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We propose a trial wave function for the quantum Hall bilayer system of total filling factor $\nu_T = 1$ at a layer distance d to magnetic length ℓ ratio $d/\ell = \kappa_{c1} \approx 1.1$, where the lowest charged excitation is known to have a level crossing. The wave function has two-particle correlations which fit well with those in previous numerical studies, and can be viewed as a Bose-Einstein condensate of free excitons formed by composite bosons and anti-composite bosons in different layers. We show the free nature of these excitons indicating an emergent SU(2) symmetry for the composite bosons at $d/\ell = \kappa_{c1}$, which leads to the level crossing in low-lying charged excitations. We further show the overlap between the trial wave function and the ground state of a small size exact diagonalization is peaked near $d/\ell = \kappa_{c1}$, which supports our theory.

The two-dimensional (2D) electron quantum Hall bilayer exhibits a rich physics [1, 2]. When the system has a total filling factor $\nu_T = 1$ and a negligible interlayer hopping, an exciton superfluid phase arises at small layer distances d [3], which exhibits a perfect Coulomb drag effect [4–7] and a greatly enhanced interlayer tunneling [8]. The electrons and holes in the lowest Landau levels (LLLs) of different layers are bounded into excitons, and form an exciton superfluid which has a U(1) symmetry breaking and a charge gap [9–17]. As the layer distance d over the magnetic length ℓ exceeds a critical value $d/\ell > \kappa_{c2} \approx 1.8$, the system enters a compressible phase without superfluidity if the bilayer is symmetric [3], which is equivalent to two copies of the $\nu = 1/2$ composite fermi liquid (CFL) as $d \to \infty$ [18–20].

Constructing trial wave functions for quantum Hall systems proves to be a powerful and successful method [1, 2, 21–28]. In the limit $d \to 0$, the ground state of the $\nu_T = 1$ bilayer is known to be the Halperin (111) state [9, 16, 24]. Meanwhile, the ground state of the system at intermediate layer distances $0 < d/\ell < \kappa_{c2}$ is still unsettled [29–40], which is a major obstacle in understanding the transition from exciton superfluid to CFL. Numerical calculations are employed to reveal the nature of the ground state [37–44], and a charge gap closing is indeed observed at $d/\ell \approx 1.8$. In particular, the calculations of both density matrix renormalization group (DMRG) and exact diagonalization (ED) have identified a level crossing between the first and second charged excitations at a layer distance $d/\ell = \kappa_{c1} \approx 1.1$ [39, 40], which is not yet well understood.

In this letter, we propose a trial wave function for the exciton superfluid of the $\nu_T = 1$ bilayer at the level crossing layer distance $d/\ell = \kappa_{c1}$, and show its two-particle correlations fit well with the previous DMRG results [39]. The wave function can be viewed as a Bose-Einstein condensate (BEC) of free excitons formed by composite bosons (CBs) and anti-CBs in different layers, based

on which we argue there is an emergent SU(2) symmetry for the CBs at $d/\ell = \kappa_{c1}$ that ensures the level crossing. In a crude estimation we obtain $\kappa_{c1} \approx \ln 4 \approx 1.4$, which is comparable to the numerical result $\kappa_{c1} = 1.1$. Lastly, we show the trial wave function has a high overlap with ground state of a small size ED calculation at $d/\ell = \kappa_{c1}$.

we shall take a simplification that all the electrons in the $\nu_T = 1$ bilayer are in the LLL of each layer, which is legitimate in the situation $e^2/\epsilon\ell \lesssim \hbar\omega_c$ [23] as is true in the experiments [3–8]. Here e is the electron charge, ϵ is the dielectric function, $\ell = \sqrt{\hbar c/eB}$ is the magnetic length, $\omega_c = eB/m_e c$ is the cyclotron frequency, B is the magnetic field, and m_e is the effective electron mass. We also assume the interlayer hopping is zero. Since the LLL has no kinetic energy, the energy of the system is solely determined by the Coulomb interactions, which take the form $V_{11}(q) = V_{22}(q) = 2\pi e^2 e^{-q^2\ell^2/2}/\epsilon q$ and $V_{12}(q) = 2\pi e^2 e^{-qd-q^2\ell^2/2}/\epsilon q$ in the Fourier space when projected into the LLL [16, 39, 40], where q is the transferred momentum, and $V_{ij}(q)$ is the interaction between two electrons in layers i and j. It is conventional to define layers 1 and 2 as pseudospins $s_z = +1/2$ and $s_z = -1/2$, respectively, so the Hamiltonian of the system has a U(1) pseudospin rotational symmetry about the z axis.

We first briefly review the ground state of $\nu_T = \nu_1 + \nu_2 = 1$ bilayer in the limit $d \to 0$, namely, the Halperin (111) state:

$$\Psi_{111} = \mu(z, w) \prod_{i<j}^{N} (z_i - z_j) \prod_{k<l}^{M} (w_k - w_l) \prod_{i,k}^{N,M} (z_i - w_k), \quad (1)$$

where $z_i = x_i^{(1)} + iy_i^{(1)}$ and $w_k = x_k^{(2)} + iy_k^{(2)}$ are the complex coordinates of the i-th of the N electrons in layer 1 and the k-th of the M electrons in layer 2, respectively, and $\mu(z, w) = \prod_i e^{-|z_i|^2/4\ell^2} \prod_k e^{-|w_k|^2/4\ell^2}$ is the Landau level Gaussian factor in the symmetric gauge. The total number of electrons $N + M$ is equal to the Landau level degeneracy, while $\nu_1 = \frac{N}{N+M}$ and

$\nu_2 = \frac{M}{N+M}$ are the filling factors of layers 1 and 2, respectively. Note that when $d \to 0$, the Coulomb interactions $V_{11}(q) = V_{12}(q) = V_{22}(q)$ become independent of the layer indices, so the pseudospin rotational symmetry of the system is enhanced to SU(2). As a result, the ground state should reduce to a $\nu = 1$ monolayer integer quantum Hall (IQH) state if the layer indices are omitted, which indeed holds for state Ψ_{111}.

Unlike most bilayer quantum Hall states, the filling factor difference $\nu_1 - \nu_2$ in Ψ_{111} is not fixed, which also holds in the entire exciton superfluid phase. [39, 40]. This leads to a U(1) symmetry breaking order parameter $\Delta(\mathbf{r}) = \langle c_1^\dagger(\mathbf{r})c_2(\mathbf{r})\rangle \neq 0$ and a charge neutral Goldstone mode, where $c_j(\mathbf{r})$ is the electron annihilation operator at position $\mathbf{r}$ in layer j [9, 14, 45]. Note that $\Delta(\mathbf{r})$ can be viewed as the pairing amplitude of an electron and a hole in different layers, so it marks the occurrence of an exciton superfluid. In a symmetric bilayer with $\nu_1 = \nu_2 = 1/2$, the pseudospin is polarized in the x-y plane at an angle $\arg \Delta(\mathbf{r})$ from the x axis.

The exciton superfluidity of state Ψ_{111} can be seen more clearly via a particle-hole transformation in the LLL of layer 2 [46, 47], after which the state Ψ_{111} becomes a wave function of N electrons in layer 1 and N holes in layer 2 as shown in Ref. [16]:

$$\Psi_0 = \mu(z, w) \sum_\sigma \text{sgn}(\sigma) \prod_{i=1}^N e^{z_i w_{\sigma_i}^*/2\ell^2} = \det M_{ij} \quad (2)$$

plus a full electron LLL in layer 2, where $\mu(z,w) = \prod_{i=1}^N e^{-(|z_i|^2+|w_i|^2)/4\ell^2}$ is again the Gaussian factor except that w_i now are the coordinates of holes, σ is the permutation of 1 through N with $\text{sgn}(\sigma)$ being its sign, and $\det M_{ij}$ is the determinant of the N by N matrix with elements $M_{ij} = e^{-(|z_i|^2 - 2z_i w_j^* + |w_j|^2)/4\ell^2}$. Both the electrons and holes in such a wave function are in the LLLs at a filling factor ν_1. Note that $|M_{ij}|^2 = e^{-|z_i - w_j|^2/2\ell^2}$, so one can view each M_{ij} as a bound state of the i-th electron and j-th hole, i.e., an exciton wave function. Therefore, Ψ_0 can be understood as a BEC of N *free* excitons (similar to the Slater determinant state of N free fermions). The free nature of the excitons is exactly due to the enhanced SU(2) symmetry as $d \to 0$. As shown in Fig. 1(e), the Coulomb interaction between two excitons is approximately

$$V_E(q) = V_{11} + V_{22} - 2V_{12} = 2V_{11}(q)(1 - e^{-qd}), \quad (3)$$

so the exciton interaction $V_E(q)$ vanishes as $d \to 0$ [16].

At intermediate layer distances $0 < d/\ell < \kappa_{c2}$, the excitons become interacting. Since the LLLs have no kinetic energy, the exciton superfluid is strongly correlated and barely understood. A prominent feature revealed by numerical studies is a level crossing between the lowest two charged excited states at $d/\ell = \kappa_{c1} \approx 1.1$ for $\nu_1 = \nu_2 = 1/2$ [39, 40]. Besides, unlike Ψ_{111} where

the overlap probability of interlayer electrons is zero, the ground state at $d > 0$ is shown to have a nonzero interlayer overlap probability [39], which enlarges the intralayer electron spacing and lowers the total energy when $V_{11}(q) > V_{12}(q)$.

We propose here a wave function for N electrons and N holes in layers 1 and 2 respectively at certain intermediate layer distances $0 < d/\ell < \kappa_{c2}$:

$$\Psi_{1,\alpha} = \mu(z, w) \prod_{i<j}^N (z_i - z_j)(w_i^* - w_j^*) \sum_\sigma \prod_{i=1}^N e^{\alpha z_i w_{\sigma_i}^*/2\ell^2}$$

$$= \prod_{i<j}^N (z_i - z_j)(w_i^* - w_j^*) \text{perm } M_{ij}(\alpha) \,,$$

$$(4)$$

where α is a real parameter satisfying $0 \le \alpha \le 1$, and perm $M_{ij}(\alpha)$ is the permanent [48] of a N by N matrix $M(\alpha)$ with elements $M_{ij}(\alpha) = e^{-(|z_i|^2 - 2\alpha z_i w_j^* + |w_j|^2)/4\ell^2}$. In particular, we shall show that the state $\Psi_{1,1/2}$ with $\alpha = 1/2$ is a good trial wave function for the exciton superfluid in the symmetric $\nu_T = 1$ bilayer at $d/\ell = \kappa_{c1}$, and gives an explanation for the level crossing.

The electron (hole) filling factor ν_1 of the wave function $\Psi_{1,\alpha}$ is controlled by the parameter α. To see this, we first note the matrix element $M_{ij}(\alpha)$ can be rewritten as $M_{ij}(\alpha) = e^{-[(1-\alpha)(|z_i|^2+|w_j|^2)+\alpha|z_i-w_j|^2]/4\ell^2 + i\phi_{ij}}$, where $\phi_{ij} = \alpha(z_i w_j^* - z_i^* w_j)/4i\ell^2$ is real. Ignoring the phase factor $e^{i\phi_{ij}}$ in $M_{ij}(\alpha)$, one can view the translationally invariant part $e^{-\alpha|z_i-w_j|^2/4\ell^2}$ as an exciton bound state, while regard the other part $e^{-(1-\alpha)(|z_i|^2+|w_j|^2)/4\ell^2}$ as the Gaussian factor for a residual magnetic field $(1 - \alpha)B$ felt by the electron and the hole. With a Jastrow factor of power 1 in $\Psi_{1,\alpha}$, one would expect the electrons (holes) to have the same density as that of a $\nu = 1$ IQH state in a reduced magnetic field $(1-\alpha)B$, yielding $\nu_1 = 1-\alpha$. This is verified by our small size Markov chain Monte Carlo (MCMC) calculations [45] for state $\Psi_{1,\alpha}$. As shown in Fig. 1(a), the filling factor for $N = 6, 10$ and 15 fits very well with $\nu_1 = 1 - \alpha$ as the system size N increases. Fig. 1(b) shows the electron (hole) density $\rho(r) = \langle c_1^\dagger(\mathbf{r})c_1(\mathbf{r})\rangle = \langle c_2(\mathbf{r})c_2^\dagger(\mathbf{r})\rangle$ as a function of the radius $r = |\mathbf{r}|$ in units of the fully occupied Landau level density for $N = 6$ and different α, which has a flat droplet shape with an overshoot near the edge similar to that of the Laughlin states [49, 50].

The two-particle correlations of $\Psi_{1,\alpha}$ can also be extracted out in our MCMC calculations, which are defined as $g_{ee}(r) = \langle c_1^\dagger(\mathbf{r}_1)c_1(\mathbf{r}_1)c_1^\dagger(\mathbf{r}_2)c_1(\mathbf{r}_2)\rangle/\nu_1^2$ between two electrons and $g_{eh}(r) = \langle c_1^\dagger(\mathbf{r}_1)c_1(\mathbf{r}_1)c_2(\mathbf{r}_2)c_2^\dagger(\mathbf{r}_2)\rangle/\nu_1^2$ between an electron and a hole for $r = |\mathbf{r}_1 - \mathbf{r}_2|$. The correlations for different values of α are plotted in Fig. 1(c), where the higher and lower curves are $g_{eh}(r)$ and $g_{ee}(r)$, respectively. By transforming holes in layer 2 back to electrons [45], one can

show the interlayer electron-electron correlation $g_{12}(r) = \langle c_1^\dagger(\mathbf{r}_1)c_1(\mathbf{r}_1)c_2^\dagger(\mathbf{r}_2)c_2(\mathbf{r}_2)\rangle/\nu_1\nu_2$ for $r = |\mathbf{r}_1 - \mathbf{r}_2|$ is

$$g_{12}(r) = \nu_2^{-1}[1 - \nu_1 g_{eh}(r)] . \quad (5)$$

The resulting $g_{12}(r)$ for different α are shown in the inset of Fig. 1(c). In particular, for $\alpha = 1/2$ where both ν_1 and ν_2 are $1/2$, the intralayer and interlayer correlation functions $g_{11}(r) = g_{ee}(r)$ and $g_{12}(r)$ fit remarkably well with previous DMRG results at $d/\ell = \kappa_{c_1}$ [39], as shown in Fig. 2(a). This means the energy of the state $\Psi_{1,1/2}$ is quite close to the ground state energy of the system, and thus strongly suggests $\Psi_{1,1/2}$ may be a good approximation to the true ground state at $d/\ell = \kappa_{c1}$. For $\kappa_{c1} = 1.1$, the state $\Psi_{1,1/2}$ yields a Coulomb energy per electron $E_0 = \sum_{ij} \frac{\nu_i\nu_j}{2\ell^2} \int r V_{ij}(r)[g_{ij}(r) - 1]\mathrm{d}r \approx -0.35 e^2/\epsilon\ell$ [51].

The wave function $\Psi_{1,1/2}$ can be better understood in the picture of CBs [52–55]. Here a CB (anti-CB) is defined as an electron (hole) bound with a 2π statistical flux relative to the other electrons (holes), which obeys bosonic statistics. In the basis of CBs in layer 1 and anti-CBs in layer 2, the state $\Psi_{1,1/2}$ can be rewritten as:

$$\Psi_{1,1/2}^{CB} = \mathrm{perm}\ \widetilde{M}_{ij} , \quad (6)$$

where $\widetilde{M}_{ij} = e^{-(|z_i|^2 - 2z_i w_j^* + |w_j|^2)/8\ell^2}$, while the Jastrow factor in $\Psi_{1,1/2}$ is absorbed by the 2π fluxes bound to CBs and anti-CBs [27, 52–54]. In analogy to M_{ij} in Eq. (2), $\widetilde{M}_{ij}$ is a wave function of a composite exciton formed by the i-th CB and the j-th anti-CB, where the magnetic field is reduced to $B/2$ due to their bound fluxes. Thus, the state $\Psi_{1,1/2}^{CB}$ can be viewed as a BEC of N free composite excitons. This leads us to conjecture that the level crossing point $d/\ell = \kappa_{c1}$ is exactly where the composite excitons become free. An understanding of this is as follows. As shown in Fig. 1(f), the interaction between two composite excitons is determined by the intralayer and interlayer interactions $V'_{ij}(q)$ between CBs (anti-CBs). Due to the fluxes bound to the CBs, the intralayer interaction $V'_{11}(q) = V'_{22}(q)$ is generically largely screened compared to that of electrons [56]. As a crude estimation, $V'_{11}(q)$ approximately equals to the Coulomb potential between two $e/2$ ($-e/2$) charges, namely, $V'_{11}(q) \approx V_{11}(q)/4$, since the fluxes roughly counteract one half of the gauge potential. In contrast, the interlayer interaction remains $V'_{12}(q) = V_{12}(q)$, since a CB and an anti-CB in different layers do not have a mutual statistical flux. The approximate interaction between two composite excitons is then

$$V'_E(q) = V'_{11} + V'_{22} - 2V'_{12} \approx 2V_{11}(q)\left(\frac{1}{4} - e^{-qd}\right). \quad (7)$$

If we substitute $q = 1/\ell$ into the formula as a characteristic momentum, we find $V'_E(q)$ vanishes at $d/\ell = \ln 4 \approx 1.4$, which is rather close to the numerical value $\kappa_{c1} \approx 1.1$ considering the roughness of this estimation. A precise
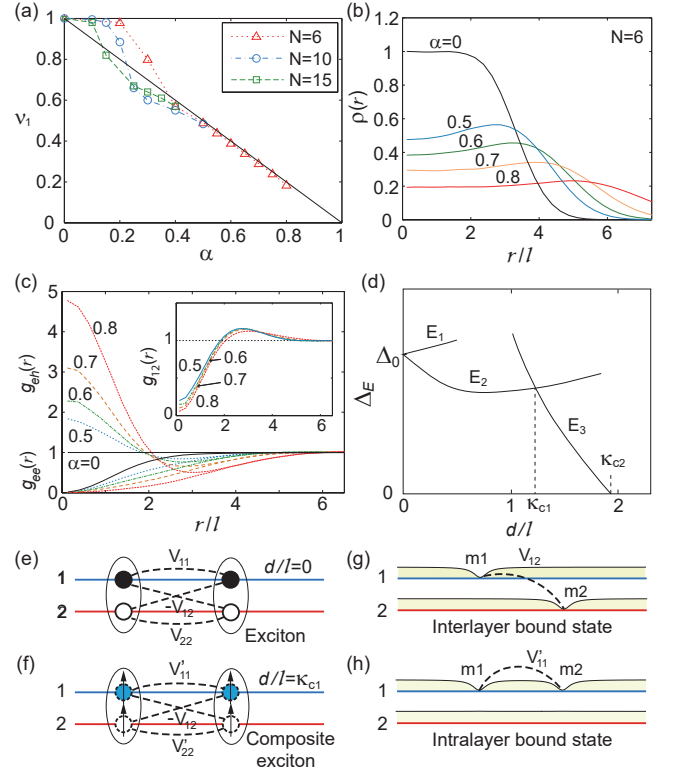

FIG. 1: (color online) (a) The filling factor ν_1 of $\Psi_{1,\alpha}$ as a function of α for size $N = 6, 10$ and 15 from MCMC, in good agreement with $\nu_1 = 1 - \alpha$. (b) The density profile $\rho(r)$ of $\Psi_{1,\alpha}$ for different α and $N = 6$. (c) The two-particle correlations $g_{ee}(r)$ and $g_{eh}(r)$ of $\Psi_{1,\alpha}$ for different α, and the interlayer correlation $g_{12}(r)$ plotted in the inset. (d) The low-lying charged excitation spectrum expected in our theory, where a level crossing occurs at both $d = 0$ and $d/\ell = \kappa_{c1}$. (e)-(f) Interaction between two excitons formed by electrons and holes (e) or CBs and anti-CBs (f). (g)-(h) Illustration of the interlayer and intralayer bound states of two CB merons, which we expect to be excitations E_2 and E_3 in (d), respectively.

determination of κ_{c1} would call for a more careful calculation of $V'_{ij}(q)$ in the future.

Similar to the case for electrons at $d = 0$, the vanishing of $V'_E(q)$ implies an emergent pseudospin SU(2) symmetry for CBs at $d/\ell = \kappa_{c1}$. In condensed matter systems, an emergent symmetry usually leads to extra degeneracies in the energy spectrum [57–60]. We claim here this emergent SU(2) symmetry is responsible for the level crossing of lowest charged excitations at $d/\ell = \kappa_{c1}$, and should yield approximate degeneracies at higher energies as well. For the same reason, we expect the level crossing to also occur at $d = 0$. This is understood as follows. The minimal charged excitation in the bilayer exciton superfluid is known to be the meron, which has an electrical charge $\pm e/2$ and pseudospin up or down in the core, and evolves into an in-plane pseudospin vortex with vorticity ± 1 away from the core [14, 61]. In particular, a

meron with a given vorticity can have either charge $e/2$ or $-e/2$, and the charge can be localized in either layer depending on the core pseudospin direction [14, 45]. A single meron has a logarithmically diverging energy with the system size, thus is not a low energy excitation. However, it is believed that the lowest charged excitation of charge $\pm e$ is a bound state of two merons with the same charge but opposite vorticities [14, 32, 55, 62, 63]. There are two kinds of such bound states (BSs) competing: one interlayer BS and two degenerate intralayer BSs (in either layer), where the charges of two merons are in different layers (Fig. 1(g)) and in the same layer (Fig. 1(h)), respectively. At $d = 0$, these two kinds of BSs are degenerate since they have identical interactions $V_{11}(q) = V_{12}(q)$. Similarly, at $d/\ell = \kappa_{c1}$, we expect the lowest charged excitations to be the interlayer and intralayer bound states of CB merons (Fig. 1(g) and (h)) [45], which also have identical interactions $V'_{11}(q) \approx V'_{12}(q)$ and are thus degenerate. As a result, we arrive at the low-lying charged excitation spectrum with a level crossing at both $d = 0$ and $d/\ell = \kappa_{c1}$ as shown in Fig. 1(d). In particular, we expect the lower excited state E_2 at small d/ℓ to adiabatically evolve from the interlayer bound state of merons to that of CB merons, while states E_1 and E_3 to be the intralayer bound states of merons and CB merons, respectively. We note the level crossing cannot be avoided, since any matrix element between the interlayer BS and the intralayer BS must involve a fractional charge $e/2$ interlayer hopping, which cannot be a local operator and must be zero in the thermodynamic limit. Our results also suggest the intralayer BSs E_3 may play a key role in the exciton superfluid-CFL transition at $d/\ell = \kappa_{c2}$.

A disadvantage of the above electron-hole formulation of $\nu_1 = \nu_2 = 1/2$ bilayer is that it is asymmetric between the two layers. As a result, when holes in layer 2 are transformed back into electrons, the wave function $\Psi_{1,1/2}$ transforms into a bilayer wave function $\Psi_{1/2}$ of N electrons per layer, which is not exactly symmetric between two layers [45]. A layer swapping yields its mirror state $\Psi^M_{1/2} = (-1)^N \Psi_{1/2}(z_i \leftrightarrow w_i)$. Numerical calculations show their overlap $|\langle\Psi_{1/2}|\Psi^M_{1/2}\rangle|^2 \propto N^{-\alpha}$ with $\alpha \approx 0.5$ [45], so the two states are orthogonal in the thermodynamic limit. However, a power law decay of overlap indicates the two states $\Psi_{1/2}$ and $\Psi^M_{1/2}$ are quite alike each other. In contrast, we find the overlap between $\Psi_{1/2}$ and the Halperin state Ψ_{111} decays exponentially as $|\langle\Psi_{1/2}|\Psi_{111}\rangle|^2 \propto e^{-gN^\beta}$ with $\beta \approx 1.5$ [45], indicating they are rather distinct states. Therefore, it is reasonable to believe that $\Psi_{1/2}$ and $\Psi^M_{1/2}$ differ only by some low-energy gapless Goldstone modes [9, 12], and are both close to the ground state at $d/\ell = \kappa_{c1}$. We then propose their symmetric superposition $\Psi^S_{1/2} = \Psi_{1/2} + \Psi^M_{1/2}$ as an improved trial wave function, which respects the mirror symmetry between two layers.

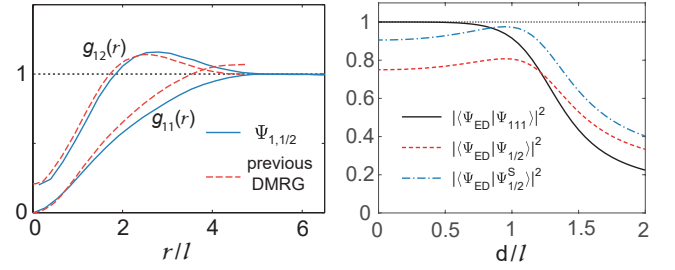To further test the validity of our theory, we run a

FIG. 2: (color online) (a) Two-particle correlations $g_{11}(r)$ and $g_{12}(r)$ of state $\Psi_{1,1/2}$ (solid blue lines) obtained on disk and those at $d/\ell = \kappa_{c1}$ reproduced from DMRG calculations on torus in Ref. [39] (dashed red lines). (b) Overlap of the ground state Ψ_{ED} from ED calculations for $N = 2$ electrons per layer with trial wavefunctions Ψ_{111}, $\Psi_{1/2}$ and $\Psi^S_{1/2}$, respectively, as a function of d/ℓ.

small toy size ED calculation for $\nu_1 = \nu_2 = 1/2$ with $N = 2$ electrons per layer at different d/ℓ, and calculate the overlap between the ED ground state Ψ_{ED} and three trial wave functions Ψ_{111}, $\Psi_{1/2}$ and $\Psi^S_{1/2}$ in the same total angular momentum sector[45]. As shown in Fig. 2(b), the overlaps of both $\Psi_{1/2}$ and $\Psi^S_{1/2}$ with Ψ_{ED} are indeed peaked at $d/\ell \approx \kappa_{c1}$ as expected in our theory, and the peak value of $|\langle\Psi_{ED}|\Psi^S_{1/2}\rangle|^2$ is as high as 0.95. In contrast, the overlap between Ψ_{111} and Ψ_{ED} monotonically decays with d/ℓ. A larger size ED calculation is desired in the future to further verify this result.

Finally, we mention that wave function $\Psi_{1,\alpha}$ can be generalized into a larger class of wave functions

$$\Psi_{m,\alpha} = \prod_{i<j}^N (z_i - z_j)^m (w_i^* - w_j^*)^m f\left[M_{ij}(\alpha)\right], \quad (8)$$

where $f\left[M_{ij}(\alpha)\right]$ is defined as $\det M_{ij}(\alpha)$ for m even, and perm $M_{ij}(\alpha)$ for m odd, with $M_{ij}(\alpha)$ defined as in Eq. (4). These states admit a similar physical picture of CBs or composite fermions [45], and may describe certain filling-imbalanced $\nu_T = 1$ bilayers or electron-hole bilayers [64, 65].

In conclusion, we have shown a trial wave function $\Psi_{1,1/2}$ shares many features with the numerical ground state of $\nu_T = 1$ quantum Hall bilayer at $d/\ell = \kappa_{c1} = 1.1$, and is likely to characterize the ground state well. The wave function implies an emergent SU(2) symmetry for CBs at $d/\ell = \kappa_{c1}$, which gives an straightforward explanation of the excited state level crossing found therein. These results suggest the $\nu_T = 1$ bilayer at intermediate d/ℓ may have an easier understanding in terms of CBs.

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