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# Gravitational Waves from Oscillons with Cuspy Potentials

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We study the production of gravitational waves during oscillations of the inflaton around the minimum of a cuspy potential after inflation. We find that a cusp in the potential can trigger copious oscillon formation, which sources a characteristic energy spectrum of gravitational waves with double peaks. The discovery of such a double-peak spectrum could test the underlying inflationary physics.

*Introduction.* Gravitational waves (GWs) play an important role in the context of inflationary cosmology. A stochastic background of GWs, produced during inflation and subsequent preheating/reheating after inflation, carries useful information about the inflationary dynamics and inflaton decay (see [1] for a recent review). Detecting such a stochastic background of GWs, whether directly or indirectly, can provide us with a unique opportunity to test theories of inflation.

During inflation, quantum fluctuations of the tensor modes of the spacetime metric were stretched by the accelerated expansion of the Universe, and were then nearly frozen on super-Hubble scales. Since these GWs can result in B-mode polarization of the cosmic microwave background (CMB) anisotropies, their spectrum is in principle measurable by CMB polarization experiments. Current CMB data alone already put an upper bound on the tensor-to-scalar ratio  $r < 0.09$  at 95% confidence level [2], and when combined with the constraints on the scalar spectral index, have been effective in discriminating inflationary models. For example, the cubic and quartic potentials are strongly disfavored, and the quadratic potential is moderately disfavored by the Planck 2015 data [3], while axion monodromy inflation with a linear potential [4] or fractional powers [5] are compatible with the current Planck results. Further advances in axion monodromy inflation have suggested potentials with even more possible powers [6, 7]. Moreover, it has recently been shown that stringy effects can lower the power of a quadratic axion monodromy potential to less than linear [8]. Thus, axion monodromy inflation represents an interesting class of large field inflationary models that are compatible with data.

Besides vacuum fluctuations during inflation, another source of GWs is parametric resonance during preheating [9]. During preheating after inflation, the Fourier modes of a scalar matter field  $\chi$  coupled to the inflaton grow exponentially by parametric resonance, driven by the oscillating inflaton. The resonant modes are quickly pumped up to a large amplitude. Such highly pumped modes correspond to large, time dependent density inhomogeneities in position space, ensuring that the matter distribution

has a non-trivial quadrupole moment, which can source a significant spectrum of GWs [10]. The present peak frequency of such GWs is proportional to the energy scale of inflation [11], while the present amplitude of GWs is independent of the energy scale of inflation [12]. In hybrid inflation the stochastic background produced during preheating is expected to be directly detected by future GW detectors [13].

In this Letter, we shall investigate the production of GWs during oscillations of the inflaton after inflation with a cuspy potential

$$V(\phi) = \lambda M_{\text{pl}}^{4-p} |\phi|^p \quad (1)$$

with  $p = 1, 2/3, 2/5$ , and  $M_{\text{pl}} \equiv (8\pi G)^{-1/2}$  is the reduced Planck mass. In string theory, axion monodromy can be introduced by space-filling wrapped branes leading to a linear potential [4]. Inflationary potentials with powers of  $2/3$  and  $2/5$  arise in compactifications on manifolds with metric flux such as Nil manifolds which contain tori twisted over circles [5]. More generally, monodromy generated by fluxes can lead to potentials with more varieties of power [6, 7]. Here we hasten to add that the powers of these potentials are expected only at large field values, due to the coupling of the inflaton to high scale physics. At the end of inflation, i.e., for small  $\phi$ , these potentials for axion monodromy become quadratic. Nonetheless, cuspy potentials can arise in other inflationary contexts, e.g., through non-standard kinetic terms or as a result of integrating out the dynamics of other fields that couple to the inflaton. Thus, we use these specific potentials as benchmarks to illustrate the novel GW signatures that can arise when the potential has a cuspy behavior at the end of inflation. Assuming the potential in eq. (1) applies to both the inflationary era and at the end of inflation, the value of  $\lambda$  in this simple class of models can be fixed by the estimated amplitude of scalar perturbations from the CMB data. For powers of  $p = 1, 2/3, 2/5$ ,  $\lambda \approx 3, 4, 5 \times 10^{-10}$ , the predicted scalar spectral index  $n_s \approx 0.970, 0.973, 0.976$ , and the predicted tensor-to-scalar ratio  $r \approx 0.08, 0.05, 0.03$ , respectively, assuming the number of e-folds  $N = 50$ . These predictions are in agreement with the recent CMB data. In the reheating

scenario, the inflaton  $\phi$  oscillates near the minimum of its potential after inflation and decays into elementary particles. However, due to the cusp of the potential, the oscillating behavior of the inflaton is very different from that of smooth potentials like  $\phi^2$  and  $\phi^4$ . It has recently been shown that an efficient parametric resonance can occur during preheating for an inflaton potential of the form of eq. (1) with  $0 < p \leq 2$ , if the inflaton is coupled to a scalar matter field  $\chi$  via an interaction term  $\phi^2\chi^2$  [14]. However, the production of GWs has not been studied to our knowledge. In this Letter, we are interested in the production of GWs during oscillations of the inflaton after inflation with cuspy potentials. We find that the non-smooth oscillations can trigger amplification of fluctuations of the inflaton itself at the moment when  $\phi(t) = 0$ , so that oscillons copiously form after inflation. As in the models with a symmetric smooth potential [15] and an asymmetric smooth potential [16], the oscillon formation in the models with cuspy potentials sources a stochastic background of GWs, on which the characteristic size of the oscillons is imprinted. Interestingly, these cuspy potentials result in a characteristic energy spectrum of GWs with double peaks, which can be distinguished from smooth potentials by probing the shape of the energy spectrum of GWs.

*Model.* GWs are described by the transverse-traceless gauge-invariant tensor perturbation,  $h_{ij}$ , in a Friedman-Robertson-Walker (FRW) metric,

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j. \quad (2)$$

The perturbed Einstein equation reads

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\text{pl}}^2 a^2}\Pi_{ij}^{\text{TT}}, \quad (3)$$

where  $\Pi_{ij}^{\text{TT}}$  is the transverse-traceless projection of the anisotropic stress tensor  $T_{ij}$ . In our model we assume that the inflaton is weakly coupled to other fields during preheating. GWs are sourced by the inflaton  $\phi$ , i.e.,  $\Pi_{ij}^{\text{TT}} = (\partial_i \phi \partial_j \phi)^{\text{TT}}$ . The energy density of GWs is

$$\rho_{\text{GW}} = \frac{M_{\text{pl}}^2}{4} \langle \dot{h}_{ij} \dot{h}^{ij} \rangle, \quad (4)$$

where  $\langle \dots \rangle$  denotes a spatial average over the volume. It is commonly parameterized by the dimensionless density parameter per logarithmic frequency interval,  $\Omega_{\text{GW}} = d\rho_{\text{GW}}/d\ln k/\rho_c$ , where  $\rho_c$  is the critical density of the Universe. The energy density spectrum of the inflaton is defined as

$$k^3 \rho_k = \frac{1}{2} k^3 (|\partial_\tau \varphi_k|^2 + \omega_k^2 |\varphi_k|^2), \quad (5)$$

where  $\omega_k = \sqrt{k^2 + a^2 \langle V'' \rangle - \partial_\tau^2 a/a}$ ,  $\varphi_k$  are the Fourier modes of  $a\phi$ , and  $\tau$  is the conformal time.

To understand oscillon formation during oscillations of the inflaton after inflation with a cuspy potential, we now investigate the evolution behavior of inflaton fluctuations. To first order, the equation of motion for the Fourier modes of fluctuations of the inflaton  $\phi$  is

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left[\frac{k^2}{a^2} + V''(\phi)\right]\delta\phi_k = 0. \quad (6)$$

To solve this equation, we neglect the expansion of the Universe for the moment, thus the friction term drops out of the equation of motion. Moreover, since we are interested in large-scale modes, the  $k^2$  term can be dropped. The equation becomes

$$\delta\ddot{\phi}_k + V''(\phi)\delta\phi_k = 0. \quad (7)$$

For illustrative purposes, in what follows let us consider the linear potential. Since the inflaton potential  $V(\phi) = \lambda M_{\text{pl}}^3 |\phi|$  has a cusp at  $\phi = 0$ , its derivative with respect to  $\phi$  is a step function and its second order derivative is a delta function  $V''(\phi) = 2\lambda M_{\text{pl}}^3 \delta(\phi)$ . We focus on the evolution behavior of the  $\delta\phi_k$  modes near the point  $\phi(t) = 0$ . It is convenient to define  $t$  such that  $\phi(t = 0) = 0$ . The solution to the equation of motion for the inflaton is  $\phi(t) = |\dot{\phi}_m|t + \lambda M_{\text{pl}}^3 t^2/2$  when  $t < 0$  and  $\phi(t) = |\dot{\phi}_m|t - \lambda M_{\text{pl}}^3 t^2/2$  when  $t > 0$ , where  $\dot{\phi}_m$  is the maximum value of  $\dot{\phi}$  at  $t = 0$ . Since  $\phi(t) \approx |\dot{\phi}_m|t$  is a good approximation in a small vicinity of  $\phi = 0$ , we find  $\delta\dot{\phi}_k(t = 0^+) - \delta\dot{\phi}_k(t = 0^-) = -2\lambda M_{\text{pl}}^3 \delta\phi_k(t = 0)/|\dot{\phi}_m|$ , which implies that  $\delta\dot{\phi}_k$  jumps suddenly when  $\phi$  crosses the cusp of the potential. Such periodic jumps of  $\delta\dot{\phi}_k$  result in periodic, rapid increases of the energy density  $\rho_k$ . We show the time evolution of the energy density for a linear potential (orange) in an expanding Universe in Fig. 1. In the cases of  $p = 2/3$  and  $p = 2/5$ ,  $|V'(\phi)|$  becomes infinite when  $|\phi|$  tends to zero. To avoid this singularity, the potential with a cut-off  $|\phi| > \mathcal{O}(10^{-3})$  is adopted in our numerical calculations. We also show the time evolution of the energy density for the  $\phi^{2/3}$  potential (blue) and  $\phi^{2/5}$  potential (green) in Fig. 1. Similar to the linear potential, the energy density suddenly increases near the points at which  $\phi(t) = 0$ . However, the sudden increase is followed by a sudden decrease near  $\phi(t) = 0$  after several oscillations of the field  $\phi$ . The energy density always increases after each oscillation of the field  $\phi$ . We have checked that the increase in energy density is independent of the choice of the cut-off. As a result, oscillon formation occurs when the inflaton oscillates near the minimum of its potential, which sources a stochastic background of GWs.

*Simulation Results.* Using a modified version of LATTICEASY [17], we simulate the production of GWs during preheating in the models (1) with cuspy potentials. LATTICEASY has been developed for more generally calculating the evolution of interacting scalar fields

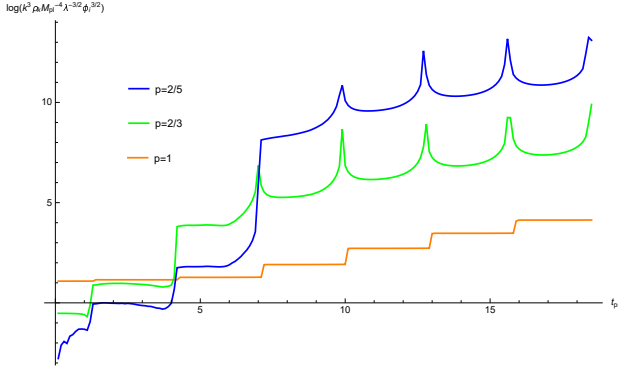


FIG. 1: Evolutions of  $\rho_k$  for cuspy potentials with  $p = 1$  and  $k = 1.0 \times 10^{-3} M_{\text{pl}}$  (orange),  $p = 2/3$  and  $k = 1.0 \times 10^{-5} M_{\text{pl}}$  (blue),  $p = 2/5$  and  $k = 1.3 \times 10^{-4} M_{\text{pl}}$  (green), respectively.

in an expanding Universe. The spectral method can directly solve the GW equation (3) in Fourier space [18]. Actually, one can first evolve the tensor perturbations in configuration space and then apply the transverse-traceless projector to the real physical  $h_{ij}$  in Fourier space [19]. Another method is based on the Green's functions in Fourier space to calculate numerically the energy spectrum of GWs generated well inside the horizon [20]. In our lattice simulations we adopt the configuration-space method for solving the following evolution equation of the tensor perturbations

$$\ddot{u}_{ij} + 3H\dot{u}_{ij} - \frac{1}{a^2}\nabla^2 u_{ij} = \frac{2}{M_{\text{pl}}^2 a^2} T_{ij}. \quad (8)$$

Therefore, the transverse-traceless tensor perturbations can be written as  $h_{ij}(t, \mathbf{k}) = \Lambda_{ij,lm}(\hat{\mathbf{k}}) u_{lm}(t, \mathbf{k})$ , where  $\Lambda_{ij,lm}(\hat{\mathbf{k}})$  is the transverse-traceless projection operator and  $u_{lm}(t, \mathbf{k})$  is the Fourier transform of the solution to the equation (8). The energy density of GWs can be expressed in terms of  $u_{ij}$  as

$$\rho_{\text{GW}} = \frac{M_{\text{pl}}^2}{4L^3} \int d^3\mathbf{k} \Lambda_{ij,lm}(\hat{\mathbf{k}}) \dot{u}_{ij}(t, \mathbf{k}) \dot{u}_{lm}^*(t, \mathbf{k}). \quad (9)$$

We perform three-dimensional lattice simulations with  $256^3$  points in a box with periodic boundary conditions assuming  $\lambda = 1.26 \times 10^{-12}$  in the linear potential model. We set the initial values of the inflaton, its derivative and the scale factor as  $\phi_i = 0.75 M_{\text{pl}}$ ,  $\dot{\phi}_i = 6.8 \times 10^{-4} M_{\text{pl}}^2$  and  $a_i = 1$ . The inflaton fluctuations and its derivative are initialized by quantum vacuum fluctuations, while the tensor fluctuation and its derivative are initialized as zero. We stop the simulation when the energy spectrum of GWs does not grow significantly. We assume that reheating ends at the end of the simulation. After that, the Universe enters into the radiation-dominated era. The energy spectrum and its frequency at the end of simulations are converted to the present values. Fig. 2 shows

the time evolution of the energy density as a function of position on a two-dimensional slice through the simulation from  $a(t) = 6.13$  (top-left), to 6.81 (top-right), to 7.43 (bottom-left) and 13.4 (bottom-right) in the linear potential model. We can see that at the beginning, oscillons copiously form and then decay during oscillations of the inflaton. In this model, the rapid growth of oscillons results in the production of GWs with  $\Omega_{\text{GW}} h^2 \sim 2 \times 10^{-9}$  today. Our lattice simulation results show that there appear two peaks in the energy spectrum of GWs, a feature very distinct from that of other models. Therefore, our model can be distinguished from the production of GWs during preheating by future GW detectors. While this paper was in preparation, a phenomenological study of GWs produced from preheating with a time dependent resonance parameter  $q(t)$  was recently undertaken [23]. For some choices of  $q(t)$ , one also finds a GW spectrum with multiple peaks due to non-linear effects. The double peak of GWs in [23] arises due to the parametric resonance in the preheating phase, while in our work, it is not the case, the double peak is due to the copious oscillon formation triggered by the cusp in the potential.

The evolution of the spectrum goes through two different stages, the linear growth stage and nonlinear growth stage. In the first stage, as shown in Fig. 3, the small- $k$  modes of the field  $\phi$  exponentially grow due to the cusp of the potential until the turning point  $a(t) = 7.30$ . The linear growth is more efficient than those driven by the symmetric potential [15] and asymmetric potential [16]. This leads to the left peak in the energy spectrum of GWs, which is characteristic of the cuspy potential. In the second stage, from Fig. 3 we see that the small- $k$  modes begin to drop and the large- $k$  modes continue to grow. It implies that the energy flows from the small- $k$  modes to the large- $k$  modes, as discussed in detail in [19]. This leads to the right peak in the energy spectrum of GWs.

Moreover, from the lattice simulations of preheating in the models (1) with  $p = 2/3$  and  $p = 2/5$ , we find that the energy spectra of GWs peak at around  $\Omega_{\text{GW}} h^2 \sim 1.2 \times 10^{-9}$  and  $\Omega_{\text{GW}} h^2 \sim 4 \times 10^{-10}$ , respectively, which are lower than the one in the linear potential model.

In our analysis, we have neglected the interactions between the inflaton  $\phi$  and other matter fields. If the inflaton is coupled to a matter field  $\chi$ , broad parametric resonance leads effectively to a fast growth of the fluctuations of  $\chi$  [14]. However, our numerical simulations confirm that the growth of the inflaton fluctuations themselves triggered by the cusp in its potential is more effective than that of the field  $\chi$  by parametric resonance. Therefore, GWs are sourced mainly by the inflaton fluctuations, even if a parametric resonance for the field  $\chi$  occurs in this model.

*Observational Implications.* As found in [12, 13], the peak amplitude of the energy spectrum of GWs needs not depend on the energy scale of inflation, while the peak

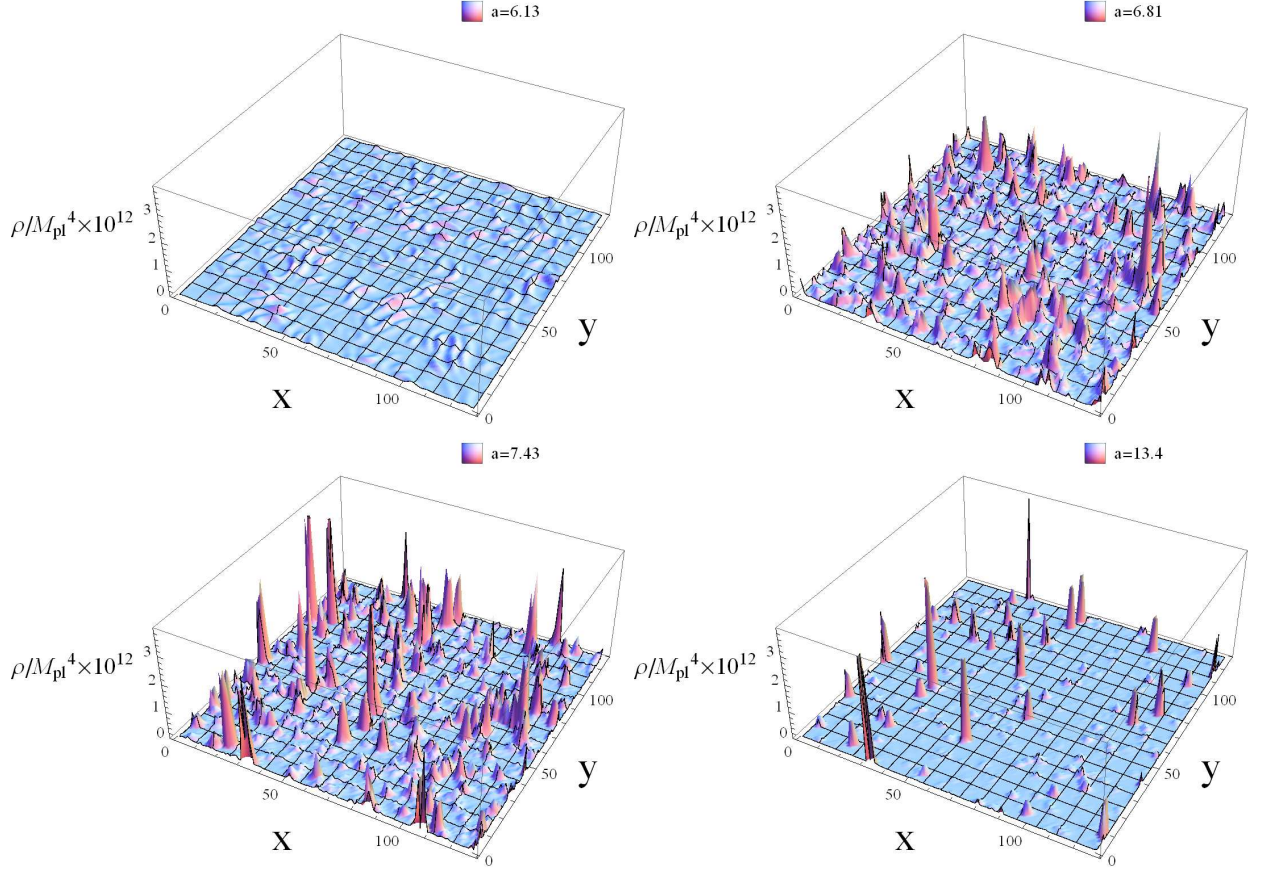


FIG. 2: Energy density  $\rho$  on a two-dimensional slice through the simulation, when  $a(t) = 6.13$  (top-left),  $6.81$  (top-right),  $7.43$  (bottom-left) and  $13.4$  (bottom-right), in the linear potential model.

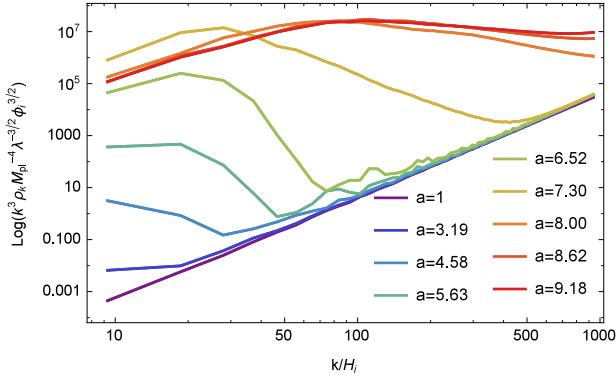


FIG. 3: Evolution of the energy density spectrum of the field  $\phi$  for the linear potential. The yellow line corresponds to a turning point  $a(t) = 7.30$ .

frequency scales inversely with the energy scale of inflation. In the single-field slow-roll inflationary scenario, if  $\lambda \approx 3 \times 10^{-10}$  is fixed by the amplitude of the primordial curvature perturbation  $A_s = 2.2 \times 10^{-9}$  [3], the peak frequency of GWs today is fixed to be  $f \sim 10^9 \text{ Hz}$ , many orders of magnitude beyond the frequencies that can be

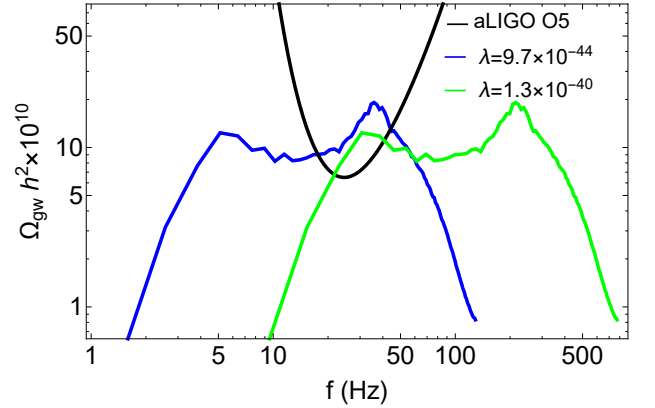


FIG. 4: Energy spectra of GWs today, predicted by the linear potential with  $\lambda = 9.7 \times 10^{-44}$  (blue) and  $\lambda = 1.3 \times 10^{-40}$  (green). The black curve is the expected sensitivity curve of the fifth observing run (O5) of the aLIGO-Virgo detector network.

reached by current GW detection experiments.

If the model parameter  $\lambda$  is not fixed by the amplitude of the primordial curvature perturbation, the sensitivity of advanced LIGO (aLIGO) is expected to be



significantly improved, which allows us to possibly observe GWs produced during oscillations of inflaton after inflation. For example, in the hybrid inflationary scenario [21], since  $\phi$  is not necessarily the inflaton itself,  $\lambda$  becomes essentially a free parameter. In this case we have plotted in Fig. 4 the present-day energy spectra of GWs produced during oscillon formation in the linear potential model (1) with  $\lambda = 9.7 \times 10^{-44}$  (blue) and  $\lambda = 1.3 \times 10^{-40}$  (green), the peaks of which lie above the expected sensitivity curve of the fifth observing run (O5) of the aLIGO-Virgo detector network [22]. Fig. 4 shows that there are two peaks in the energy spectrum of GWs, which differ from other spectra of GWs produced during preheating. A detection of the second peak may require corroboration from other GW detectors such as the Big Bang Observatory.

To summarize, we have studied the production of GWs during oscillon formation after inflation with cuspy potentials. At the end of inflation, oscillon formation can be triggered by the particular oscillations of the inflaton around the minimum of its potential, which sources a characteristic double-peak spectrum of GWs. The discovery of such a background would open a new observational window into inflationary physics.

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