Microwave Properties of Superconductors Close to the Superconductor-Insulator Transition

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Microwave properties of superconductors close to SIT

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Strongly disordered pseudogapped superconductors are expected to display arbitrarily high values of kinetic inductance close to the superconductor-insulator transition (SIT) that makes them attractive for the implementation of large dissipationless inductance. We develop the theory of the collective modes in these superconductors and discuss associated dissipation at microwave frequencies. We obtain the collective mode spectra dependence on the disorder level and conclude that collective modes become a relevant source of dissipation and noise in the outer proximity of SIT.

A piece of superconductor is characterized by the phase of the order parameter, φ. Because the order parameter $\Psi = |\Psi| e^{i\varphi}$, the state of the superconductor does not change when $\varphi \to \varphi + 2\pi$ even if it is connected to other superconductors by Josephson junctions. However, for a superconductor that is also connected to others by a very long superconducting wire, the change of the phase by 2π leads to the states that are distinguishable even though the energy due to the phase variations along the wire might be vanishingly small. In this system a plethora of new physical effects becomes possible such as formation of Bloch states in the Josephson potential, current Shapiro steps, etc. All these effects require that the phase change by 2π leads to a state of the same energy but distinguishable from the original one. Quantitatively, the superconducting wire can be characterized by the energy $E = (1/2)E_L\varphi^2$ where φ is the phase difference and $E_L = \hbar^2/(e^2 L)$ and L is the effective inductance. The energy E_L should be much less than all relevant energy scales, for a typical problem this translates into $L \gtrsim 1\mu$H. Such a *superinductor* should be dissipationless, and as such it should contain no low energy modes, in particular it should not form a low frequency resonator. This limits the geometrical size of the superinductor to a few μm, for realistic thin film wire the width is limited by $w \gtrsim 20$ nm that translates into $L_\square \gtrsim 10$ nH for the inductance per unit area. The question is if such superinductors are physically possible?

An attractive candidate for superinductors is the superconductor close to the superconductor-insulator transition (quantum critical point). One expects that at the transition the superfluid stiffness $\rho_S = 0$ ($\rho_S = \hbar^2/e^2 L_\square$), so if this transition leads to an insulating state with a large gap, in the vicinity of it the superfluid stiffness can be arbitrarily small corresponding to arbitrarily large superinductances. Generally, there are two mechanisms for the destruction of the superconductivity by disorder that lead to a quantum critical point where ρ_S is exactly zero (for recent reviews see[1,2]). The first (fermionic) mechanism attributes the suppression of the superconductivity to the increase of the Coulomb interaction that results in the decrease of the attraction between electrons and their eventual depairing.[3] In this mechanism the state formed upon the destruction of the superconductor is essentially a poor conductor. This mechanism clearly does not lead to the formation of the superinductance. The alternative (bosonic) mechanism attributes superconductivity suppression to the localization of Cooper pairs that remain intact even when superconductivity is completely suppressed. The theory of the bosonic mechanism has a long history: this scenario of the superconductor-insulator transition was suggested long ago[4–7] but was not developed further until recently[2,8] when experimental data indicated it might indeed occur in InO.[9–12]

In this letter we show that as the bosonic SIT is approached the collective modes are pushed down to low energies. In BCS theory the critical temperature of the superconductor or its low energy gap does not depend on the disorder. In the simplest model of the bosonic SIT the critical temperature does not depend on the disorder until the latter exceeds some critical value. At larger values of the disorder the transition temperature decreases quickly and eventually becomes zero while single electron gap, Δ_P remains constant.[2] It is natural to associate the regime where the transition temperature depends on the disorder with the critical regime of the SIT in the bosonic model. As we show below, the collective modes are pushed to low energies even outside the critical regime. This severely limits the possible values of the kinetic inductances that can be achieved in the strongly disordered superconductors close to SIT.

Before we give the details of the model of the bosonic SIT and its low energy properties we discuss its main physical assumptions and materials in which such physics might be realized. The main assumption of the bosonic model is that Coulomb repulsion does not lead to the electron depairing. This might occur if it is screened by the electrons far from the Fermi surface. In other words, the Coulomb interaction between superconducting electrons is small due to a large effective dielectric constant of the material. Empirically, in this case one expects that superconductivity occurs against the background of the insulating $R(T)$. This is the situation in InO that displays strong insulating temperature behavior that is followed by superconductivity at very

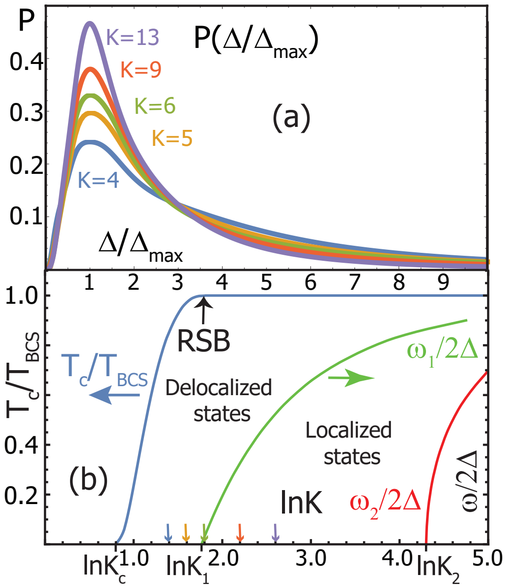

Figure 1. Schematics of the phase diagram, order parameter distribution function and collective mode spectra at low T of strongly disordered superconductors obtained from the solution of model (1) in cavity approximation. At large disorder, $K < K_1$, the distribution of the order parameter becomes anomalously broad (upper panel) and T_c is rapidly suppressed and becomes $T_c = 0$ at $K < K_c$ (lower panel). In the regime of the critical suppression of T_c, $K < K_1$, delocalized collective modes exist for all frequencies. For smaller disorder, $K_1 < K < K_2$ very low frequency modes are localized. The modes $\omega = 0$ disappear completely only $K_2 < K$. The numerical values of K shown here correspond to the interaction constant $g = 0.129$ that gives $T_{BCS} \approx 10^{-3} E_F$. Arrows indicate the values of K for which distribution is shown in the upper plot.

low T.[9–11] Large dielectric constants, $\kappa \gtrsim 10^3$ are expected in superconductors derived from high-κ host[13], $SrTiO_3$, such as $SrTiO_3$-$LaAlO_3$ interfaces[14,15] or Nb-doped $SrTi_{1-x}Nb_xO_3$[16]. In the material where Coulomb interaction is completely suppressed by large κ one expects that T_c and Δ_P initially increase with disorder due to the electron wave function localization before the effects of the suppression of Cooper pair tunneling suppress T_c and ρ_S leading to SIT while the single electron gap Δ_P remains large everywhere. Such unusual behavior (with the maximum of T_c) was indeed observed in $SrTiO_3$-$LaAlO_3$ system.[15,17,18] The increase of T_c followed by abrupt transition to the insulating state was also observed in Li_xZrNCl crystals[19] as well as in slightly oxidized aluminum wires (aka granular aluminum), in the latter the suppression of the superfluid density is not accompanied by a significant dissipation at high frequencies[20], pointing towards the bosonic mechanism. Finally, a likely candidate for this physics are superconducting semiconductors with low density of carriers, such as In-doped $Pb_zSn_{1-z}Te$[21,22] The distinguishing feature of the bosonic SIT is the different behavior of the tunneling and conductivity gaps which allows their experimental identification[23–26].

An excellent probe for the absence of the low energy modes is provided by the appearance of the coherent phase slips that are expected in the wires made from thin films with large Δ_P and small ρ_S. This was indeed observed[27] in InO wires and other strongly disordered superconductors that retain significant single electron gap: NbN and TiN.[28] However, in all these materials the quality factor remains low indicating a significant intrinsic dissipation. While expected for fermionic suppression mechanism in NbN[29–31] and TiN[32,33] that leads to the formation of the subgap states, the reason for the dissipation in InO remains unclear.

Model. We consider a simplified model of a pseudogapped superconductor where single-particle excitation are totally absent so that all electronic degrees of freedom can be represented in terms of Anderson pseudospins[34] that describe population and hopping of localized electron pairs. In other words, we assume that Δ_p is larger than all relevant energy scales of the problem. The low energy physics is described by

$$H = \sum_i 2\xi_i s_i^z - \sum_{(ij)} (J_{ij} s_i^+ s_j^- + h.c.) \qquad (1)$$

where indices i, j enumerate localized single-electron states, notation (i, j) indicates a pair of connected sites, ξ_i represent their energies, and spin-$\frac{1}{2}$ operators $\mathbf{s}_i$ are related with electron creation/annihilation operators $a^+_{i,\sigma}, a^-_{i,\sigma}$ by $2s_i^z = a^+_{i,\uparrow}a_{i,\uparrow} + a^+_{i,\downarrow}a_{i,\downarrow} - 1$, $s_i^+ = a^+_{i,\uparrow}a^+_{i,\downarrow}$ and $s_i^- = a_{i,\downarrow}a_{i,\uparrow}$. Matrix elements J_{ij} that describe hopping of localized Cooper pairs are determined by single-electron wavefunctions $\psi_i^2(\mathbf{r})$ which are supposed to be localized at relatively long spatial scale: $J_{ij} = \tilde{g} \int d^3\mathbf{r}\, \psi_i^2(\mathbf{r})\, \psi_j^2(\mathbf{r})$. In a 3D pseudogapped superconductor typical value of matrix element J_{ij} depends in a non-trivial way on the energy difference between the participating states: $\epsilon_{ij} = |\xi_i - \xi_j|$, see Ref.[2]; this dependence is due to fractal nature of nearly-critical (in terms of Anderson localization) electron eigenfunctions. An effective number Z of localized electron states $j(i)$ coupled to a given state i by hopping matrix elements J_{ij} depend on the difference between Fermi energy E_F and localization threshold E_c; increase of disorder moves E_F further into the localized part of the spectrum, decreasing Z. The model (1) neglects the effect of the long range Coulomb interaction that is inconsistent with bosonic mechanism (see Supplemental material).

Solution. In order to obtain the analytical solution we simplify further the model (1). Namely, we assume that all the sites i, j where spins $\mathbf{s}_i$ are located, belong to a Bethe lattice with coordination number $Z = K + 1$ and all nonzero couplings J_{ij} are equal and connect each spin with its Z nearest neighbors: $J_{ij} = 2g/K$, such normalization is used to allow for a well-defined limit of $K \to \infty$. Random variables ξ_i are distributed independently over sites i with the flat density $P(\xi) = \frac{1}{2}\theta(1-|\xi|)$. Within this model, increase of disorder corresponds to the decrease of K. We have shown previously[35] that within such a model a standard BCS-type phase transition takes place at very large $K \geq g\exp(1/g)$, while at lower (but still large) values of K spatial fluctuations of superconducting order parameter become large and eventually lead to an unusual kind of a quantum $T=0$ phase transition from superconducting to insulating state.

In the present Letter we concentrate upon the low-temperature properties of a superconducting state at moderately large values of K in the range $K_c < K_1 < K \leq K_2$, where $g \ll 1$ and

$$K_c = ge^{1/(eg)}; \;\; K_1 = ge^{1/2g}; \;\; K_2 = \frac{g}{4}e^{1/g} \qquad (2)$$

The region $K > K_1$ is known[35] to possess a usual BCS-like temperature-controlled superconducting transition with $T_c = T_{c0}(g) = \frac{4e^{\mathrm{C}}}{\pi}e^{-1/g}$ and low-temperature amplitude of the order parameter $\Delta(T=0,g) = 2e^{-1/g}$. At smaller K superconducting transition temperature $T_c(g,K)$ is suppressed with respect to $T_{c0}(g)$ and eventually vanishes at $K = K_c$. In the range $K_c < K < K_1$ local values Δ_i of the order parameter fluctuate strongly[35], with a "fat tail" extending to the range of Δ_i much larger than its typical value $\Delta_{\rm typ} = \exp(\langle \ln|\Delta_i|\rangle)$ that also vanishes at $K \to K_c + 0$. At larger $K > K_1$ the order parameter follows BCS relation and its spatial fluctuations are relatively weak.

Contrary to expectations at $K > K_1$ there is a whole band of delocalized low-lying collective excitation modes with a lower cutoff of their energies $\omega_1(K)$ growing upon the increase of K. Moreover, we find a band of localized collective modes with $\omega < \omega_1(K)$ which extends down to zero energy as long as $K \leq K_2$.

We start the derivation of our results by writing the action for low-ω transverse fluctuations $b_i(\omega)$ of the order parameter. These fluctuations are parameterized via phase rotation of the mean-field solution: $\Delta_i(\omega) = \Delta_i e^{i\varphi(\omega)} \equiv \Delta_i + b_i(\omega)$ with action

$$\mathcal{A} = -\sum_{i,j} b_i(\omega)\hat{J}^{-1}_{ij}b_j(\omega) + \sum_i \frac{b_i^2(\omega)\sqrt{\xi_i^2+\Delta_i^2}}{\xi_i^2+\Delta_i^2-\bar{\omega}^2} \qquad (3)$$

where $\bar{\omega} \equiv \omega/2$. At $K > K_1$, $\Delta_i \approx \Delta = 2e^{1/g}$. The action (3) is directly applicable for $\omega \ll \Delta$; at energies comparable to Δ antisymmetric coupling (neglected in (3) between transverse mode and longitudinal (gapful) mode might become relevant. Equation for the collective mode can be obtained as an extremum of the action (3) with respect to $b_i(\omega)$:

$$b_i(\omega) = \sum_j J_{ij}b_j(\omega)\eta_j(\omega) \text{ where } \eta(\omega) \equiv \frac{\sqrt{\xi_j^2+\Delta^2}}{\xi_j^2+\Delta^2-\omega^2} \qquad (4)$$

At $\omega = 0$ it is satisfied automatically for $b_i = \mathrm{const}\cdot\Delta$ due to self-consistency equations for local order parameters Δ.

Eqs. (3,4) are general, below we study eigenfunctions of (4) defined on the Bethe lattice and employ the method developed in the seminal paper[36]. To use this method we need to introduce the self-adjoint linear operator $\hat{L}$ related to (4), its matrix elements are $C_{ij} = J_{ij}\left[\eta_i(\omega)\eta_j(\omega)\right]^{1/2}$. Eqs.(4) possess delocalized solutions if the expansion for the imaginary part of the Green function $\hat{G} = \left(\hat{1} - \hat{C} + i\delta\right)^{-1}$ in powers of $\hat{C}$ is singular. This singularity is indicated by the nonzero value of typical imaginary part $(\Im G_{ii})_{\rm typ}$ of the local Green function in the limit of $\delta \to 0$. We look for the singularity threshold within the "forward path" approximation[35,37] equivalent to the "Anderson upper limit" condition[36], i.e. we neglect self-energy corrections for the Green function $G_{ii}(\omega)$. Each path over Bethe lattice that contribute to $(\Im G_{ii})$ is traversed twice (forward and backward). Therefore summation over the paths is equivalent to calculation of partition function $Z_{DP}(N)$ for the N-links directed polymer (DP) model with weights $w_{ij} = J_{ij}^2\eta_i(\omega)\eta_j(\omega)$ defined on nearest-neighbor links: $Z_{DP}(N) = \sum_P \prod_{\{l(P)\}} w_{ij}$.

We need to find an extensive part of the DP free energy $F_{DP}(N) = \ln Z_{DP}(N) \approx Nf$ at $N \to \infty$; localization threshold is determined by the condition $\langle f\rangle = 0$ where averaging is over distribution of random ξ_i. An equivalent way to calculate f is to use modified weights $\tilde{w}_{ij} = J_{ij}^2\eta_j^2$; the difference between corresponding partition functions Z_{DP} and $\tilde{Z}_{DP}$ is concentrated at the end points of each contributing path and thus does not contribute to $f = \lim_{N\to\infty}\frac{1}{N}F_{DP}(N)$.

The shortest method to calculate f is to use replica trick as described in[35,37]. It gives:

$$e^{xf(x)} \equiv K\int_0^1 d\xi \left[\frac{g}{K}\frac{\sqrt{\xi^2+\Delta^2}}{\xi^2+\Delta^2-\bar{\omega}^2}\right]^{2x} = 1, \; \frac{\partial f}{\partial x} = 0 \qquad (5)$$

Here $0 < x < 1$ is an anomalous exponent that measures the degree of Replica Symmetry Breaking (RSB) for the DP problem (within usual mean-field theory $x = 1$ and second equation in (5) is absent). The condition $\partial f/\partial x|_{x_0} = 0$ selects typical Green functions of the operator $\hat{C}$ introduced above; the first equation in (5) then leads to $f(x_0) = 0$ which indicates a critical point between localized domain for $f(x_0) < 0$ where typical Green function decays upon iterations, and extended domain, which corresponds then to $f(x_0) > 0$, where linear iterations diverge and nonlinear terms should be taken into account to get stable distribution.

At $K = K_1 = ge^{1/2g}$ and $\omega = 0$ the system of equations (5) can be solved exactly (up to relative corrections $\sim e^{-1/g} \ll 1$), with $x = 1/2$. At slightly large $K > K_1$ and low energies $\bar{\omega} = E\Delta$ we look for the solution assuming $2x - 1 \equiv \epsilon \ll 1$ and $E \ll 1$. Expanding the integral in (5) up to the 2nd order in ϵ and up to the 1st order in $\delta K = K - K_1$, we find (the term $\propto E^2$ can be omitted in the second of Eqs.(5)):

$$E^2 = \epsilon\frac{\delta K}{K_1} - \frac{\epsilon^2}{24g^2}, \quad \epsilon = 12g^2\frac{\delta K}{K_1} \tag{6}$$

leading to the result for the threshold energy in the main order expansion over $\delta K/K_1 \ll 1$:

$$\frac{\omega_1}{2\Delta} \equiv E(K) = \sqrt{6}g\frac{K-K_1}{K_1} \tag{7}$$

Eigenfunctions with $\omega > \omega_1$ are extended, while those with lower energies are localized. Numerically obtained delocalization line for $\omega_1(K)$ is shown in green in Fig. 1 for specific choice of $\Delta = 10^{-3}$, that corresponds to $g = 0.129$ and $K_1 = 5.85$.

To find the domain of existence of *localized* eigenfunctions with low energies $\omega \ll \Delta$, we use another criterion based upon (5). Namely, we look for solutions of the equation $\partial f/\partial x|_{x_0} = 0$ such that $x_0 < 1$ and $f(x_0) < 0$. The condition $x_0 < 1$ guarantees RSB that implies the different behavior of typical and average of Green functions. Namely, in the limit in the limit of $\delta \to 0$ the average imaginary part of the Green function has a finite value which implies that the density of states is non-zero in this regime. The condition $f(x_0) < 0$ implies that the wave function decreases, so this regime corresponds to the localized states. This band of localized state ends when x_0 coincides with unity: at this point typical average of the imaginary part of the Green function $\langle\Im G(\omega)\rangle_{typ}$ becomes equal to the simple average, $\langle\Im G(\omega)\rangle = \pi\rho(\omega)$. Because at the same time $f(x_0 = 1) < 0$, $\langle\Im G(\omega)\rangle$ decays upon iterations over the Bethe lattice, and $\rho(\omega) = 0$ at the stationary point of these iterations. Therefore, the boundary of the parameter region with $\rho(\omega) > 0$ is given by the solution of the equation $\partial f/\partial x|_{x_0=1} = 0$, where $f(x) \equiv f(x, \omega, K)$ is defined in (5). At $\omega = 0$ a straightforward calculation leads to the result (2); in deriving it we used the equality $\int_0^\infty \frac{dt}{\cosh t}\ln\cosh t = \frac{\pi}{2}\ln 2$. At $\omega > 0$ the same procedure provides the dependence of the spectrum boundary ω_2 on K in the region $\omega \ll \Delta$:

$$K_2(\omega) = K_2\frac{\Delta}{\sqrt{\Delta^2 - \bar{\omega}^2}} \approx K_2\left[1 + \frac{1}{2}\left(\frac{\omega}{2\Delta}\right)^2\right] \tag{8}$$

Numerical solution of the equation $\partial f/\partial x|_{x_0=1} = 0$ gives the red line in Fig. 1b. Qualitatively, the appearance of K_2 as one of characteristic value for coordination number $Z = K + 1$ in our model can be understood by noticing that at $K \gg K_2$ the total number of neighbors in which local energies $\xi_i \sim \Delta$ becomes large, so at these K the system becomes similar to conventional Ginzburg-Landau superconductor.

Experimentally observable properties. The spectrum shown in Fig. 1b translates into microwave properties of the superconductors. In the vicinity of the transition the spectrum of delocalized collective modes extends to zero frequency. Even for $K > K_1$, at which the critical temperature of the superconductor does not experience the suppression due to the quantum critical point, the low energy modes are delocalized at relatively low frequencies $\Delta > \omega > \omega_1(K)$ resulting in a relatively large intrinsic dissipation of the superconductors at these frequencies. The resonators made from such superconductors exhibits low quality factors. As the disorder is decreased the delocalized modes are shifted to higher frequencies. At $\omega < \omega_1(K)$ the oscillation with frequency ω excite only long-living localized states, so that the dissipation in the superconductor is suppressed. However, the localized modes extend down to zero frequencies for $K < K_2$. At any non-zero temperatures these low frequency bosonic modes are excited. Because the relaxation of these modes is slow, their occupation numbers fluctuate slowly with time. This, together, with the mode-mode interaction implies that the frequency of the high energy modes experience significant jitter in the range $K_1 < K < K_2$. The microwave properties described above can be compared with the other predictions of the model (1). Namely, one expects broadening of the distribution function at $K < K_1$ sketched in Fig. 1a that was observed in[12]. Another experimentally measurable characteristic is the behavior of superfluid stiffness that is proportional to the Δ^2 in the whole range of K considered here.[38] Finally, we note that fluctuational conductivity is given by a slightly modified[39] Aslamazov-Larkin formula above T_c for $K > K_2$ which can serve as yet another verification of the applicability of the theory; similarly one can estimate the value of K_2 from ultrasound attenuation measurements that are expected[40] to become exponentially low only at $K > K_2$. Notice that these different regimes happens *within* the pseudogapped regime where localization of single electron function leads to the formation of preformed Cooper pairs.[2]. Such materials are expected to have normal-state resistivity R_n only several times below the critical value R_c. Experimentally, for moderately thin films the value of $R_c \sim 10$ kΩ. Assuming that ρ_s for the film is suppressed by a factor of $2 - 5$ compared to BCS formula $\rho_{BCS} = \pi\Delta/R_\square$[27,28,38] we conclude that for the films with $\Delta \sim 1 - 2K$ and $R_\square \sim 1 - 2$ kΩ one should be able to reach $L_\square \sim 10$ nH as required for superinductor. However to achieve this goal the material should be tuned into the regime where resistance is already large but not too large so that effective $K > K_2$. Notice that very small gap in the microwave experiment was reported recently in strongly disordered NbN films, see Fig. 3d in[25]. We also mention recent complimentary approach clarifying classical Mattis-Bardeen theory of microwave conductivity for strongly disordered superconductors[41,42].

Conclusion. We demonstrated theoretically the presence of low-lying collective modes in disordered superconductors in the outer proximity of the SIT, and formulated the conditions for realization of a dissipationless superinductors.

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