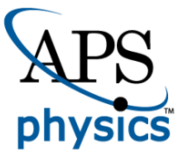

Clean Floquet Time Crystals: Models and Realizations in Cold Atoms

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Clean Floquet Time Crystals: Models and Realizations in Cold Atoms

Biao Huang[1],* Ying-Hai Wu[2], and W. Vincent Liu[1,3,4†]
[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh PA 15260, USA*
[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany*
[3] *Wilczek Quantum Center, School of Physics and Astronomy and T. D. Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*
[4]*Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China*

Time crystals, a phase showing spontaneous breaking of time-translation symmetry, has been an intriguing subject for systems far away from equilibrium. Recent experiments found such a phase both in the presence and absence of localization, while in theories localization by disorder is usually assumed a priori. In this work, we point out that time crystals can generally exist in systems without disorder. A series of clean quasi-one-dimensional models under Floquet driving are proposed to demonstrate this unexpected result in principle. Robust time crystalline orders are found in the strongly interacting regime along with the emergent integrals of motion in the dynamical system, which can be characterized by level statistics and the out-of-time-ordered correlators. We propose two cold atom experimental schemes to realize the clean Floquet time crystals, one by making use of dipolar gases and another by synthetic dimensions.

Introduction — The recent realizations of Floquet (or discrete) time crystals have drawn much attention [1–10]. A common feature of these systems is that certain physical observable $\hat{O}$ shows a rigid reduced periodicity $\langle\hat{O}\rangle(t+nT) = \langle\hat{O}\rangle(t), n \geq 2$, compared with the Floquet driving period T of the Hamiltonian $H(t+T) = H(t)$. As originally conceptualized in Ref. [11–13], "time-crystals" are regarded as a new addition to the concept of spontaneous symmetry breaking, for the temporal translation symmetry missing for nearly a century.

Early discussions of time crystals [12–15] concluded with a no-go theorem [16] forbidding such a phase in equilibrium. Consequently, a new generation of periodically-driven models were proposed [1–5], with results that challenge our understanding of dynamical interacting systems. Unlike the usual quasi-static examples such as charge pumping [17–19] or lattice shaking [20, 21], the Floquet time crystal lives in the regime with large driving amplitude and resonant frequencies, surprisingly robust against chaotic behaviors, such as in turbulence [22–24]. It is therefore natural to ask what serves as the stabilizer against butterfly effects and heating.

A key strategy in recent theories is to employ non-ergodic systems to resist trivialization of dynamics due to thermalization [2–5]. Besides the fine-tuned integrable Hamiltonians, many-body localized (MBL) systems consist of the most well-studied examples showing robust non-ergodicity. As such, it is assumed *a priori* in most theories that stable time-crystal phase can only occur in the MBL regime with strong spatial disorder [3, 5, 9]. However, a recent experiment on nitrogen-vacancy (NV) centers performed by Choi *et. al.* demonstrated an alternative possibility [10], where time crystals formed regardless of the delocalization by the three-dimensional spin-dipolar interactions. It was also emphasized that the system is not in a pre-thermal regime [6, 10]. The experimental breakthrough indicates the tantalizing possibility of seeking for stable time-crystals without the aid of localization, and the theoretical need to understand the time-crystal phase in this regime.

The purpose of this work is to demonstrate through a simple model that stable time crystals can exist in the strongly interacting regime *completely without disorder*. These Floquet-ladders we propose represent a large class of models including, as special cases, the quenched Ising chain [2–5] discussed before. Within certain parameter *regions*, the persisting double-periodic oscillation modulates with time spans that scale exponentially with system sizes. Unlike the "MBL time crystal" [2–5] which inherits integrability from a static MBL-Hamiltonian, these "clean time crystals" exhibit emergent integrability through dynamics and is a property of the Floquet evolution operator. Such a character is illustrated by the level statistics and out-of-time-ordered-correlators (OTOC) in different parameter regimes. Moreover, these phenomena even survive when the interactions are modified to those that can be readily realized in current cold atom experiments. The generality of our results clearly suggests an exciting field of studying time crystals in various clean systems with more intriguing properties.

Definition of time crystal—Periodic motions exist widely in dynamical systems, ranging from Rabi oscillations [25] to Josephson effects [26] and Zitterbewegung [27]. More generally, if one picks an arbitrary initial state, the unitary time-evolution $e^{-iHt/\hbar} = \sum_n |n\rangle e^{-iE_n t/\hbar}\langle n|$ may fairly endow the evolved state certain oscillations. Therefore, restrictions must be applied to screen out some periodic motions that are already well-understood without involving a new name. Here we give a phenomenological definition of non-equilibrium time crystal by selecting oscillations that are emergent from many-body dynamics. Specifically, there should exist a physical observable $\hat{O}$ and a class of initial states $|\psi\rangle$, such that

$$f(t) = \lim_{L\to\infty} \langle\psi|\hat{O}(t)|\psi\rangle \tag{1}$$

satisfy *all* of the three conditions: **(A)** Time-translation-symmetry-breaking, which means $f(t+T) \neq f(t)$ while the Hamiltonian has $H(t+T) = H(t)$; **(B)** Rigidity: $f(t)$ shows a fixed oscillation frequency without fine-tuned Hamiltonian parameters. **(C)** Persistence: the non-trivial oscillation with fixed frequency must persist to infinitely-long time when first

taking system size L to the thermodynamic limit.

The above definition is inspired by making analogy to the familiar charge-density-wave (CDW). Condition (A) rules out oscillations trivially following the external drive, which functions as "temporal lattice potentials". The rigidity of frequency in condition (B) requires many-body origins, resembling the rigidity of wave-vector for density-modulation in CDW given by Fermi-surface nesting [28]. Condition (C) is added to distinguish a stable time crystal from a quasi-stable, i.e. a pre-thermal one [6], or accidental oscillations lasting for short periods. See also Ref. [2–5, 7] emphasizing different aspects of the definitions respectively.

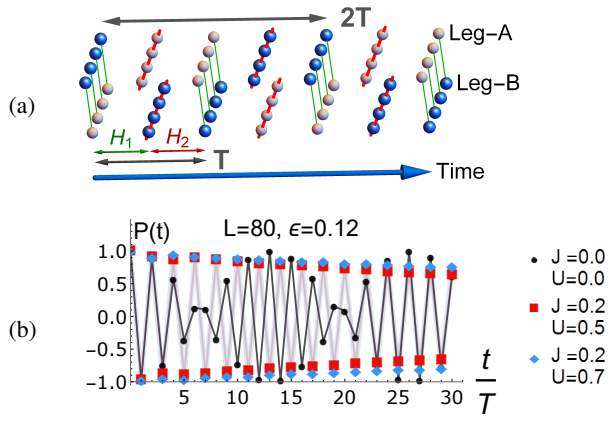

FIG. 1. (a) Schematic plot for the Floquet-ladder. Green and red lines indicate the inter-chain (Eq. (2)) and intra-chain couplings (Eq. (3)) respectively, which alternate during the binary drive. Blue dots represent occupied sites. In time crystal regime, density distribution in two chains shows rigid reduced periods $2T$. (b) DMRG result for density polarization $P(t)$ (Eq. (5)) under perturbation $\varepsilon = 0.12$ at stroboscopic time (lines are guides to the eyes). The interaction U rigidifies the $2T$ periodicity, signifying a time-crystal phenomenon. Here the lattice size is $L = 80$ for each chain, $\Delta = 0.1$, and the open boundary condition is used.

Model—We introduce a clean Floquet-ladder model that turns out to satisfy all of the conditions (A)-(C). The Hamiltonian is under binary quench with periodicity $T = t_1 + t_2$, where during

$$t_1: \quad H_1 = -J' \sum_{i=1}^{L} (a_i^\dagger b_i + b_i^\dagger a_i), \qquad \frac{J' t_1}{\hbar} = \frac{\pi}{2} + \frac{t_1}{\hbar}\varepsilon, \tag{2}$$

$$t_2: \quad H_2 = -J \sum_{i=1}^{L} (a_{i+1}^\dagger a_i + b_{i+1}^\dagger b_i + h.c.) + U \sum_{i=1}^{L} (n_i^A n_{i+1}^A + n_i^B n_{i+1}^B) + \Delta \sum_{i=1}^{L} (n_i^A - n_i^B). \tag{3}$$

See Fig. 1(a) for illustrations. Here $a_i^\dagger$ ($b_i^\dagger$) creates a particle in leg-A (-B), $n_i^{A,B} = a_i^\dagger a_i$ (or $b_i^\dagger b_i$) is the particle number operator, and L is the number of sites in each leg. The evolution operator at stroboscopic time is

$$U(nT) \equiv (U_F)^n = \left(e^{-iH_2 t_2/\hbar} e^{-iH_1 t_1 \hbar}\right)^n \tag{4}$$

where U_F is the Floquet operator. The physics is controlled by dimensionless parameters $(\varepsilon t_1, U t_2, J t_2, \Delta t_2)/\hbar$, which will be denoted simply as $(\varepsilon, U, J, \Delta)$ later on. To compare with previous works using an Ising chain [2–5], we note that for either spinless fermions or hard-core bosons, our model maps to *two* coupled spin-1/2 XXZ chains, and is therefore generically different (in additional to the lack of disorder) except in the special limit $J = 0$ and $n_i^A + n_i^B = 1$ [29].

The general characters of our model are as follows. Dynamics during t_1 resembles single-particle Rabi oscillations of particles between two chains $U_1 = e^{-iH_1 t_1/\hbar}$, i.e. $U_1^\dagger a_j^\dagger U_1 = i\cos(\varepsilon) b_j^\dagger - \sin(\varepsilon) a_j^\dagger$, and $U_1^\dagger b_j^\dagger U_1 = i\cos(\varepsilon) a_j^\dagger - \sin(\varepsilon) b_j^\dagger$. During t_2, each chain is experiencing nearest-neighbor interactions separately. Define the physical observable as the density polarization $P(t)$ between two chains,

$$P(t) = \frac{1}{L} \sum_i \langle \psi(t) | \hat{P}_i | \psi(t) \rangle, \qquad \hat{P}_i = a_i^\dagger a_i - b_i^\dagger b_i. \tag{5}$$

When $\varepsilon = 0$, its periodicity is strictly $2T$ regardless of H_2. But the period of $P(t)$ is unstable against perturbations ε to the "Rabi frequency": see the example of $J = U = 0$ in Fig. 1(b). The essential feature is that the dynamics during t_2, though keeping $P(t)$ unchanged, functions as a many-body synchronizer for the $2T$ periodicity of $P(t)$ and rigidifies the temporal ordering, as we shall see.

Time crystal signatures—We first seek for solutions in a large system using density-matrix-renormalization-group (DMRG) method. Remarkably, time crystal behaviors show up in a parameter *region* where the interaction strength U is large enough (in units of $\hbar/t_2$) and J/U is small, completely without disorder or fine-tuning. Two examples with different $U = 0.5$ and 0.7 for $J = 0.2, \Delta = 0.1$ are presented in Fig. 1(b) for the system size $L = 80$ on each chain. Here we consider hard-core bosons, with the initial state that one of the two legs is fully occupied, i.e. $|\psi_i\rangle = \prod_i a_i^\dagger |0\rangle$. When the "Rabi frequency" is perturbed by $\varepsilon = 0.12$, the oscillation frequency is indeed locked to $2T$. In supplementary materials we checked the longer time behavior for a smaller system ($L = 20$) using DMRG, which shows that the amplitudes cease to decay around $t/T \in [35, 45]$ and remain almost a constant. We have also checked that a slight variation of Hamiltonian parameters or the initial state does not change the $2T$ periodicity [29]. Thus, conditions (A) (B) are both met.

To further understand the DMRG result and to access late time behaviors, we next turn to exact diagonalization for the same initial state with periodic boundary condition. A dramatic contrast for systems in and out of the time crystal regime is found in their finite-size scalings. Starting from isolated Rabi oscillators $H_2 = 0$, a chaotic regime is reached immediately upon turning on weak interactions U, see Fig. 2(a1). After an initial period $t/T \approx 10$, the many-body physics sets in and the oscillation becomes non-universal for different L. Especially, for weak drive $J' = 0.22$, the oscillation amplitude decays for larger L, signifying a thermalizaing behavior. However, for drivings near $J' = \pi/2 + \epsilon$, further increasing interaction strength U leads to a time-crystal regime with fixed

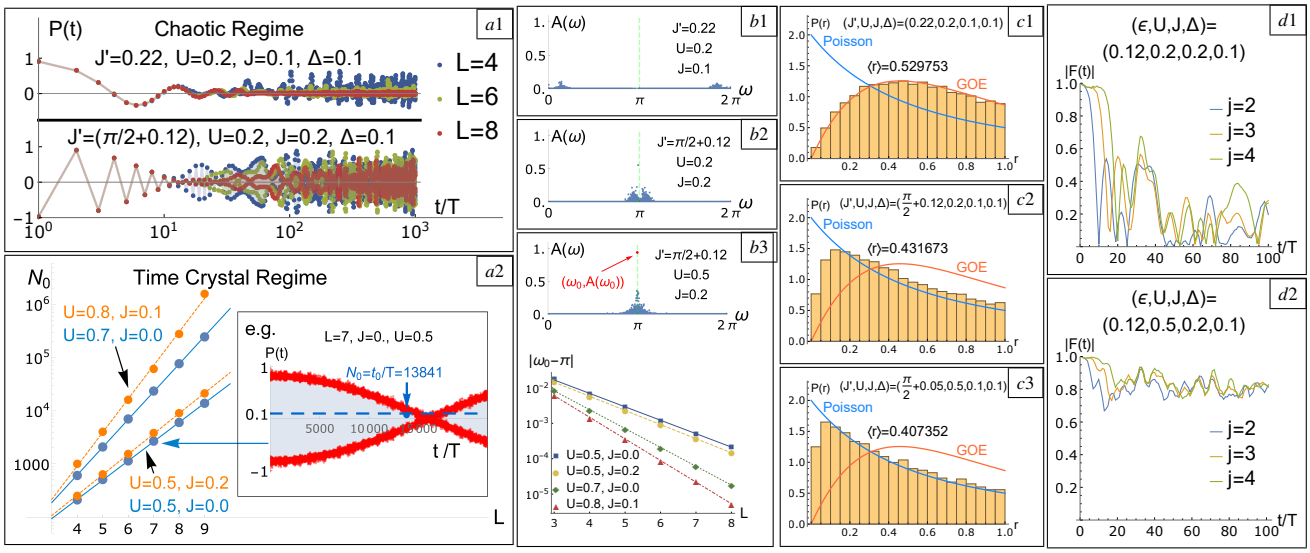

FIG. 2. **(a1)-(a2):** Histogram of $P(t)$. In time-crystal regime, $P(t)$ shows an envelope modulation for the amplitude of $2T$-periodic oscillations. The modulation length $N_0 \equiv t_0/T$ (set by $P(t_0)$ decreasing below 10% of the initial value) scales exponentially with system size. **(b1)-(b3):** Spectral weight $A(\omega)$ for temporal correlation functions, where ω carries the unit $1/T$. (We plotted $L = 6$ for example, and $\Delta = 0.1$). **(c1)-(c3):** Distribution of level spacing ratios ($L = 9$). It crosses from a GOE type deep in thermalizing regime (c1) to the Poisson limit in time crystal regime (c3). **(d1)-(d2):** OTOC with site $i = 1$ and for different sites j's. The system size is $L = 7$ with periodic boundary condition. The initial state is that one of the two chains is fully occupied.

period-$2T$ oscillations, consistent with DMRG results. For much later time, the oscillation amplitude shows an overall envelope shape (Fig. 2(a2) inset). This is because 1) the finite size effects lead to a tiny deviation of oscillation period from $2T$ [4], and 2) the oscillation amplitudes are only plotted at stroboscopic time [29]. As expected, the envelope's length expands exponentially with increasing system size (Fig. 2), indicating an exact $2T$ periodic oscillation in the thermodynamic limit and fulfilling the requirement (C).

The real-time evolution can only be performed up to a finite time span, and one may wonder whether different characters would show up in the next moment. Thus, as complements, we consider the correlation function in frequency domain [39]:

$$C(\omega) = \sum_{N=-\infty}^{\infty} \frac{e^{-i\omega NT}}{2\pi} \sum_n \langle\omega_n|\hat{\mathcal{P}}(NT)\hat{\mathcal{P}}(0)|\omega_n\rangle \qquad (6)$$

$$= \sum_{mn} \delta(\omega - \omega_{mn})A(\omega_{mn}). \qquad (7)$$

Here $\hat{\mathcal{P}} = \frac{1}{L}\sum_i \hat{P}_i$, $U_F|\omega_m\rangle = e^{i\omega_m T}|\omega_m\rangle$, and the spectral function $A(\omega_{mn}) = |\langle\omega_m|\hat{\mathcal{P}}|\omega_n\rangle|^2$, $\omega_{mn} = \omega_m - \omega_n$. We emphasize that a direct calculation of spectral function $A(\omega_{mn})$ at arbitrary Floquet eigenstates gives us *infinite* time response characters to arbitrary initial states. The time-crystal phase is highlighted by a strong peak of $A(\omega_0)$ at $\omega_0 T = \pi$ (Fig. 2(b3)) corresponding to $2T$ periodic motions of $P(t)$, compared with no or weak peaks in other regimes (Fig. 2(b1)-(b2)). For finite-size systems, the shrinking deviation $|\omega_0 T - \pi| \sim e^{-\alpha L}$ (Fig. 2(b3)) corresponds to the expanding modulation length N_0 for $P(t)$.

Finally, let us summarize the effects of various parameters. Interaction U serves to restore the $2T$ periodic oscillation perturbed by ε. The intrachain tunneling J helps restore the major oscillation frequency to π/T, but also enhances the amplitude of oscillations at other frequencies. [40] This is why J/U needs to be small. Finally, Δ is essentially introduced to break the Hamiltonian integrability at the special limit $J = 0, n_i^A + n_i^B = 1$ [29]. In practice, the system is very insensitive to the change of Δ so it is set to a fixed value throughout this work.

Emergent Floquet-integrability—The coupling between two chains H_1 breaks the integrability of H_2, and the linear combinations $\alpha H_1 + \beta H_2$ should exhibit thermalizing behaviors in late-time dynamics if localization is absent. Then, how do we understand the non-trivial dynamics in the time-crystal regime? The key point is that when the system is under strong drive, i.e. the Hamiltonian parameters are no longer much smaller than Floquet driving frequencies, the Magnus expansion of U_F is no longer dominated by the linear terms of static Hamiltonians, and it turns out that emergent Floquet integrability shows up in the time crystal regime as a property of U_F.

We first look at level statistics as a diagnostics of integrability [41]. Arrange the Floquet quasi-energies $\alpha_m \in (0, 2\pi)$: $U_F|\alpha_m\rangle = e^{i\alpha_m}|\alpha_m\rangle$ such that $\alpha_{m+1} > \alpha_m$, define the level spacings $\delta_m = \alpha_{m+1} - \alpha_m$ and further the ratios $r_n = \max(\delta_m, \delta_{m+1})/\min(\delta_m, \delta_{m+1})$, we typically end up with two distributions of r_n with probability $P(r_n)$. In the integrable limit, such as in MBL systems, we expect a Poisson distribution $P(r) = 2/(1+r)^2$ with mean values $\langle r\rangle \approx 0.386$. Contrarily for thermalizing systems, level repulsion gives a Gaussian orthogonal ensemble (GOE) for $P(r) = (27/4)(r+r^2)/(1+r+$

$r^2)^{5/2}$ with the mean value $\langle r\rangle \approx 0.536$. From Fig. 2(c1)-(c3), we see that as one goes from thermalizing regime (c1),(c2) to deep in the time-crystal regime (c3), the distribution gradually crosses from the GOE type to the Poisson limit.

To further understand the emergent integral of motion, we compute the out-of-time-order correlators (OTOC),

$$F(t) = \frac{\langle W_i^\dagger(t)V_j^\dagger(0)W_i(t)V_j(0)\rangle}{\langle W_i^\dagger(t)W_i(t)\rangle\langle V^\dagger V\rangle}. \tag{8}$$

Here i, j are site indices, and operators W_i, V_j are both chosen as local density polarization P_i, P_j, for reasons specified later. The average is taken on the state of interest, i.e. the initial state. Such a correlator has the intriguing property of quantifying quantum chaos, and has been used extensively in recent works ranging from gravity theories [42] to quantum many-body systems [43–45]. Several experimental measurements [46] have also been performed recently.

For isolated Rabi oscillators with $H_2 = 0$, $W_i(t)$ remains local and commutes with $V_{j\neq i}$ for all time, giving a constant $|F(t)|$. In contrast, OTOC in thermalizing systems should decay to and remain a small value [44]. But if the system possesses integrals of motion with W_i, V_j having large overlap with them, $F(t)$ would remain close to unity. Accordingly, we find a sharp contrast of OTOC in and out of the time-crystal phases, as shown in Fig. 2(d1) and (d2) respectively. The fact that $|F(t)|$ for P_i remains a large value prompts us to suggest the possible form for emergent Floquet-integral of motion $I^\alpha = \sum_i k_i^\alpha \hat{P}_i$: $\{U_F, \hat{I}^\alpha\}_+ = O(e^{-L}) \xrightarrow{L\to\infty} 0$, when the parameters are within time-crystal regime, where $\hat{P}_i$ is defined in Eq. (5). As we do not have localizations, the configuration for the proportionality coefficients $\{k_i^\alpha \in \mathbb{C}\}$ can be extended in space.

Two caveats are in order. First, the integrals of motion in our system may not be complete, as can be reflected in the imperfect Poisson distribution in Fig. 2(c3) and an irregular pattern of $\langle r\rangle$ when system sizes change. This resembles the "partial thermalization" as in mobility edge of MBL [47, 48] or in quantum disentangled liquids [49–51]. Second, the characters we show differ from the typical description of "pre-thermal time crystals" in Ref. [6], where oscillations cease to exist within fixed time regardless of system size and a longer thermalization time relies on weaker interactions. However, our time crystal phase requires strong interactions, and the temporal correlator in Fig. 2(b3) with a dominant peak clearly dictates persisting oscillations to infinite time, as one can verify that the same histogram in the inset of Fig. 2(a2) repeats with modulation periods N_0.

Experimental realization and generality—Since the time-crystal phase does not rely on the integrability of static Hamiltonians, we expect such phases to persist when the models in Eqs. (2)-(3) are generalized. This is verified by the following results for experimental proposals using dipolar gases or alkaline-earth atoms with spin-SU(N) symmetry.

Dipolar atoms [52–54] or molecules [55–59] have been successfully trapped in current cold atom experiments. In our case, the interaction within each chain can be written as [29]

$$V_{\rm dip} = \sum_{ij}\left(U_{\rm dip}/x_{ij}^3\right)(n_i^A n_j^A + n_i^B n_j^B) \tag{9}$$

where x_{ij} is the distance between lattice sites i, j along a chain, and $U_{\rm dip}$ is the interaction strength. This term replaces the nearest-neighbor interaction proportional to U in Eq. (3). In particular, one can polarize the dipolar gases along suitable directions by electric fields such that there is vanishingly small interaction between two chains [29].

Alternatively, using SU(N) fermions [60, 62–64], one can engineer an "infinite-ranged" interaction

$$V_{\rm SU(N)} = U\sum_{m<m'}(n_m^A n_{m'}^A + n_m^B n_{m'}^B), \tag{10}$$

where the particle at each "site" m interacts with all particles at other "sites" m'. Here we have exploited the concept of "synthetic dimensions" [60, 61] where one uses the internal degree of freedom, i.e. spins $m = -S, -S+1, \ldots, S$, to play the role of different lattice sites. For atom species trapped in current experiments, the spin S can be $9/2$ for ^{87}Sr [62, 63], or $5/2$ for ^{131}Yb [60, 64]. The SU(N) particle gains its name as the interaction (10) among different spin species preserve the SU(N) symmetry. One therefore only needs a tight double-well potential accommodating totally $N = (2S+1)$ particles in its lowest orbital state if we have half-filling in the initial state.

We refer the readers to Supplemental Material for details regarding lattice set-up, quench process, and parameter estimations. Here we present a phase diagram for each of these two cases in Fig. 3(a) and (b) respectively. We clearly see that time crystal phases are stabilized by strong interactions.

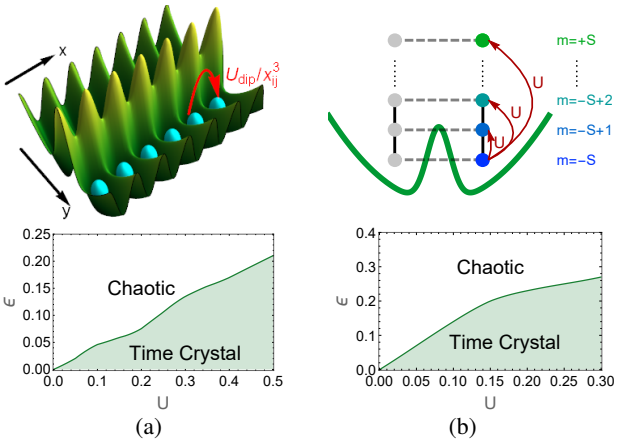

FIG. 3. The experimental set-ups and phase diagrams for fermionic (a) dipolar gases ($J = 0.4U$, $L \to \infty$ extrapolated using $L = 4, 5, 6, 7$ data [29]) and (b) SU(N) particles ($J = 0, L = 10$ for ^{173}Yb) with open boundary conditions. The phase boundary is set to that the "envelope" height of the oscillation, as shown in the inset of the Fig. 2(a), remains above (or below) 50% for the time crystal (or chaotic) phase during the first 200 periods.

Conclusion — We have shown through explicit models that a stable time crystal phase exists without the need for fine tun-

ing or localization by disorder. The exponential scaling of the modulation length with respect to system size, together with the dependence on *strong* interaction strength, imply that the clean-Floquet time crystal phase is different from the usual pre-thermal state [6]. The existence of such a phase is of genuine dynamical origin, where certain integrals of motion emerge in the Floquet operator instead of being in the static Hamiltonian. Therefore, it points to a tantalizing possibility of using dynamical process to preserve quantum information. Finally, as being confirmed in the experimental proposals, the time-crystal behavior is not restricted to a specific model. Thus, it is intriguing to generalize the present discussions to systems with more complexity in parallel to usual spatial crystals. Studying time crystals in various clean systems will surely yield new principles and phenomena of non-equilibrium nature.

Additional notes—Around the time our preprint appeared online, we noted two independent works for Floquet systems with 3D dipolar [66] and 1D [67] infinite-ranged interactions, both of which also found time crystal signatures.

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* phys.huang.biao@gmail.com
† wvliu@pitt.edu

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