbers. Thus the non-quantized energy pump may be thought of as an average of the topological winding number over both gaps and disorder realizations [41].

This argument is consistent with the histogram of E_{abs} in this crossover region (Fig. 3c), which shows broadening from a perfectly quantized δ-function peak at $E_{\text{abs}} = \hbar\Omega$ towards statistical ensemble that will eventually be non-topological ($\overline{E}_{\text{abs}} = 0$). Importantly, this breakdown by a proliferation of Berry monopoles is precisely the mechanism that leads to the loss of charge pump quantization in disordered systems [44, 45]. Thus the crossover behavior in our system likely falls into the same class as this undriven case.

Many-body localization. Finally, let us see that our results hold in the presence of many-body localization. We test this by adding nearest neighbor interactions

$$H_{\text{int}} = U \sum_j \left(n_j - \frac{1}{2}\right)\left(n_{j+1} - \frac{1}{2}\right) \tag{8}$$

throughout the cycle and simulate the dynamics via exact diagonalization [46]. In Fig. 4a, we map out the phase diagram as a function of interaction and disorder strengths. The data confirm that the energy absorption remains beautifully quantized in the topological phase (Fig. 4b). We note that, in the absence of disorder, the system is expected to heat to infinite temperature, and thus approach $E_{\text{abs}} = 0$ for $N_\lambda \to \infty$. The remarkable quantization we see is likely a prethermal phenomenon. Interestingly, the data indicate that weak interactions also stabilize the topological phase. While this may be due to a trivial microscopic effect such as shortening of the localization length due to interactions, it leaves open the tantalizing possibility that interactions stabilize the phase and lead to an energy pump that is again topologically protected.

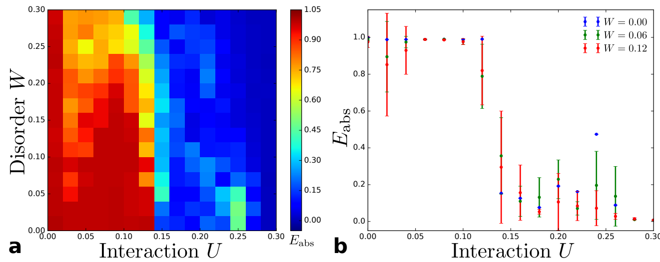

Figure 4. Phase diagram of energy pumping in an interacting many-body localized (MBL) system for $\alpha = 0$, $L = 16$, $N_\lambda = 64$, and $N_c = 256$. Error bars in (b) show standard error over a fixed number of disorder configurations. Interestingly, the error bars become smaller – indicating increased stability of the MBL energy pump – in the presence of weak nonzero interactions. E_{abs} is in units of $\hbar\Omega$, while W and U are in units of hopping amplitude J.

Experiments. The topological energy pump is directly amenable to being realized experimentally, requiring hopping models in one dimension similar to those recently realized in optical lattice charge pumps [6–8]. Instead of measuring local charge, these experiments would simply have to measure local force, ρ^F_x. This should be readily realized by combining adiabatic pump protocols with systems that enable site-resolved measurement, such as optical lattice microscopes [47, 48], trapped ion arrays [49], and other engineered platforms [50–52], where ρ^F_x is simply the measurable local current operator during steps 2 and 4 [53]. In addition to the pulsed multi-step protocols discussed in this work, which are quite natural in such engineered systems, we will show elsewhere that the topological pumping may also occur in monochromatically driven models, such as a driven version of the Rice-Mele model [41, 54]. This opens the intriguing possibility to directly measure the back-action on the drive lasers. For instance, if the periodic driving is realized by a pair of Raman lasers with frequency difference Ω, adiabatic cycling of the pump parameter λ would result in quantized transfer of ν photons from one Raman beam to the other. If one further quantizes the Floquet drive photons, for instance by use of a high-Q cavity, then each adiabatic cycle would directly back-act on the cavity photons. This can, for example, lead to either quantized absorption/emission of cavity photons, whose behavior at low photon number represents an interesting quantum limit of our problem.

Conclusion. We have introduced a novel topological energy pump which exhibits a new type of topologically protected response with no equivalent in undriven systems. The pump is inspired by a dimensional reduction scheme from the anomalous Floquet insulator, but features fundamentally different topological protection and transport properties. We note that other topological energy pumps recently introduced in the driven qubit systems derive instead from reducing the Thouless charge pump to zero dimensions, replacing momentum with a magnetic field angle [38] or the phase of a second incommensurate drive [55]. This suggests a number of fascinating future directions from dimensional reduction of other entries in the Floquet periodic table [12, 13], such as the Floquet generalization of the $\mathbb{Z}_2$ pump [56, 57] or fractionalized systems [58]. Furthermore, studying the backaction of our topological pump on a classical or a quantum drive represents an interesting quantum adiabatic limit on statistical mechanics, where pumping of bosonic objects such as the drive photons is a long sought-after goal [59, 60].

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plicated potential acting on a point particle due to the fermions in the lattice, then $-\langle\psi|\partial_x H|\psi\rangle$ is the force acting on x. Note that this is true for arbitrary state ψ, whether or not in equilibrium.

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[39] In follow up work, we will prove many of our claims in the extreme adiabatic limit $\left|\dot{\lambda}\partial_\lambda\epsilon_F\right| \ll \Delta^2_{\mathrm{miniband}}$, where $\Delta_{\mathrm{minibands}} \sim e^{-L/\xi}$ is the exponentially-small gap between disorder minibands. In the numerics, we are nowhere near this limit, but nevertheless find surprising robustness of our results. We postulate therefore that adiabaticity should be in reference to other characteristic energy scales of the system, such as the hopping amplitude J and the drive frequency Ω. This is equivalent to the statement that $N_c \gg 1$.

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