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# Quasi-phase-matched supercontinuum-generation in photonic waveguides

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Supercontinuum generation in integrated photonic waveguides is a versatile source of broadband light, and the generated spectrum is largely determined by the phase-matching conditions. Here we show that quasi-phase-matching via periodic modulations of the waveguide structure provides a useful mechanism to control the evolution of ultrafast pulses during supercontinuum generation. We experimentally demonstrate quasi-phase-matched supercontinuum to the TE<sub>20</sub> and TE<sub>00</sub> waveguide modes, which enhances the intensity of the SCG in specific spectral regions by as much as 20 dB. We utilize higher-order quasi-phase-matching (up to the 16<sup>th</sup> order) to enhance the intensity in numerous locations across the spectrum. Quasi-phase-matching adds a unique dimension to the design-space for SCG waveguides, allowing the spectrum to be engineered for specific applications.

Supercontinuum generation (SCG) is a  $\chi^{(3)}$  nonlinear process where laser pulses of relatively narrow bandwidth can be converted into a continuum with large spectral span [1–3]. SCG has numerous applications, including self-referencing frequency combs [4–6], microscopy [7], spectroscopy [8], and tomography [9]. SCG is traditionally accomplished using bulk crystals or nonlinear fiber, but recently, “photonic waveguides” (on-chip waveguides produced using nanofabrication techniques) have proven themselves as a versatile platform for SCG, offering small size, high nonlinearity, and increased control over the generated spectrum [10–19]. The spectral shape and efficiency of SCG is determined by the input pulse parameters, the nonlinearity of the material, and the refractive index of the waveguide, which determines the phase-matching conditions.

Specifically, when phase-matching between a soliton and quasi-continuous-wave (CW) light is achieved, strong enhancements of the intensity of the supercontinuum spectrum can occur in certain spectral regions. These spectral peaks are often referred to as a dispersive waves (DWs) [2, 20–22], and they are often crucial for providing sufficient spectral brightness for many applications. The soliton-DW phase-matching condition is typically satisfied by selecting a material with a favorable refractive index profile and engineering the dimensions of the waveguide to provide DWs at the desired wavelengths [15, 16]. However, there are limitations to the refractive index profile that can be achieved by adjusting only the waveguide cross-section. Quasi-phase-matching (QPM) takes a different approach, utilizing periodic modulations of the material nonlinearity to achieve an end-result similar to true phase-matching [23–26]. QPM is routinely employed to achieve high conversion efficiency for nonlinear processes such as second harmonic generation and different frequency generation, and QPM can also be used to

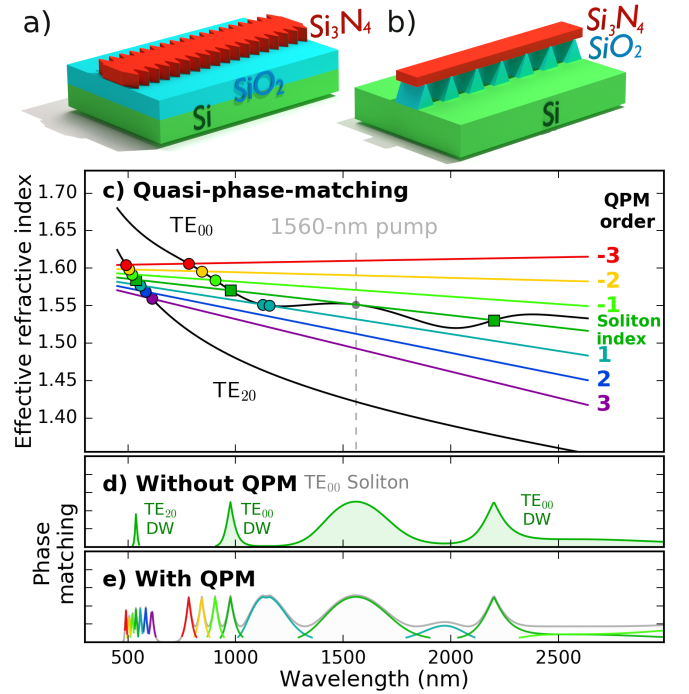


FIG. 1. a,b) Quasi-phase-matching (QPM) of supercontinuum generation in on-chip photonic waveguides can be achieved via waveguide-width modulation (a) or cladding-modulation (b). c) When the effective index of the soliton in the TE<sub>00</sub> mode (“soliton index”) intersects the effective index of a waveguide mode (black curves), phase matching to dispersive waves (DWs) is achieved (green squares) and a spectrum with several peaks (d) is generated. The periodic modulation of the waveguide can enable numerous QPM orders, both positive and negative, which can allow QPM-DW generation to the fundamental mode and to higher-order modes (circles), producing a spectrum with many peaks (e). Note: the index curvature is exaggerated to better show phase-matching.

satisfy the phase-matching conditions for DW generation [27–31].

Here we show that periodic modulations of the effective mode area can enable QPM of DW generation in photonic silicon nitride ( $\text{Si}_3\text{N}_4$ ) waveguides, enhancing the intensity of the supercontinuum in specific spectral regions determined by the modulation period. Experimentally, we utilize a sinusoidal modulation of the waveguide width to enable first-order QPM to the  $\text{TE}_{20}$  mode. Additionally, we demonstrate that periodic  $\text{SiO}_2$  under-cladding provides numerous orders of QPM to both the  $\text{TE}_{20}$  and  $\text{TE}_{00}$  modes. This quasi-phase-matched dispersive-wave (QPM-DW) scheme provides a fundamentally different approach to phase-matching in SCG, allowing light to be generated outside the normal wavelength range, and providing separate control over the group-velocity dispersion (GVD) of the waveguide (which influences soliton propagation) and DW phase-matching, capabilities that are desirable for many applications of SCG.

In the regime of anomalous GVD, the nonlinearity of the material can balance GVD and allow pulses to propagate while remaining temporally short. Such solitons can propagate indefinitely, unless perturbed [2, 3]. However, in the presence of higher-order dispersion, some wavelengths of quasi-CW light may propagate at the same phase velocity as the soliton. Light at these wavelengths can “leak out” of the soliton, in the form of DW radiation [2], which is also referred to as “resonant radiation” [20, 21] or “optical Cherenkov radiation” [22]. In the SCG process, higher-order solitons undergo soliton fission and can convert significant amounts of energy into DW radiation [2]. The phase-matching condition for DW generation (in the absence of QPM) is simply [22]

$$n(\lambda_s) + (\lambda - \lambda_s) \frac{dn}{d\lambda}(\lambda_s) + \gamma p \lambda = n(\lambda), \quad (1)$$

where  $\lambda$  is wavelength,  $\lambda_s$  is the center wavelength of the soliton,  $n$  is the effective index of the waveguide,  $\frac{dn}{d\lambda}(\lambda_s)$  is the slope of the  $n$ -versus- $\lambda$  curve evaluated at  $\lambda_s$ ,  $\gamma$  is the effective nonlinearity of the waveguide, and  $p$  is the peak power. The left side of Eq. 1 represents the effective index of the soliton while the right side represents the effective index of the DW. This equation has a simple graphical interpretation; because all wavelengths in the soliton travel with the same group velocity, the effective index of the soliton is simply a straight line (Fig. 1c). Where this line crosses the refractive index curve for any waveguide mode (black lines in Fig. 1c), DW generation is phase matched.

Periodic modulations of the waveguide change the effective area of the mode, modulating both  $\gamma$  and the GVD, enabling QPM-DW generation (see Supplemental Materials, section IV). The contribution of a modulation (with period  $\Lambda$ ) to the wavevector phase-mismatch will be  $k_{\text{QPM}} = 2\pi q/\Lambda$ , where  $q$  is the QPM order and can be any positive or negative integer. Thus, the phase-

matching condition for dispersive wave generation, in the presence of QPM is

$$\underbrace{n(\lambda_s) + (\lambda - \lambda_s) \frac{dn}{d\lambda}(\lambda_s) + \gamma p \lambda_s}_{\text{Soliton index}} = \underbrace{n(\lambda)}_{\text{DW index}} + \underbrace{\frac{2\pi q \lambda}{\Lambda}}_{\text{QPM}}. \quad (2)$$

This phase-matching condition has a similar graphical interpretation; an additional line is drawn for each QPM order, with curve-crossings indicating QPM of DWs (Fig. 1c).

In addition to satisfying the QPM condition (Eq. 2), three additional requirements must be met for efficient DW generation. (1) The QPM order  $q$  must be a strong Fourier component of the periodic modulation. (2) There must be overlap between the spatial mode of the soliton and the mode of the DW. (3) The DW must be located in a spectral region where the soliton has significant intensity, a requirement that applies regardless of the phase-matching method.

Experimentally, we explore two different approaches for QPM-DW generation in  $\text{Si}_3\text{N}_4$  waveguides: width-modulated waveguides [32] and cladding-modulated waveguides. The width-modulated  $\text{Si}_3\text{N}_4$  waveguides (Fig. 1a) are fully  $\text{SiO}_2$ -clad and have a thickness of 750 nm, a maximum width of 1500 nm, and an overall length of 15 mm. Over a 6-mm central region, the width is modulated sinusoidally from 1250 to 1500 nm. Multiple waveguides are fabricated on the same silicon chip, and each waveguide has a different width modulation period, which ranges from 5.5 to 6.5  $\mu\text{m}$ . Each cladding-modulated waveguide (Fig. 1b) consists of a 700-nm-thick  $\text{Si}_3\text{N}_4$  waveguide that is completely air-clad, except for underlying  $\text{SiO}_2$  support structures. The support structures are placed every 200  $\mu\text{m}$  along the waveguide, and each one contacts the  $\text{Si}_3\text{N}_4$  waveguide for approximately 20  $\mu\text{m}$ . For the cladding-modulated waveguides, the modulation period is kept constant, but several waveguide widths are tested, ranging from 3000 to 4000 nm.

We generate supercontinuum by coupling  $\sim 80$  fs pulses of 1560-nm light from a compact 100 MHz Er-fiber frequency comb [33]. The power is adjusted using a computer-controlled rotation-mount containing a half-waveplate, which is placed before a polarizer. The polarization is set to horizontal (i.e., parallel to the Si-wafer surface and along the long dimension of the rectangular  $\text{Si}_3\text{N}_4$  waveguide), which excites the lowest order quasi-transverse-electric ( $\text{TE}_{00}$ ) mode of the waveguide. We record the spectrum at many increments of the input power using an automated system [34] that interfaces with both the rotation mount and the optical spectrum analyzers. The waveguide modes (and their effective indices) are calculated using a vector finite-difference mode-solver [35, 36], using published refractive indices for  $\text{Si}_3\text{N}_4$  [37] and  $\text{SiO}_2$  [38]. Further experimental details are found in the SM.

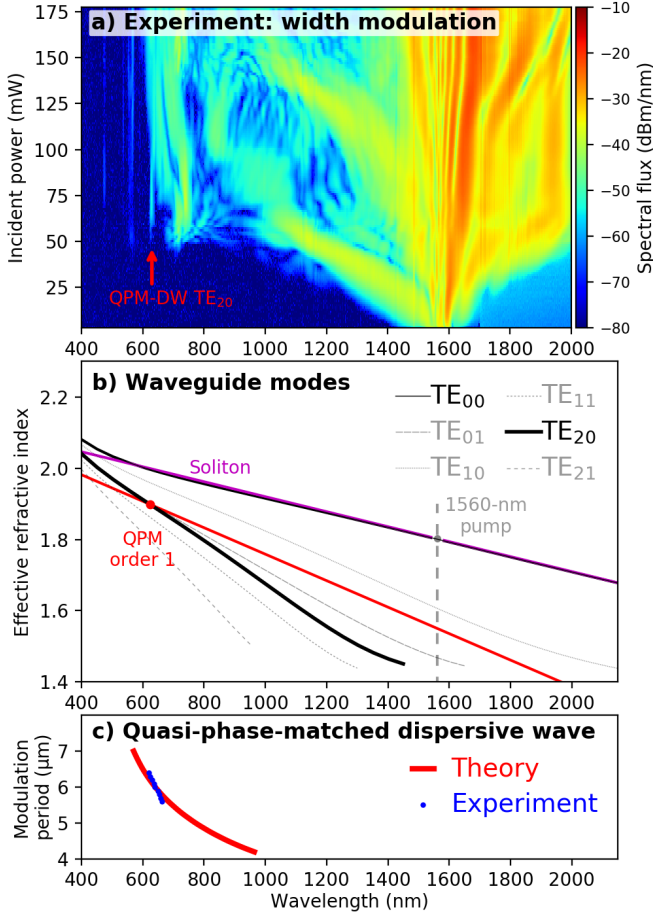


FIG. 2. a) The spectrum of supercontinuum generation from a width-modulated waveguide as a function of input power. The arrow indicates the quasi-phase-matched dispersive wave (QPM-DW) to the TE<sub>20</sub> mode. b) The effective refractive index of various modes of the waveguide as a function of wavelength. When the index of the soliton including a first order grating effect from the 6.2- $\mu\text{m}$  width-modulation (red line) crosses the TE<sub>20</sub> mode, a QPM-DW is generated. c) The calculated spectral location of the TE<sub>20</sub> QPM-DW as a function of the width-modulation period is in agreement with experimental results. The slight difference in slope may arise from irregularities in the dimensions of the waveguides.

For the width-modulated waveguides, a narrow peak appears in the spectrum in the 630-nm region (Fig. 2a), and the location of this peak changes with the width-modulation period (Fig. 2c). By calculating the refractive index of the higher order modes of the waveguide, and including the QPM effect from the periodic width modulation (Fig. 2b and Eq. 2), we find the QPM-DW generation to the TE<sub>20</sub> mode is a likely mechanism for the appearance of this peak (Fig. 2c). The preference for QPM-DW generation to the TE<sub>20</sub> mode is a result of the modal overlap [3, 39] (Fig. 3) between the TE<sub>20</sub> mode at the DW wavelength ( $\sim 630$  nm) and the TE<sub>00</sub> mode at the soliton wavelength (1560 nm). In general, modes that are symmetric in both the vertical and horizontal (such

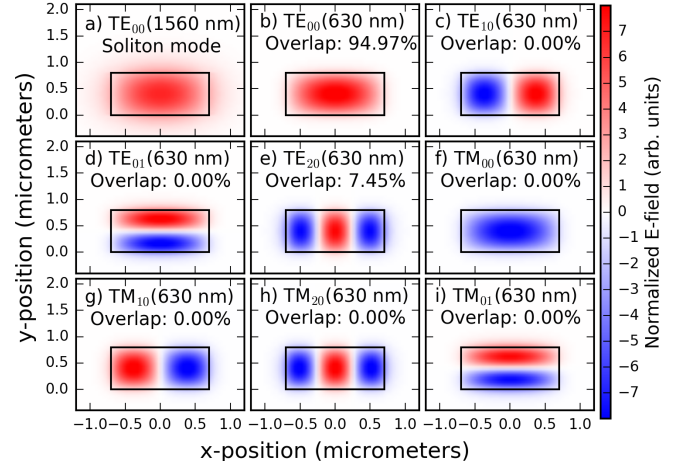


FIG. 3. Electric field profiles for the waveguide modes for a fully SiO<sub>2</sub>-clad Si<sub>3</sub>N<sub>4</sub> waveguide. a) The TE<sub>00</sub> mode at 1560 nm, which is the expected mode of the soliton. b-i) The electric field for various higher order modes at 630 nm, which is the approximate wavelength for the QPM-DW observed for the width-modulated waveguides. The result of the overlap integral of each mode with the TE<sub>00</sub> mode at 1560 nm is listed. Only the TE<sub>00</sub> and TE<sub>20</sub> modes have overlap integrals that are not vanishingly small. Note: for TM modes,  $E_y$  is shown, while  $E_x$  is shown for TE modes.

as the TE<sub>20</sub> mode) will have much higher overlap to the fundamental mode than antisymmetric modes (see SM, section III), and are consequently the most commonly used for model phase-matching schemes [40].

For the cladding-modulated waveguides, many QPM-DW peaks are seen in the supercontinuum spectrum (Fig. 4a). In some cases, the enhancement in the spectral intensity is as high as 20 dB. Similarly to the width-modulated waveguides, an analysis of the refractive index profile indicates that the TE<sub>20</sub> mode is responsible for the QPM-DW generation (Fig. 4). Interestingly, peaks are observed corresponding to both odd- and even-order QPM, and effects up to the 16<sup>th</sup> QPM order are detectable. This situation differs from the preference for low, odd-ordered QPM effects in typical QPM materials (such as periodically poled lithium niobate, PPLN [26]), which usually employ a 50-percent duty-cycle modulation. In contrast, the cladding-modulated Si<sub>3</sub>N<sub>4</sub> waveguides have short regions of oxide cladding, followed by long regions of fully air-clad Si<sub>3</sub>N<sub>4</sub>. This high-duty-cycle square wave is composed of both even- and odd-order harmonics, and consequently provides both even- and odd-order QPM. The simulated QPM-DW positions are in good agreement with the experiment for all grating orders and waveguide widths. Importantly, we see that QPM can still produce strong DWs with QPM orders of 8 or more, indicating that QPM for strongly phase-mismatched processes could be achieved with higher-order QPM instead of short modulation periods, potentially avoiding fabrication difficulties and scattering

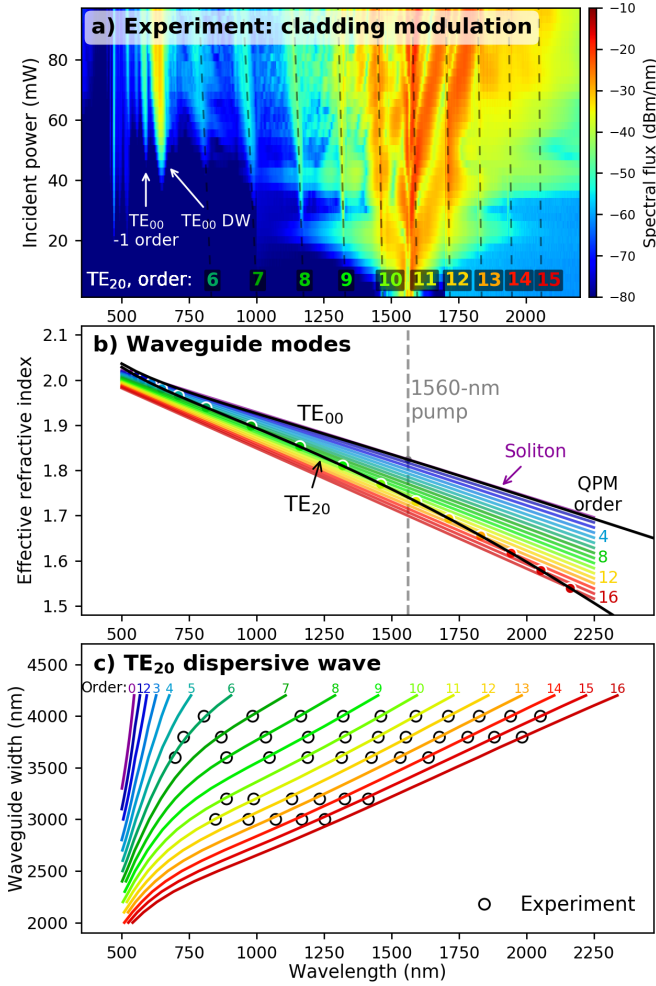


FIG. 4. a) Supercontinuum generation from a 4000-nm-width cladding-modulated waveguide, showing many QPM-DW peaks resulting from QPM orders 6-to-15 to the TE<sub>20</sub> mode. The locations of the QPM-DWs predicted by theory (dashed lines) agree with the experiment. DWs corresponding to phase-matching to the soliton and to -1 order QPM to the TE<sub>00</sub> mode are indicated with arrows. b) The effective index of the TE<sub>00</sub> and TE<sub>20</sub> modes, compared with the effective index of the soliton for various QPM orders (colorful lines). The dots indicate the location of the QPM-DW in the TE<sub>20</sub> mode. c) Calculations indicate that the locations of the QPM-DWs change as a function of waveguide width, in agreement with experiment. Note: the sharp peaks in the 530-nm region of (a) are a result of third-harmonic generation to higher-order spatial modes [17].

loss. We also observe QPM-DW generation to the TE<sub>00</sub> mode (Fig. 4a), which is reproduced by numerical simulations using the nonlinear schrödinger equation [41–43] (See Supplementary Material, Fig. S2).

This is the first demonstration of QPM to produce DWs in on-chip waveguides, but it is interesting to note that the QPM-DWs have been observed in a variety of situations. Indeed, the “Kelly sidebands” [44] seen in laser cavities and sidebands seen during soliton propaga-

tion in long-distance fiber links [45] are both examples of QPM-DWs. QPM has also been demonstrated for both modulation-instability as well as soliton-DW phase-matching using width-oscillating fibers [27, 29, 31] and for fiber-Bragg gratings [46–50]. QPM has also been seen in Kerr frequency comb generation [51]. On-chip waveguides provide a powerful new platform for QPM-DW generation, offering straightforward dispersion engineering, access to a range of modulation periods, scalable fabrication, and the ability to access well-defined higher-order modes.

In this first demonstration, the spectral brightness of the QPM-DWs was limited by several factors. First, most of the QPM-DWs were generated in the TE<sub>20</sub> mode, which doesn’t have optimal overlap with the TE<sub>00</sub> mode. Indeed, in the case where a -1-order QPM-DW is generated in the TE<sub>00</sub> mode (Fig. 4a), the intensity of the light is higher. Second, for the cladding-modulated waveguides, we rely on a QPM structure with a high duty cycle, which effectively spreads the available QPM efficiency over many QPM orders, sacrificing efficiency in one particular order. Third, our waveguides only made modest changes to the effective mode area, and stronger QPM could be likely achieved with a deeper width modulation or stronger change in the cladding index. Finally, the QPM-DWs are often produced far from the pump wavelength, in a spectral region where the soliton is dim. In future designs, optimized strategies for QPM-DW generation could utilize somewhat longer modulation periods, allowing QPM to the TE<sub>00</sub> mode, thereby maximizing mode-overlap and allowing the QPM-DWs to be located closer to the soliton central wavelength. Additionally, the waveguide modulation could be designed such that the efficiency of  $\pm 1$  order QPM is optimized. All of these parameters can be modeled using software that calculates the modes of the waveguide. This fact, combined with the massive scalability of lithographic processing, should allow for rapid progress in designing optimized photonic waveguides for SCG.

Currently, designers of waveguide-SCG sources work in a limited parameter space: selecting materials and selecting the dimensions of the waveguide cross section. QPM opens a new dimension in the design-space for photonic waveguides, one that is largely orthogonal to the other design dimensions. This orthogonality exists both in real-space, since the QPM-modulations exist in the light-propagation direction, but also in the waveguide-design-space, as it provides a simple vertical shift of the phase-matching conditions with no bending of the index curve (Fig. 1c). Consequently, it allows the spectral location of DWs to be modified with minimal effect on the GVD at the pump wavelength, which enables the soliton propagation conditions to be controlled separately from the DW phase-matching conditions. For example, using QPM, DWs could be produced even for purely anomalous GVD, greatly relaxing the requirements for



material dispersion and waveguide cross section. Importantly, since the GVD at the pump is known to affect the noise properties of the SCG process [2], the ability to manipulate the locations of the DWs separately from the GVD could enable SCG sources that are simultaneously broadband and low-noise. In addition, since similar phase-matching conditions apply to SCG with picosecond pulses or continuous-wave lasers [2], QPM of the SCG process is likely not restricted to the regime of femtosecond pulses.

In summary, here we demonstrated that quasi-phase-matching is a powerful tool for controlling the supercontinuum generation process in on-chip photonic waveguides. We experimentally verified that a periodic modulation of either the waveguide width or cladding can allow  $\text{Si}_3\text{N}_4$  waveguides to produce dispersive wave light at tunable spectral locations. By allowing dispersive waves to be quasi-phase-matched without significantly modifying the dispersion at the pump wavelength, this approach provides independent control over soliton compression and the spectral location of dispersive waves. Thus, quasi-phase-matching provides a new dimension in the design-space for on-chip waveguides and allows supercontinuum sources to be tailored for the specific needs of each application.

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