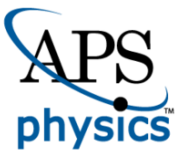

Light-Induced Fractional Quantum Hall Phases in Graphene

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Light-induced fractional quantum Hall phases in graphene

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We show how to realize two-component fractional quantum Hall phases in monolayer graphene by optically driving the system. A laser is tuned into resonance between two Landau levels, giving rise to an effective tunneling between these two synthetic layers. Remarkably, because of this coupling, the interlayer interaction at non-zero relative angular momentum can become dominant, resembling a hollow-core pseudo-potential. In the weak tunneling regime, this interaction favors the formation of singlet states, as we explicitly show by numerical diagonalization, at fillings $\nu = 1/2$ and $\nu = 2/3$. We discuss possible candidate phases, including the Haldane-Rezayi phase, the interlayer Pfaffian phase, and a Fibonacci phase. This demonstrates that our method may pave the way towards the realization of non-Abelian phases, as well as the control of topological phase transitions, in graphene quantum Hall systems using optical fields and integrated photonic structures.

Introduction. The fractional quantum Hall (FQH) effect is a fascinating phenomenon, where electron-electron interactions and a magnetic field lead to strong correlations [1–3]. Soon it was realized [4–7] and experimentally confirmed [8, 9] that the electron spin plays an important role at several fillings. More generally, multicomponent FQH phases [10] occur in systems with subbands, as wide quantum wells [11–14], with layers, as double wells [15, 16], or with degenerate valleys, as AlAs quantum well [17] or graphene [18–21]. Much effort has been made towards engineering system parameters like tunneling, in order to realize different phases. Here we propose a new method based on light-matter interactions which enables flexible control in a synthetic FQH bilayer.

Interactions between light and graphene quantum Hall samples have been subject of many theoretical [22–25] and experimental [26–29] studies. FQH phases in integrated GaAs quantum well-cavity structures have also been explored experimentally [30]. A distinctive feature of graphene is the linear dispersion, resulting in non-equidistant Landau levels (LLs) [31] which can selectively be coupled with resonant light.

The present letter explores this possibility. While in the absence of light a large gap freezes out all but one LL, resonant light coupling to an empty level provides an effective tunneling to this new degree of freedom. The coupled LLs can then be viewed as two layers of a physical bilayer. Depending on the tunneling rate, which is tunable via the laser intensity, the system either polarizes in the lower dressed LL, or it realizes a singlet phase. Analysis of the Coulomb interaction between different LLs shows that the repulsion between singlet pairs becomes particularly small when first and second LLs are coupled, resembling a hollow-core Haldane pseudo-potential [2, 32]. Such interaction favors the formation of a many-body singlet phase, which we confirm explicitly by exact diagonalization (ED), at filling $\nu = 1/2$ and $\nu = 2/3$. We identify the polarized phases as a composite Fermi sea ($\nu = 1/2$) [33], and a quasi-hole conjugate 1/3 Laughlin state ($\nu = 2/3$) [34]. The singlet phase at $\nu = 1/2$ has good overlap with the Haldane-Rezayi phase [35], an intriguing gapless quantum Hall phase [36–39]. Some evidence of non-Abelian quantum Hall singlets are found at $\nu = 2/3$, including the Fibonacci phase [40] and the interlayer Pfaffian phase [41, 42], which are interesting candidates for topological quantum computing [43].

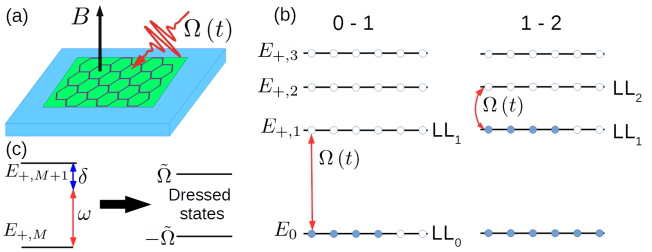

FIG. 1. (a) A single graphene layer driven by light at Rabi frequency Ω. (b) LL structure with partial filling and optical transitions LL_{0-1} and LL_{1-2}. (c) Formation of dressed states due to coupling between two LLs.

System. We consider a monolayer of graphene under a perpendicular magnetic field, in the quantum Hall regime [31]. We restrict ourselves to a single valley, and assume that the electron spin is fully polarized. The single-particle states are given by spinors of the form $\Psi_{\gamma,n,j}(z) = (-\gamma C_n^- \phi_{n-1,j}(z), C_n^+ \phi_{n,j}(z))^T$, where $C_n^\pm = \sqrt{(1 \pm \delta_{n,0})/2}$ are coefficients, $z = x - iy$ are spatial coordinates, and $\phi_{n,j}(z)$ are the (gauge-dependent) non-relativistic Landau level (LL) wave functions, char-

acterized by the LL index $n \geq 0$, and a second quantum number $j \geq 0$ [31]. In symmetric gauge, j specifies the z-component of angular momentum, while in Landau gauge, it defines momentum along one direction in the plane. In graphene, a third quantum number $\gamma = \pm 1$, distinguishes between states at positive and negative energy, $E_{\gamma,n} = \gamma\omega_c\sqrt{n}$, where $\omega_c = \sqrt{2}v_F/l_B$ and $l_B = \sqrt{c/eB}$ is the magnetic length. The magnetic field strength is B and the Fermi velocity is v_F. In the following, we drop the index γ, and assume $\gamma = +1$, without loss of generality.

As illustrated in Fig. 1(a,b), we consider a coupling between the partially filled $n = M$ level at the Fermi surface to the empty LL $n = M+1$, described by ($\hbar = 1$):

$$\mathcal{H}_{\rm coup} = \sum_{j,j'} \Omega_{j,j'}(t) c^\dagger_{M+1,j} c_{M,j'} + \text{H.c.}. \tag{1}$$

Here, $c^\dagger_{M,j}$ and $c_{M,j}$ are the creation and annihilation operators in LL_M with the (angular) momentum quantum number j. For simplicity, we assume a plane wave drive, which acts uniformly on all orbitals: $\Omega_{j,j'}(t) = 2\Omega\delta_{j,j'}\cos(\omega t)$, with ω the drive frequency, and the Rabi frequency Ω. Within the rotating frame, transformed to by $U = \exp\left[-\frac{i}{2}\omega t \sum_j \left(c^\dagger_{M,j}c_{M,j} - c^\dagger_{M+1,j}c_{M+1,j}\right)\right]$, a rotating-wave approximation (RWA) removes the time-dependence from the coupling. The effective single-particle Hamiltonian then reads

$$\mathcal{H}_{\rm sp} = \sum_j -\frac{\delta}{2}\tau_z^{(j)} + \Omega\tau_x^{(j)}, \tag{2}$$

with δ the detuning of the light from the LL resonance, i.e. $\delta = E_{M+1} - E_M - \omega$. The notation of Eq. (2), using Pauli operators $\tau_z^{(j)} \equiv |M,j\rangle\langle M,j| - |M+1,j\rangle\langle M+1,j|$, and $\tau_x^{(j)} \equiv |M,j\rangle\langle M+1,j| + |M+1,j\rangle\langle M,j|$, captures the analogy to a spin-1/2 system, if the n quantum number is interpreted as the z-component of spin, or to a bilayer system if n is associated with a layer index. The first term in Eq. (2) corresponds to a Zeeman term (in the spin picture), while the second term mimics interlayer tunneling (in the bilayer picture). Both terms are independently tunable. The single-particle eigenstates are dressed LLs at energies $\pm\tilde{\Omega} = \pm\sqrt{\frac{\delta^2}{4} + \Omega^2}$, see Fig. 1(c). While strong coupling and/or far detuning lead to polarization in the lower dressed level, both manifolds are occupied if the gap between dressed states becomes small compared to the interaction strength, $e^2/\epsilon l_B$, i.e. if Ω and δ are sufficiently small. The transition occurs near $\Omega \sim 10^{-2}$ (in units of $e^2/\hbar\epsilon l_B$), above the threshold required for thermalization in the rotating frame Hamiltonian, $\Omega > 10^{-4}$, as estimated below.

Applying RWA to the interactions, the many-body Hamiltonian reads $\mathcal{H} = \mathcal{H}_{\rm sp} + \mathcal{H}_{\rm int}$, where

$$\mathcal{H}_{\rm int} = \sum_{\{n,j\}} A^{n_1,j_1,n_2,j_2}_{n_3,j_3,n_4,j_4} \delta_{n_1+n_2,n_3+n_4} c^\dagger_{n_1,j_1} c^\dagger_{n_2,j_2} c_{n_3,j_3} c_{n_4,j_4}. \tag{3}$$

The interaction matrix elements $A^{n_1,j_1,n_2,j_2}_{n_3,j_3,n_4,j_4}$ are the same as without light, but the RWA enforces conservation of single-particle energy, i.e. $\delta_{n_1+n_2,n_3+n_4}$.

Results. Before numerically solving $\mathcal{H}$ for small systems, we gain some intuition by decomposing the interactions into Haldane pseudopotentials [32]. These pseudopotentials describe the interaction strength V_j of two particles at fixed relative angular momentum j. In our case, we distinguish between intra-layer processes $V_j^{(n)}$ within LL_n, and inter-layer processes, $V_j^{\uparrow\downarrow,\downarrow\uparrow}$ and $V_j^{\uparrow\downarrow,\uparrow\downarrow}$, where the index $\uparrow$ ($\downarrow$) shall denote the LL_{M+1} (LL_M). Clearly, the difference between $V_j^{(M+1)}$ and $V_j^{(M)}$ breaks the $\mathbb{Z}_2$ symmetry usually present in a system of two equivalent layers. However, as seen from Fig. 2(a), this breaking is weak, since only potentials at odd j contribute to the intra-LL scattering of fermions, whereas the strongest n-dependence occurs for $V_0^{(n)}$. A more important difference to standard bilayer systems stems from the interaction $V_j^{\uparrow\downarrow,\uparrow\downarrow}$ where scattering particles exchange their LL index, while in standard bilayers only density-density-type interactions $V_j^{\uparrow\downarrow,\downarrow\uparrow}$ occur between two layers. Both types of inter-LL processes can conviently be accounted for by a single pseudopotential $V_j^{\rm inter}$. Therefore, we switch to a singlet/triplet basis, $|\pm\rangle \sim |\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle$, where the corresponding pseudopotentials are $V_j^{\pm} = (V_j^{\uparrow\downarrow,\downarrow\uparrow} \pm V_j^{\uparrow\downarrow,\uparrow\downarrow})/2$. Since $|+\rangle$ ($|-\rangle$) is even (odd) under particle exchange, it requires odd (even) j, and it is sufficient to consider

$$V_j^{\rm inter} = \left[V_j^{\uparrow\downarrow,\downarrow\uparrow} + (-1)^j V_j^{\uparrow\downarrow,\uparrow\downarrow}\right]/2. \tag{4}$$

As seen from Fig. 2(b), these inter-LL pseudopotentials $V_j^{\rm inter}$ are dominated by $j = 0$ for a coupling between LL_0 and LL_1 (denoted LL_{0-1}). In contrast, the repulsion between singlets at $j = 0$ is suppressed for a coupling between LL_1 and LL_2 (denoted LL_{1-2}), and $V_1^{\rm inter}$ becomes the dominant contribution. This behavior leads to the general expectation that coupling LL_{1-2} favors singlet phases and could give rise to bilayer quantum Hall phases which are derived from a hollow-core Hamiltonian. In the following, we will test this expectation at filling factors $\nu = 1/2$ and $\nu = 2/3$ using ED on torus [1, 44], sphere, and disk.

$\nu = 1/2$. Since the discovery of FQHE understanding the physics of a half filled LL has been a challenge. Early generalizations of the Laughlin wave functions to systems with spin provide an Abelian spin singlet state at $\nu = 1/2$, known as the (331)-Halperin state [4]. However, in most systems, no quantum Hall plateaux are observed

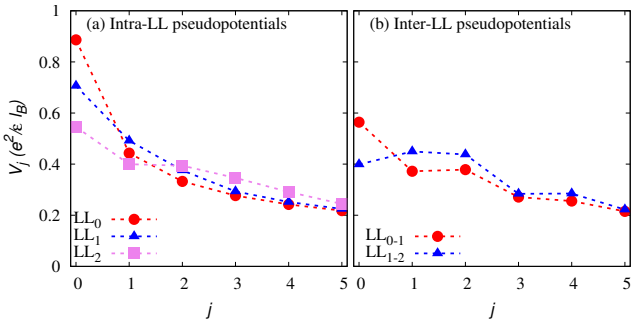

FIG. 2. (a) Pseudopotentials for scattering of two particles in the same graphene LL, $n = 0$, $n = 1$, and $n = 2$. (b) Pseudopotentials for scattering in different LLs, as defined in Eq. (4). If $n = 1$ is coupled to $n = 2$, $V_j^{\rm inter}$ is dominated by the contribution $j = 1$.

	Sphere	Disk	Torus
$\nu = 1/2$	0.85 ($N = 6$)	0.97	0.83 ($\mathbf{K} = \mathbf{0}$)
(HR)	0.75 ($N = 8$)	($N = 6, L = 24$)	0.72 ($\mathbf{K} \neq \mathbf{0}$)
	0.72 ($N = 10$)		($N = 8$)
$\nu = 2/3$	0.99 ($N = 4$)	0.81 ($N = 6, L = 18$)	
(IP)	0.55 ($N = 8$)	0.63 ($N = 8, L = 36$)	
	0.39 ($N = 12$)		

TABLE I. Overlaps of ground states in different geometries, for weak LL$_{1-2}$ coupling ($\Omega = 10^{-3}$ and $\delta = 0.02$), with HR state ($\nu = 1/2$), and with interlayer Pfaffian (IP) state ($\nu = 2/3$). At $\nu = 2/3$, fast decay of the overlap with N suggests a different phase, possibly a Fibonacci phase (see discussion), however we are not aware of unique trial wave functions to test the overlaps with this phase.

at $\nu = 1/2$. This fact has been explained by Halperin, Lee, and Read through a theory which attaches all magnetic fluxes to composite fermions [33]. As a consequence, these fermions do not feel a magnetic field, and may form a compressible Fermi liquid. In an alternative scenario, the composite fermions undergo BCS pairing which, due to the Meissner effect, leads to incompressibility [36, 45]. The most prominent paired state is the Moore-Read Pfaffian state. It involves p-wave pairing, and is spin polarized. In contrast, a spin singlet state can be obtained via d-wave pairing, and is known as the Haldane-Rezayi (HR) state [35]. Evidences of non-Abelian excitations have been discussed for both states [37]. The HR phase has been identified as a critical phase between strong and weak pairing [36], providing an example for a gapless FQH system. Hollow-core two-body interactions, i.e. pseudopotentials given by $V_j^{\rm intra} \sim \delta_{j,1}$ and $V_j^{\rm inter} \sim \delta_{j,1}$, yield a parent Hamiltonian for the HR state.

Accordingly, given the pseudopotential structure of coupled LLs discussed above, the HR phase becomes a likely candidate for coupling LL$_{1-2}$. Indeed, for sufficiently weak Rabi frequencies, numerical results support this expectation: In all three geometries, the ground state is a singlet, having large overlaps with the HR state (see Table I). We have also evaluated the overlap with the Jain singlet, which is known to have a large overlap with the ground state of pseudopotential $V_0 \simeq V_1$ [46]. However, since this overlap decreases rapidly with the system size, we excluded the Jain singlet as a possible candidate [47]. For the observed singlet phase, the topological degeneracy on the torus is $4q$-fold with ground states at high-symmetry points $\mathbf{K} = (0, 0)$, $\mathbf{K} = (0, N/2)$, $\mathbf{K} = (N/2, 0)$, and $\mathbf{K} = (N/2, N/2)$. While this is compatible with a (331)-phase, no sizable overlap with this phase are found in any geometry. The HR phase, as obtained from the hollow-core model, exhibits ground states at the same high-symmetry $\mathbf{K}$-points, but has two linearly independent ground states $\mathbf{K} = (0, 0)$. This $5q$-fold degeneracy of the HR phase has been discussed as a consequence of its criticality [36, 37], leading to a zero mode which can be either occupied or empty. However, the torus degeneracy of the HR state in the hollow-core model differs from the number of sectors in the underlying conformal field theory which is $4q$ [38], suggesting that the fifth ground state is not crucial for realizing the HR phase. In light of this point and based on the strong numerical evidence, the HR phase appears as the likely description of the observed singlet phase.

Upon increasing the Rabi frequency, a crossing of energy levels indicates a second-order phase transition (at $\Omega \approx 0.025$ and $\delta = 0.02$ in units $e^2/\epsilon l_B$, for $N = 8$ electrons on the torus). The ground state on the strong-coupling side is fully polarized in one LL, and the system exhibits Fermi sea behavior, indicated by ground states at finite angular momentum on the sphere, and at non-zero pseudomomenta on the torus. A Fermi liquid phase is also found for coupling LL$_{0-1}$ where this behavior extends to $\Omega \to 0$. For LL$_{0-1}$, increasing Ω only rotates the LL polarization from $\langle \sum_j \tau_z^{(j)} \rangle = N$ and $\langle \sum_j \tau_x^{(j)} \rangle = 0$ for $\Omega \to 0$, to $\langle \sum_j \tau_z^{(j)} \rangle = 0$ and $\langle \sum_j \tau_x^{(j)} \rangle = N$ for $\Omega \to \infty$. This pseudospin rotation is understood on the single-particle level by assuming that the ground state always remains polarized in the lower dressed LL.

$\nu = 2/3$. At filling fractions $1/q$ with q odd, electrons can anti-correlate by forming a Laughlin state [34]. Similarly, a Laughlin state of holes provides a good trial wave function at $\nu = 1 - 1/q$, including $\nu = 2/3$. In a bilayer at $\nu = 2/3$, various singlet phases compete with the polarized Laughlin state. Similar to the $\nu = 1/2$ case, Halperin (mmn)-states [4] are possible, including the (112)-state and the (330)-state, the latter being two uncorrelated copies of the 1/3 Laughlin states. Apart from these Abelian phases, there are also different non-Abelian phases. It has been argued that tunneling between the layers can transform the (330)-state into a phase supporting Fibonacci anyons [40]. These anyons are defined by simple fusion rules, but still allow for universal quantum

computing [43]. Other non-Abelian phases are obtained via p-type pairing, either between particles within a layer or between all particles, leading to intra- and the interlayer Pfaffian wave functions [41, 42]. Recently, extensive numerical works have revealed some of these phases if interactions are properly modified [48–50]. In particular, studies on the thin torus [40] as well as exact numerics [49] point towards a Fibonacci phase if the short-range contribution to the interlayer interactions is weakened.

In both coupling scenarios, LL_{0-1} and LL_{1-2}, ED on torus and sphere gives clear hints for a hole-conjugate Laughlin phase when the Rabi frequency is sufficiently strong. If the Laughlin state is formulated in a dressed LL basis, overlaps with this state reach close to 1, see Fig. 3(c,d). As already observed at $\nu = 1/2$, the two coupling scenarios show different behavior when Ω is decreased. Again, while for LL_{0-1} tuning the Rabi frequency only rotates the spin, a transition into a singlet phase occurs for LL_{1-2}, see Fig. 3(e,f). In contrast to $\nu = 1/2$, where the transitions occurs between two gapless phases, we now observe a transition between gapped phases, and the gap vanishes only at the critical point, see Fig. 3(b). Also, at $\nu = 2/3$, the transition does not affect the symmetry of the ground state ($\mathbf{K} = (0, 0)$ on both sides).

The identification of the singlet phase at weak LL_{1-2} coupling is challenging. On the sphere, where our numerics extend up to 12 electrons, we find large gaps for $N = 8$ and $N = 12$, but tiny gaps for $N = 6$ and $N = 10$, suggesting a tetra-periodic system behavior. While an intralayer Pfaffian state, requiring $\mathrm{mod}(N, 4) = 0$, would explain this pattern, the overlap with this state is zero (for $N = 8$ on sphere and disk). In contrast, significant overlaps are obtained with the interlayer Pfaffian state (see Table I). However, the corresponding $(3q)$-fold torus degeneracy is not seen for 8 or 10 electrons. Lacking obvious ground state degeneracies beyond the q-fold center-of-mass degeneracy, an Abelian phase such as Jain's spin-singlet state seems possible [3, 46, 51], but only infitesimal overlap is found. Given the relative weakness of V_0^{inter}, we shall also consider the Fibonacci phase. On the torus, it is characterized by $2q$ ground states at $\mathbf{K} = (0, 0)$ [49]. While we obtain the second and the third state at $\mathbf{K} = (0, 2)$ and $\mathbf{K} = (2, 0)$ on an isotropic torus, squeezing the torus changes this pattern, and the lowest two eigenstates indeed become singlets at $\mathbf{K} = (0, 0)$. Moreover, they have large overlaps with the corresponding eigenstates of the hollow-core Hamiltonian (0.76 and 0.81 on an isotropic torus), previously identified as representatives of the Fibonacci phase [49]. This makes the Fibonacci phase more likely than other candidate phases, although a final conclusion is impossible based on the available numerical results.

Thermalization. In this work, we have assumed that the electronic system thermalizes to the ground state in the rotating frame of the optical drive field. To estimate

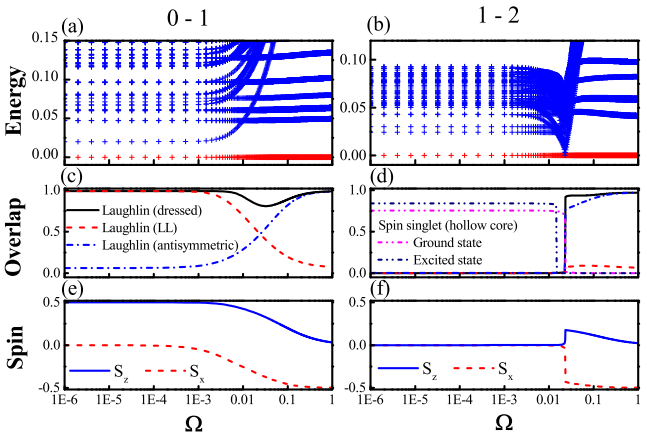

FIG. 3. (a,b) Energy levels (above ground state in units of $e^2/\epsilon l_B$) vs. Rabi frequency Ω, for coupling LL_{0-1} (a), and LL_{1-2} (b). (c,d) Ground state overlaps with trial wave functions (particle-hole conjugate 1/3 Laughlin state and a singlet phase obtained from hollow-core model). Trial states are constructed in three different bases: (1) *LL basis.* All the electrons reside in the lower LL. (2) *Dressed basis.* All electrons reside in lower eigenstates of Eq. (2), i.e. $|j\rangle \propto \left(\delta\sqrt{\delta^2 + 4\Omega^2}\right)|M+1, j\rangle + 2\Omega|M, j\rangle$. (3) *Antisymmetric basis.* All electrons reside in the singlet state, i.e., $|j\rangle \propto -|M+1, j\rangle + |M, j\rangle$. (e,f) Spin polarization $S_\alpha = \frac{1}{2N}\sum_j \langle \sum_j \tau_\alpha^{(j)} \rangle$ of the ground state vs. Ω for LL_{0-1} (e), and LL_{1-2} (f). Data in all panels (a–f) was obtained for 8 electrons on the torus, and $\delta = 0.02$.

the validity of this approximation, we must compare the timescale for relaxation of the optically excited Landau levels to the timescale for thermalization of the electronic system with the lattice. The carrier lifetime of optically excited Landau levels has contributions from optical relaxation, phonon relaxation, and Auger scattering into other Landau levels [52]. In Ref. [53], it was measured at moderate magnetic fields in epitaxial graphene samples to be roughly 10 - 20 ps. Although one expects longer lifetimes in higher quality graphene samples suitable to observe the FQH effect, we can use this as an upper bound on the relaxation rate. In units $e^2/\epsilon l_B$, this timescale translates to roughly 10^{-3} to 10^{-4}, depending on the magnetic field. For LL_{0-1} coupling, the Laughlin state of the driven and the non-driven regime are adiabatically connected, and one can adiabatically prepare the system by slowly turning on the light. In contrast, the singlet states for LL_{1-2} coupling cannot be connected to the non-driven regime, which makes the thermalization problem particularly relevant. For the case of the $\nu = 2/3$ singlet phase, we can roughly estimate the thermalization time by the size of the many-body gap in the spectrum, which, from Fig. 3, is on the order of 10^{-2}. As a result, there is a large separation of timescales between the thermalization and carrier relaxation, which allows the system to remain in the rotating frame ground states before carrier relaxation. For the gapless phases at

$\nu = 1/2$ the system will still thermalize in the rotating frame, however, the timescale is more difficult to estimate as it depends on the slowest diffusive modes in the system.

A more detailed study of the thermalization dynamics in this regime is beyond the scope of the present work; however, it is worth noting that there has been recent progress in understanding of thermalization of driven isolated systems [54–56] and also thermalization of Floquet systems coupled to a bath [57, 58]. It has been pointed out that electron-phonon interaction and specific Fermi reservoirs could lead to thermalization of the system in the rotating frame, in the long time steady-state limit [58].

In conclusion, we have considered single layer graphene in the FQH regime with an optical field in resonance with a LL transition. The proposed scheme synthesizes a two-component FQH system, with the light field playing the role of tunneling between two layers. For weak tunneling between LL_1 and LL_2, a many-body singlet phase is formed at $\nu = 1/2$ and $\nu = 2/3$. In contrast, strong tunneling and/or tunneling between LL_0 and LL_1 lead to a polarized phase within the lower dressed LL. Our study gives new impetus towards experimental realization of multicomponent FQH states and *in situ* control of the phase transition using externally applied optical fields and graphene. A similar scheme could also be applied to other 2D materials with Dirac bands, such as monolayer transition metal dichalcogenides [59, 60]. Conceptually, our approach is also connected to recent quantum simulations with cold atoms in which novel topological phases are engineered in synthetic spatial dimensions which are generated by the optical coupling of internal states [61–68].

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