Squeezing on Momentum States for Atom Interferometry

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Squeezing on momentum states for atom interferometry

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We propose and analyze a method that allows for the production of squeezed states of the atomic center-of-mass motion that can be injected into an atom interferometer. Our scheme employs dispersive probing in a ring resonator on a narrow transition in order to provide a collective measurement of the relative population of two momentum states. We show that this method is applicable to a Bragg diffraction-based strontium atom interferometer with large diffraction orders. This technique can be extended also to small diffraction orders and large atom numbers N by inducing atomic transparency at the frequency of the probe field, reaching an interferometer phase resolution scaling $\Delta\phi \sim N^{-3/4}$. We show that for realistic parameters it is possible to obtain a 20 dB gain in interferometer phase estimation compared to the Standard Quantum Limit. Our method is applicable to other atomic species where a narrow transition is available or can be synthesized.

A major effort in the field of atom interferometry [1] is focused on increasing the instrument sensitivity, either by enhancing the momentum transferred by the light onto the atoms [2, 3] or by increasing the interrogation time [3–6]. For given momentum transfer and interrogation time, the instrument sensitivity is determined by the phase noise of the interferometer. By applying differential schemes, many systematics and noise sources can be efficiently rejected as common-mode effects [7, 8], and one eventually meets a fundamental limit associated with the uncorrelated phase noise of different atoms, the so-called Standard Quantum Limit (SQL) $\Delta\phi_{\rm SQL} = 1/\sqrt{N}$, where N is the atom number.

This limit can be overcome by introducing quantum correlations between the individual particles, thereby producing squeezed atomic states, potentially reaching the Heisenberg limit $\Delta\phi_{\rm H} = 1/N$ [9, 10]. For squeezing of atomic internal states, many schemes have been studied both theoretically [11, 12] and experimentally [13–21], with almost 20 dB of observed noise reduction compared to the SQL [22, 23]. The key feature of most of these schemes is the enhanced atom-light interaction in an optical resonator that enables the generation of correlations between distant atoms. The implementation of these methods with motional states in atom interferometry remains, however, a challenging task. In this Letter we propose and analyze a scheme that generates strongly squeezed momentum states [9, 24] for atom interferometry. In particular, we consider the production of squeezed states of the atomic center-of-mass motion by dispersive probing of a momentum-state superposition of atoms in an optical ring resonator. For the bosonic isotope of strontium ^{88}Sr, the interest is motivated by its expected immunity to stray fields in atom interferometers and by the possibility of attaining long coherence times in quantum interference [25, 26]. The presence of narrow intercombination transitions makes the atom well suited for squeezing experiments involving external degrees of freedom.

The proposed scheme is illustrated in Fig. 1, where two vertical counterpropagating laser beams B_1 and B_2 induce a momentum state superposition between the states

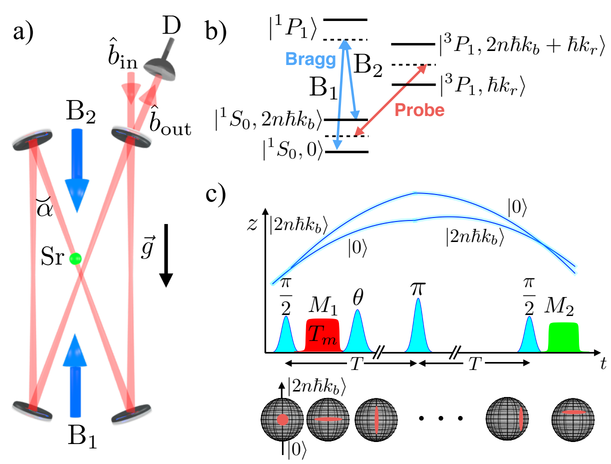

FIG. 1. a) Schematic setup with the probe field $\hat{b}_{\rm in}$ coupled to the optical cavity and interacting with the atoms (green circle) through the oblique beam with angle α with respect to gravity. The reflected field $\hat{b}_{\rm out}$ is measured by the detector D. The atomic states are manipulated through the Bragg beams B_1 and B_2. b) Level diagram for probing on the $^1S_0-^3P_1$ transition (red arrow) and for momentum state manipulation through Bragg diffraction on the $^1S_0-^1P_1$ transition (blue arrows). c) Interferometer trajectories and measurement sequence. The interferometer is formed by a $\pi/2-\pi-\pi/2$ Mach-Zehnder sequence with interrogation time T. The cavity-enhanced squeezing measurement with duration T_m is indicated as M_1. The Bragg pulse θ induces a phase-sensitive state for the interferometer and the final readout measurement is performed (M_2). Lower part: Bloch sphere representation of the state evolution.

$|^1S_0, p = 0\rangle$ and $|^1S_0, p = 2n\hbar k_b\rangle$ by n-th order Bragg diffraction [2] on the dipole-allowed $^1S_0 - {}^1P_1$ blue Sr transition at 461 nm. Here the atomic linear momentum is indicated by p and the photon momentum is denoted by $\hbar k_b$. The duration of the Bragg diffraction pulses is set in order to couple the two momentum states only. This condition is typically met by pulse durations of the order of 10 μs [27].

We consider squeezing of the atomic states by collective measurements of the relative population of the two momentum states through dispersive detection in a ring cavity (Fig. 1). This is achieved by probing for a time T_m on the red $^1S_0 - {}^3P_1$ intercombination line of strontium at 689 nm using a laser beam, of angular frequency ω_r, incident onto the cavity. Probing is performed when the free-falling atoms cross the cavity mode volume.

In the following, measurements of the cavity output field $\hat{b}_{\text{out}}$ are considered and the sensitivity to atom number fluctuations between momentum states is computed. As such measurements provide collective information about the ensemble without distinguishing between individual atoms, they project the ensemble into a collective state which corresponds to the measurement outcome [15]. This process can produce conditionally squeezed atomic momentum states that can be implemented in atom interferometers with significant metrological gain.

We treat the two momentum states as a spin-1/2 system and describe the ensemble by a total spin $S = N/2$, where N is the atom number. The population difference between the two momentum states is then $2S_z$.

We quantify the attainable metrological gain ξ_m by the ratio between the contrast squared $\mathcal{C}^2$ and the population variance $(2\Delta S_z)^2$ normalized to the atom shot noise variance $2S$ [10]:

$$\xi_m = \frac{S}{2(\Delta S_z)^2}\mathcal{C}^2. \tag{1}$$

The squeezing pre-measurement of S_z can be achieved by arranging a situation where the two momentum states are associated with opposite variations of the index of refraction and shift the cavity resonance frequency in opposite directions.

As shown in Fig. 1 b), the probe light inside the cavity couples atoms in the state $|^1S_0, p\rangle$ to the state $|^3P_1, p + \hbar k_r\rangle$, where $\hbar k_r$ is the probe photon momentum, corrected by the factor $\cos\alpha$ due to the angle between gravity and the oblique cavity beams (Fig. 1 a)). The transitions associated with the two momentum states $|p = 0\rangle$ and $|p = 2n\hbar k_b\rangle$ are then separated by the Doppler effect $2\delta\omega_r \equiv k_r \frac{2n\hbar k_b}{M} = 2\pi n \cos\alpha \times 28.6$ kHz, that is much larger than the natural linewidth $\Gamma = 2\pi \times 7.6$ kHz of the $^1S_0$-$^3P_1$ transition. For small α, the factor $\cos\alpha$ yields a small correction to the frequency splitting which we neglect in our discussion. When the cavity resonance frequency ω_c is tuned halfway between the two optical transitions, atoms in the two momentum states produce opposite shifts of the cavity resonance frequency that can be detected via the phase shift $\Delta\phi_{\text{ph}}$ of the light reflected from the cavity (Fig. 1 a)).

It can be shown that (see Supplemental Material [28])

$$\Delta\phi_{\text{ph}} = \frac{4\frac{\kappa_{\text{in}}}{\kappa} S_z \eta \mathcal{L}_d(\delta\omega_r)}{[2\frac{\kappa_{\text{in}}}{\kappa} - 1 - N\eta\mathcal{L}_a(\delta\omega_r)][1 + N\eta\mathcal{L}_a(\delta\omega_r)]} \tag{2}$$

i.e. the population difference can be detected via the phase shift of the light emerging from the cavity. Here $\mathcal{L}_d(\Delta) = -2\Gamma\Delta/(\Gamma^2 + 4\Delta^2)$ and $\mathcal{L}_a(\Delta) = \Gamma^2/(\Gamma^2 + 4\Delta^2)$ are the atomic dispersion and absorption profiles, respectively. The single-atom cooperativity is indicated as $\eta = 4g^2/(\Gamma\kappa)$, where $2g$ is the vacuum Rabi frequency, κ is the cavity mode linewidth and κ_{in} is the contribution to κ due to the input mirror transmission. The light phase measurement can be performed, for example, through the Pound-Drever-Hall technique. If the detector is at the photon shot noise level, the variance of the population difference between the two momentum states, normalized to the variance $2S$ of the atom shot noise, is given by [28]

$$\frac{2(\Delta S_z)^2}{S} = \frac{\mathcal{L}_a(\delta\omega_r)[1 + N\eta\mathcal{L}_a(\delta\omega_r)]^2}{4N\eta\epsilon_d n_{\text{sc}}[\mathcal{L}_d(\delta\omega_r)]^2}, \tag{3}$$

where ϵ_d is the detection efficiency [28], and we have expressed the measurement strength in terms of the average number n_{sc} of photons scattered per atom into free space, since the latter process constitutes the main, and fundamental, limitation on the attainable squeezing [34]. After the scattering of one photon by one atom, the momentum superposition is destroyed and the associated recoil causes the trajectory to deviate from the vertical direction. The resulting losses cause a random imbalance $2(\Delta S_z)_{\text{sc}}$ of the populations in the two momentum states. Assuming that each atom scatters at most one photon, the population variance increase is $(2\Delta S_z)^2_{\text{sc}} = 2Sn_{\text{sc}}$. By accounting for free space scattering we can then compute the optimum metrological gain, which is attained for

$$n_{\text{sc}} = \sqrt{\frac{\mathcal{L}_a(\delta\omega_r)[1 + N\eta\mathcal{L}_a(\delta\omega_r)]^2}{4N\eta\epsilon_d[\mathcal{L}_d(\delta\omega_r)]^2}}. \tag{4}$$

The resulting gain is represented by blue circles in Fig. 2 for the case where $\epsilon_d = 1$ and $N\eta = 10^4$, for varying Bragg diffraction orders n. When $N\eta$ lies in the range $10^3 - 10^4$, there is significant gain if $n > 5$, a condition typically met by large-momentum-transfer atom interferometers [3]. Indeed, for small n, the optical transitions are not sufficiently resolved in frequency space compared to the atomic linewidth, which prevents operating in the dispersive regime of atom-light interaction and leads to substantial absorption and squeezing reduction. In general, considerable gain can be observed if

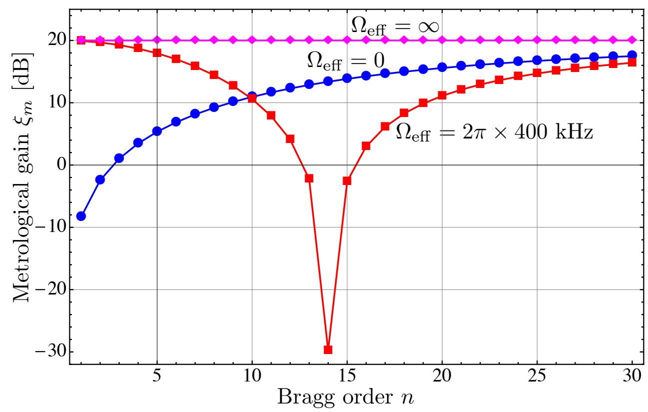

FIG. 2. Expected metrological gain ξ_m as a function of the Bragg diffraction order n, assuming perfect detection efficiency, $\epsilon_d = 1$ and for $N\eta = 10^4$. The blue circles refer to the scheme shown in Fig. 1 b). Values shown as red squares and magenta diamonds refer to the scheme in Fig. 3 a), where EIT is present with $\Omega_{\rm eff} = 2\pi \times 400$ kHz and with perfect transparency $\Omega_{\rm eff} = \infty$, respectively.

$N\eta\mathcal{L}_a(\delta\omega_r) \ll 1$, so that $n \gg 0.3 \times \sqrt{N\eta}$, in which case the gain saturates at a value $\xi_m \approx \sqrt{N\eta\epsilon_d}$, independent of the Bragg diffraction order.

We propose a scheme where it is possible to enhance the signal-to-noise ratio of momentum-state population collective measurements also at small Bragg diffraction orders n, while keeping the collective cooperativity $N\eta$, and thus the squeezing, large. As the main limitation in this regime is the spoiling of the cavity finesse by atomic absorption, we consider a scheme where coupling of the decaying $^3P_1$ state to the metastable state $^3P_0$ with a much longer lifetime results in electromagnetically induced transparency (EIT) at the original cavity resonance frequency [35–38] (see Fig. 3). The $^3P_1$-$^3P_0$ coupling is attained through two-photon Raman coupling via the $^3S_1$ intermediate state with the two copropagating Raman lasers R_1 and R_2 at 679 nm ($^3P_0$–$^3S_1$ transition) and 688 nm ($^3P_1$–$^3S_1$ transition), respectively. As discussed in detail in [28], for a large detuning of the Raman lasers from single-photon resonance, we can adiabatically eliminate the excited $^3S_1$ state and describe the system in a three-level picture formed by the states $^1S_0$, $^3P_1$ and $^3P_0$ with the effective control Rabi frequency $\Omega_{\rm eff}$ [28, 39, 40]. In Fig. 3 b) we plot the spectrum of the cavity power transmission coefficient $\mathcal{T}$ for $N\eta = 3 \times 10^3$, $\Omega_{\rm eff} = 2\pi \times 400$ kHz and diffraction order $n = 1$. This shows that the coupling to the metastable $^3P_0$ state removes the atomic absorption, allowing for a significant increase in the signal-to-noise ratio of the squeezing measurement. EIT thus results in a reduced effective linewidth which in turn allows to operate in the dispersive regime. This condition is fulfilled when $N\eta\mathcal{L}_a(\delta\omega_E) \ll 1$, or $|\delta\omega_E|/\Gamma \gg \sqrt{N\eta}/2$, where now

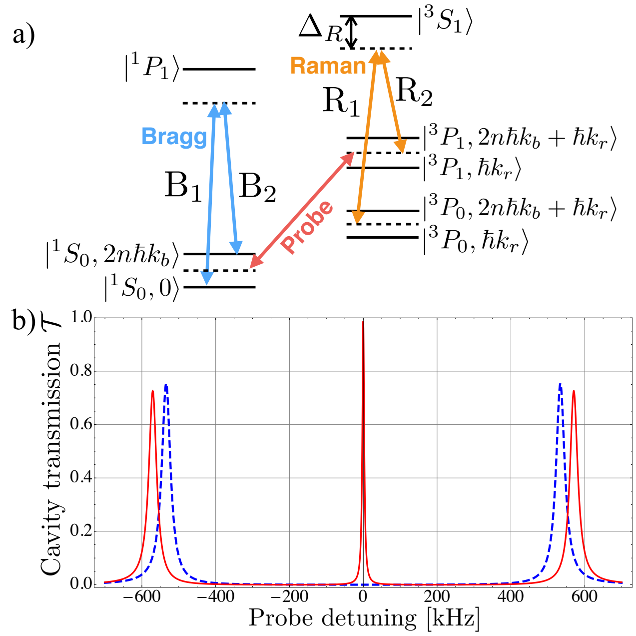

FIG. 3. a) Level diagram for the generation of squeezed momentum states enhanced by induced transparency via the $^3P_0$ state. The Bragg laser beams are indicated as B_1 and B_2. The effective control field that couples the states $^3P_0$ and $^3P_1$ is obtained by two-photon Raman coupling via the $^3S_1$ intermediate state. The two Raman beams R_1 and R_2 operate at the wavelengths 679 nm ($^3P_0$–$^3S_1$) and 688 nm ($^3P_1$–$^3S_1$), respectively, and are detuned from the transition to the $^3S_1$ state by Δ_R. b) Cavity transmission spectrum with (red solid line) and without (blue dashed line) Raman coupling to the $^3P_0$ state for $\Omega_{\rm eff} = 2\pi \times 400$ kHz, $N\eta = 3 \times 10^3$ and $n = 1$. The two lateral peaks correspond to the vacuum Rabi splitting for $\kappa = 2\pi \times 50$ kHz. The population measurement is performed at the frequency of the transparency region (at probe detuning $\delta = 0$) which corresponds to a linewidth of $\kappa_{\rm EIT} = 2\pi \times 6$ kHz.

$\delta\omega_E = \delta\omega_r - \Omega_{\rm eff}^2/(4\delta\omega_r)$. We also note that in terms of laser power of the Raman beams, this condition is less demanding for narrow transitions compared to broad dipole-allowed transitions. The corresponding metrological gain in the presence of EIT is shown in Fig. 2 as a function of diffraction order by two sets of points that correspond to a finite coupling strength $\Omega_{\rm eff} = 2\pi \times 400$ kHz (red squares) and to perfect transparency $\Omega_{\rm eff} = \infty$ (magenta diamonds).

Fig. 4 shows a comparison between the presence and the absence of the Raman coupling to the $^3P_0$ state. In terms of metrological gain, EIT is equivalent to large diffraction orders and allows to recover the signal-to-noise ratio that would be otherwise lost because of photon absorption.

An atom interferometry scheme including the squeezed source proposed here is the following (Fig. 1 c)): strontium atoms are cooled and trapped at the cavity mode

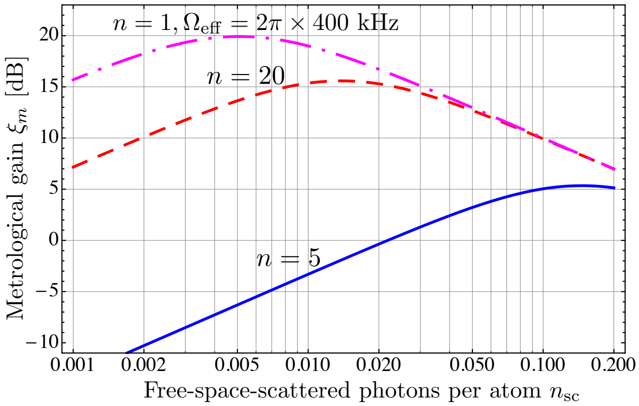

FIG. 4. Metrological gain ξ_m as a function of the number of photons scattered into free space per atom for different Bragg diffraction orders and for varying Raman coupling strength: $n = 5, \Omega_{\text{eff}} = 0$ (blue solid line); $n = 20, \Omega_{\text{eff}} = 0$ (red dashed line); $n = 1, \Omega_{\text{eff}} = 2\pi \times 400$ kHz (magenta dot-dashed line). We assume a collective cooperativity $N\eta = 10^4$ and perfect detection efficiency $\epsilon_d = 1$.

waist close to the center of the optical cavity, then, a momentum superposition is created by a Bragg $\pi/2$ pulse and immediately after that the squeezing measurement of the relative population is performed (M_1). At this stage the atomic ensemble is projected into a state with reduced relative population uncertainty. The state on the Bloch sphere is then transformed into a phase-sensitive state by applying a Bragg $\pi/2$ pulse with a phase shift of $90°$ with respect to the first pulse. Following this preparation stage, a $\pi/2{-}\pi{-}\pi/2$ Mach-Zehnder interferometer sequence of laser pulses is applied. A final measurement (M_2) is performed using, for example, fluorescence detection.

With realistic values for the various parameters, our method is applicable to strontium atoms with the current technology. Specifically, we consider an optical cavity where one of the foci has a waist $w_0 = 150\ \mu$m, at the position where the atoms cross the cavity mode volume. With a cavity finesse $F = 2.5 \times 10^4$ and at the wavelength $\lambda = 689$ nm, we get a single-atom cooperativity $\eta = 6F\lambda^2/(\pi^3 w_0^2) \approx 0.1$ [31]. We then consider $N \approx 10^5$ atoms occupying a volume with a linear size of about $30\ \mu$m. Therefore, a collective cooperativity $N\eta \approx 10^4$ is achievable. The maximum possible Bragg diffraction order with our method is set by the condition that the transit time of the wavepackets corresponding to the two momentum states through the cavity beam waist is larger than the time duration of the collective measurement. We estimate the useful transit time as the one taken by a wavepacket with speed $n\hbar k_b/M$ to cross one tenth of the effective mode waist. Because the atoms are crossing the cavity beam vertically, the effective mode waist is $w_0/\sin\alpha$. We therefore estimate the maximum Bragg diffraction order as $n_{\max} = Mw_0/(10\hbar k_b T_m \sin\alpha)$, where T_m is the measurement time duration. With $\alpha \approx 0.4$ rad and $T_m \approx 200\ \mu$s we get $n_{\max} = 10$. However, the maximum Bragg order can be made considerably larger by a suitable design of the cavity geometry, where w_0 is made larger and α is made smaller. The measurement time is set by the requirement that the number of photons scattered into free space is sufficient to provide the optimum metrological gain. To resume, by considering a collective cooperativity $N\eta = 10^4$, first-order diffraction $n = 1$, a Raman coupling strength $\Omega_{\text{eff}} = 2\pi \times 400$ kHz, a measurement time $T_m = 200\ \mu$s and a detection efficiency $\epsilon_d = 1$, we conclude that the optimum number of photons scattered into free space per atom is $n_{\text{sc}} = 5 \times 10^{-3}$, corresponding to the excited state population $P_{\text{exc}} = n_{\text{sc}}/(\Gamma T_m) = 5 \times 10^{-4}$. In this case it is possible to achieve a metrological gain of 20 dB.

In conclusion, we have proposed and analyzed a novel scheme that allows for the production of squeezed momentum states for large-momentum-transfer Bragg atom interferometers. The essence of our method is based on the ability to resolve the Doppler splitting of two momentum states by using a probe laser with frequency close to the narrow ${}^1S_0{-}{}^3P_1$ intercombination line of strontium. With realistic parameters we show that 20 dB of noise reduction in atom interferometer phase measurements can be attained compared to the Standard Quantum Limit, with less than 1 ms preparation stage. Moreover, at small diffraction orders, where cavity-enhanced absorption would limit the resolution necessary for the collective measurement, we have shown that it is possible to induce a transparency at the frequency of the probe laser by two-photon Raman coupling, thus recovering the required dispersive regime. With this method it is then possible to attain significant squeezing also for small Bragg diffraction orders and large atom numbers, with an atom number scaling for the interferometer phase resolution $\Delta\phi \sim N^{-3/4}$. Our method is applicable to atomic species where narrow transitions are available or can be synthesized through Raman coupling between hyperfine ground states, as, e.g., is possible for alkali atoms.

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