SU(5) Unification without Proton Decay

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SU(5) Unification without Proton Decay

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We construct a four-dimensional $\mathrm{SU}(5)$ grand unified theory in which the proton is stable. The Standard Model leptons reside in the 5 and 10 irreps of $\mathrm{SU}(5)$, whereas the quarks live in the 40 and 50 irreps. The $\mathrm{SU}(5)$ gauge symmetry is broken by the vacuum expectation values of the scalar 24 and 75 irreps. All non-Standard Model fields are heavy. Stability of the proton requires three relations between the parameters of the model to hold. However, abandoning the requirement of absolute proton stability, the model fulfills current experimental constraints without fine-tuning.

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Introduction.—Grand unified theories (GUTs) present an attractive way to extend the Standard Model (SM) [1–5]. In addition to being esthetically appealing, they have several nice features – they reduce the number of multiplets, exhibit gauge coupling unification and explain why electric charges of quarks and leptons are connected.

The first attempt of partial unification was based on the group $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ [6], while the seminal papers describing full unification of couplings were those proposing $\mathrm{SU}(5)$ [7] and $\mathrm{SO}(10)$ [8] gauge groups. Unfortunately, GUTs with complete gauge coupling unification constructed so far in four dimensions are plagued with proton decay and the current experimental limit [9] excludes their simplest realization. Although there exist many models extending proton lifetime to an experimentally acceptable level (see [10] and references therein, including orbifold GUTs), a theoretically interesting question remains: is it at all possible to construct a viable four-dimensional GUT based on a single gauge group with an absolutely stable proton?

In this letter we propose such a model. The main idea is simple but the realization is somewhat involved. We present our model rather as a proof of concept, anticipating a simpler realization in the future. An alternative proposal achieves proton stability by imposing gauge conditions that eliminate all non-SM fields from the theory [11], resulting in a model that, however, appears to be indistinguishable from the SM. The only other four-dimensional models with a single unifying gauge group designed to completely forbid proton decay we are aware of [12, 13] are experimentally excluded due to the presence of new light particles carrying SM charges.

The most dangerous proton decay channels in GUTs are those mediated by vector leptoquarks and arise from gauge kinetic terms in the Lagrangian. In our model those channels are absent, since the quarks and leptons live in different $\mathrm{SU}(5)$ representations. In particular, the leptons reside in the 5 and 10 irreps of $\mathrm{SU}(5)$, the right-handed (RH) down quarks are formed from a linear combination of two 50 irreps, whereas the left-handed (LH) quark doublets and the RH up quarks come from a linear combination of two 40 irreps. The $\mathrm{SU}(5)$ gauge symmetry is spontaneously broken down to the SM by vacuum expectation values (vevs) of scalar field multiplets transforming as 24 and 75 irreps. In order to obtain correct SM masses, the SM Higgs is chosen to be part of a scalar 45 irrep multiplet, and there are no proton decay channels mediated by scalar leptoquarks from the Yukawa terms.

Particle content.—The model is based on the gauge group $\mathrm{SU}(5)$. The fermion sector of the theory is composed of the 5, 10, 40 and 50 irreps, where the 40 and 50 come in two vector-like copies, making the theory anomaly-free. The scalar sector consists of Higgs fields in the 24, 45 and 75 irreps.

Fermion sector.—The fermion multiplets in the theory come in the following LH spinor field representations, listed below along with their $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ decomposition [14]:

$$\begin{aligned}
5^c \;&=\; l \oplus D_5^c \;, \\
10 \;&=\; e^c \oplus Q_{10} \oplus U_{10}^c \;, \\
40_i &= Q_{40_i} \oplus U^c_{40_i} \oplus (1,2)_{-\frac{3}{2}} \oplus (\bar{3},3)_{-\frac{2}{3}} \oplus (8,1)_1 \oplus (\bar{6},2)_{\frac{1}{6}} \,, \\
\overline{40}_i &= Q^c_{\overline{40}_i} \oplus U_{\overline{40}_i} \oplus (1,2)_{\frac{3}{2}} \oplus (3,3)_{\frac{2}{3}} \oplus (8,1)_{-1} \oplus (6,2)_{-\frac{1}{6}} \,, \\
50^c_i &= D^c_{50_i} \oplus (1,1)_2 \oplus (3,2)_{\frac{7}{6}} \oplus (6,3)_{\frac{1}{3}} \oplus (\bar{6},1)_{-\frac{4}{3}} \\
&\quad\; \oplus (8,2)_{-\frac{1}{2}} \;, \\
\overline{50^c_i} &= D_{\overline{50}_i} \oplus (1,1)_{-2} \oplus (\bar{3},2)_{-\frac{7}{6}} \oplus (\bar{6},3)_{-\frac{1}{3}} \oplus (6,1)_{\frac{4}{3}} \\
&\quad\; \oplus (8,2)_{\frac{1}{2}} \;,
\end{aligned} \qquad (1)$$

where $i = 1, 2$. The lowercase fields l, e are the LH lepton doublet and RH electron, respectively. The fields Q, U and D have the same quantum numbers as the SM's LH quark doublet q and RH quark singlets u and d, respectively.

When coupling to the 5^c, $\mathrm{SU}(5)$ gauge bosons can act to transmute an l to an anti-D^c_5, and when coupling to the 10 to transmute Q_{10} to an anti-U^c_{10}. This is the standard route for proton decay in GUTs. If, however, the 5^c multiplet is split, in that the D^c_5 mass is comparable to the GUT scale, while that of l arises from electroweak symmetry breaking, and the light d quark arises from a linear combination of the anti-$D^c_{50_i}$, then proton decay cannot proceed through this gauge boson exchange. This is an example of the realization of the mechanism we are proposing for proton stability.

Higgs sector.—The scalar sector consists of the 24, 45 and 75 irreps of SU(5). Their decomposition into SM multiplets:

$$\begin{aligned}
24_H &= (1,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-\frac{5}{6}} \oplus (\bar{3},2)_{\frac{5}{6}} \oplus (8,1)_0 \ , \\
45_H &= H \oplus (3,1)_{-\frac{1}{3}} \oplus (3,3)_{-\frac{1}{3}} \oplus (\bar{3},1)_{\frac{4}{3}} \oplus (\bar{3},2)_{-\frac{7}{6}} \\
&\quad \oplus (\bar{6},1)_{-\frac{1}{3}} \oplus (8,2)_{\frac{1}{2}} \ , \\
75_H &= (1,1)_0 \oplus (3,1)_{\frac{5}{3}} \oplus (\bar{3},1)_{-\frac{5}{3}} \oplus (3,2)_{-\frac{5}{6}} \oplus (\bar{3},2)_{\frac{5}{6}} \\
&\quad \oplus (\bar{6},2)_{-\frac{5}{6}} \oplus (6,2)_{\frac{5}{6}} \oplus (8,1)_0 \oplus (8,3)_0 \ . \qquad (2)
\end{aligned}$$

Only the Higgses in the 24 and 75 irreps develop vevs at the GUT scale, which break the SU(5) gauge symmetry down to $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ [15, 16]. The SM Higgs field H is part of the 45 irrep.

Lagrangian.—The fermion kinetic terms in the Lagrangian are:

$$\mathcal{L}_{\rm kin} = i \sum_R \mathrm{Tr}\left(\overline{R}\, \not{D}\, R\right) \ , \qquad (3)$$

where the sum is over the representations $R = 5^c, 10, 40_i, \overline{40}_i$, 50_i^c and $\overline{50_i^c}$. In the standard SU(5) GUT those terms give rise to dangerous dimension-six operators mediating proton decay. In our model such terms generating proton decay are absent, since physical states of SM quarks and leptons reside in different representations of SU(5), as shown below.

The Yukawa interactions in our model are given by:

$$\begin{aligned}
\mathcal{L}_Y &= Y_l\, 5^c 10\, 45_H^* + Y_u^{ij}\, 40_i\, 40_j\, 45_H + Y_d^{ij} 40_i\, 50_j^c\, 45_H^* \\
&+ M_{40}^{ij}\, \overline{40}_i\, 40_j + \lambda_1^{ij}\, 24_H \overline{40}_i\, 40_j + \lambda_2^{ij}\, \overline{40}_i\, 24_H 40_j \\
&+ \lambda_3^i\, 24_H 10\, \overline{40}_i + \lambda_4^{ij}\, \overline{40}_i\, 75_H 40_j + \lambda_5^i\, 75_H 10\, \overline{40}_i \\
&+ M_{50}^{ij}\, 50_i^c\, \overline{50_j^c} + \lambda_6^{ij}\, 50_i^c\, 24_H \overline{50_j^c} + \lambda_7^{ij}\, 50_i^c\, 75_H \overline{50_j^c} \\
&+ \lambda_8^i\, 75_H 5^c\, \overline{50_i^c} + \mathrm{h.c.} \ , \qquad (4)
\end{aligned}$$

with an implicit sum over $i, j = 1, 2$, the terms with $\lambda_{1,2}^{ij}$ corresponding to the two independent contractions, and the Hermitian conjugate applied to non-Hermitian terms. In Eq. (4) the coefficients of the only other allowed gauge-invariant renormalizable Yukawa terms $Y_u'^{\,i}\, 10\, 40_i\, 45_H$ were set to zero.

Since the SM leptons live only in the 5 and 10 irreps while the SM quarks live only in the 40 and 50 irreps, along with the absence of proton decay through vector gauge bosons, there is no tree-level proton decay mediated by any of the Yukawa-type terms (contrary to other GUT models [17]). To see this, consider, for example, the first term in Eq. (4): an exchange of the $(3,1)_{-\frac{1}{3}}$ of the 45 necessarily couples the light lepton doublet l to the GUT-heavy Q_{10}.

The Lagrangian of the scalar sector consists of all possible renormalizable gauge-invariant terms involving the 24, 45 and 75 representations:

$$\begin{aligned}
\mathcal{L}_H &= -\tfrac{1}{2}\mu_{24}^2 \mathrm{Tr}(24_H^2) + \tfrac{1}{4}a_1\big[\mathrm{Tr}(24_H^2)\big]^2 + \tfrac{1}{4}a_2 \mathrm{Tr}(24_H^4) \\
&\quad - \tfrac{1}{2}\mu_{75}^2 \mathrm{Tr}(75_H^2) + \tfrac{1}{4}\sum b_k \mathrm{Tr}(75_H^4)_k + M_{45}^2 \mathrm{Tr}(|45_H|^2) \\
&+ \tfrac{1}{2}\sum g_k \mathrm{Tr}(24_H^2 75_H^2)_k + \sum h_k \mathrm{Tr}(24_H^2 |45_H|^2)_k + \ldots \qquad (5)
\end{aligned}$$

where the index $k = 1, 2, 3$ corresponds to the contractions in which the two lowest representations in a given trace combine into a singlet, a two-component tensor and a four-component tensor, respectively, and a prime is added if more than one contraction in each case exists. For simplicity, we exclude cubic terms in the scalar potential by assuming a $\mathcal{Z}_2$ symmetry of the Lagrangian.

We will now show that there exists a region of parameter space for which all SM fields have standard masses at the electroweak scale and below, whereas all new fields develop large masses.

Fermion representations 5 and 50.—We first focus on the particles in the representation of the down quark. After SU(5) breaking, the corresponding Lagrangian mass terms are:

$$\mathcal{L}_{\rm mass} = \left(D_{\overline{50}_1} \;\; D_{\overline{50}_2} \right) \mathcal{M}_D \begin{pmatrix} D_5^c \\ D_{50_1}^c \\ D_{50_2}^c \end{pmatrix} \ , \qquad (6)$$

with the mass matrix elements

$$\begin{aligned}
\mathcal{M}_D^{i,1} &= \tfrac{\sqrt{2}}{3}\lambda_8^i v_{75} \ , \\
\mathcal{M}_D^{i,j+1} &= M_{50}^{ij} + c_{24}^D \lambda_6^{ij} v_{24} + c_{75}^D \lambda_7^{ij} v_{75} \ ,
\end{aligned} \qquad (7)$$

where v_{24}, v_{75} are the vevs of the representations 24, 75, respectively, $c_{24}^D = 1/(3\sqrt{30})$ and $c_{75}^D = 1/(3\sqrt{2})$. In order to switch to the mass eigenstate basis, we perform a bi-unitary transformation

$$\mathcal{M}_D^{\rm diag} = (R_D)_{2\times 2}\, \mathcal{M}_D\, (L_D)_{3\times 3}^\dagger \qquad (8)$$

and, correspondingly, the mass eigenstates are

$$\begin{pmatrix} D_5^{c\,\prime} \\ D_{50_1}^{c\,\prime} \\ D_{50_2}^{c\,\prime} \end{pmatrix}_L = L_D \begin{pmatrix} D_5^c \\ D_{50_1}^c \\ D_{50_2}^c \end{pmatrix}_L , \quad \begin{pmatrix} D'_{\overline{50}_1} \\ D'_{\overline{50}_2} \end{pmatrix}_R = R_D \begin{pmatrix} D_{\overline{50}_1} \\ D_{\overline{50}_2} \end{pmatrix}_R . \qquad (9)$$

The unitary matrices L_D and R_D are used to diagonalize the matrices $\left[(\mathcal{M}_D)^\dagger \mathcal{M}_D\right]$ and $\left[\mathcal{M}_D (\mathcal{M}_D)^\dagger\right]$, respectively. From the structure of $\mathcal{M}_D$ we immediately infer that the matrix $\left[(\mathcal{M}_D)^\dagger \mathcal{M}_D\right]$ has one of the eigenvalues equal to zero. In order to completely forbid proton decay, the corresponding eigenstate $D_5^{c\,\prime}$ cannot contain any admixture of D_5^c. This is achieved by requiring the following tuning of parameters[1]:

$$\det\left(M_{50}^{ij} + c_{24}^D \lambda_6^{ij} v_{24} + c_{75}^D \lambda_7^{ij} v_{75} \right) = 0 \ . \qquad (10)$$

---

[1] Condition (10) does not take into account terms involving the SM Higgs. With just this relation satisfied and no further fine-tuning of the electroweak terms, this would produce a tiny mixing between the heavy and light fields suppressed by $v/M_{\rm GUT}$, where v is the SM Higgs vev and $M_{\rm GUT}$ is the unification scale. This would result in proton decay with lifetime $\tau_p \approx 10^{60}$ years. However, there exists a condition more general than (10) involving also the electroweak Yukawas, which ensures that there is no mixing between the SM quarks and the heavy fields. An alternative solution would be to stay with condition (10) and simply fine-tune Y_u^{ij} and Y_d^{ij}, so that they produce exactly the SM quark mass terms, without any mixing between the light and heavy states.

Field	$c_{24} \times \sqrt{30}$	$c_{75} \times 3\sqrt{2}$
D^c_{50}	$1/3$	1
$(1,1)_2$	2	3
$(3,2)_{\frac{7}{6}}$	$7/6$	1
$(\bar{6},1)_{-\frac{4}{3}}$	$-4/3$	1
$(6,3)_{\frac{1}{3}}$	$1/3$	-1
$(8,2)_{-\frac{1}{2}}$	$-1/2$	0

TABLE I. Contribution to the masses of the fermion components of the 50^c irrep generated by the Lagrangian terms in Eq. (13).

Field	$c_{24_1} \times \sqrt{30}$	$c_{24_2} \times \sqrt{30}$	$c_{75} \times 3\sqrt{2}$
U^c_{40}	$13/9$	$1/3$	$5/9$
Q_{40}	$-7/9$	$-4/3$	$1/9$
$(1,2)_{-\frac{3}{2}}$	2	-3	1
$(\bar{3},3)_{-\frac{2}{3}}$	$1/3$	-3	$-1/3$
$(\bar{6},2)_{\frac{1}{6}}$	$1/3$	2	$-1/3$
$(8,1)_1$	$-4/3$	2	$1/3$

TABLE II. Mass contribution generated by the terms involving the scalar 24 and 75 for the fermion components of the 40 irrep.

In this case $D^{c\prime}_5$ is a linear combination solely of $D^c_{50_1}$ and $D^c_{50_2}$, and can be associated with the SM field d^c:

$$d^c = L^{12}_D D^c_{50_1} + L^{13}_D D^c_{50_2} \,, \tag{11}$$

where the matrix entries $L^{1,j+1}_D$ are functions of M^{ij}_{50}, v_{24}, v_{75}, λ^{ij}_6, λ^{ij}_7 and λ^i_8.

The condition in Eq. (10) ensures that our model has no proton decay that would involve either a component of the SM lepton doublet l or the down quark d. To our knowledge this novel model building feature has not been discussed in the literature.

If one chooses to abandon the requirement of absolute proton stability, the parameters of the model need not be tuned. Proton decay experimental constraints [9] require merely

$$L^{11}_D \lesssim 0.1 \times \sqrt{(L^{12}_D)^2 + (L^{13}_D)^2} \,. \tag{12}$$

The factor of ~ 0.1 can be easily understood: The presence of D^c_5 in $D^{c\prime}_5$ would trigger proton decay. The standard $\mathrm{SU}(5)$ model predicts proton decay at a rate roughly 100 times larger than the current experimental bound. The contribution to this rate scales like the admixture of D^c_5 squared, thus the admixture itself has to be roughly less than 10%.

Finally, one also has to show that all the fields within the 50^c irrep other than D^c_{50} are heavy. For this to be the case, it is sufficient to show that the Lagrangian terms:

$$\Delta\mathcal{L}_{\rm mass} = \lambda^{ij}_6\, 50^c_i\, 24_H \overline{50^c_j} + \lambda^{ij}_7\, 50^c_i\, 75_H \overline{50^c_j} \tag{13}$$

generate different mass contributions:

$$\Delta\mathcal{M}^{ij} = c^R_{24}\lambda^{ij}_6 v_{24} + c^R_{75}\lambda^{ij}_7 v_{75} \tag{14}$$

for those representations than for D^c_{50}, since then the equivalent of condition (10) would not be fulfilled for those representations and they would acquire GUT-scale masses. The values of c_{24} and c_{75} are presented in Table I. When combined, these fulfill our requirements. Table I shows that the contribution of the term involving the 75 irrep in Eq. (13) gives the same mass for D^c_{50} as for $(3,2)_{\frac{7}{6}}$ and $(\bar{6},1)_{-\frac{4}{3}}$. The contribution of the term involving the 24 irrep in Eq. (13) breaks this degeneracy.

Fermion representations 10 and 40.—The analysis for the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ representations with the quantum numbers of the quark doublet Q and anti-up quark U^c is a little different, since they both reside in the 40 of $\mathrm{SU}(5)$. Following the reasoning from the previous case, we arrive at the two conditions:

$$\det\Big[M^{ij}_{40} + \big(c^{U,Q}_{24_1}\lambda^{ij}_1 + c^{U,Q}_{24_2}\lambda^{ij}_2\big) v_{24} + c^{U,Q}_{75}\lambda^{ij}_4 v_{75}\Big] = 0 \,, \tag{15}$$

with the values of the coefficients provided in Table II. If these relations are fulfilled, the SM fields u^c and q are not part of the 10 irrep, preventing the proton from decaying through channels involving q, u and e. We verified that there exists a class of values for the parameters M^{ij}_{40}, $\lambda^{ij}_{1,2,4}$ fulfilling the requirement (15), thus forbidding proton decay. The SM u^c and q are given by:

$$\begin{aligned} u^c &= L^{12}_U\, U^c_{40_1} + L^{13}_U\, U^c_{40_2} \,, \\ q &= L^{12}_Q\, Q_{40_1} + L^{13}_Q\, Q_{40_2} \,, \end{aligned} \tag{16}$$

where $L^{1,j+1}_{U,Q}$ are functions of M^{ij}_{40}, v_{24}, v_{75}, $\lambda^{ij}_{1,2,4}$ and $\lambda^i_{3,5}$. The values of $c^R_{24_1}$, $c^R_{24_2}$ and c^R_{75} for the other $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ components of the 40 are given in Table II. All those representations have different sets of c^R's as compared to U^c and Q and, consequently, Eq. (15) is not satisfied in those cases. Therefore, those representations develop GUT-scale masses.

Scalar representations 24, 45 and 75.—In our model the gauge group $\mathrm{SU}(5)$ is broken down to the SM by the GUT-scale vevs of the 24 and 75 irreps, while the 45 does not develop a vev. Stability of the scalar potential is equivalent to the condition that all squared masses of the components of the 24 and 75 irreps are positive, except for one combination of $(3,2)_{-\frac{5}{6}}$ and one of $(\bar{3},2)_{\frac{5}{6}}$ [15, 16, 18], the would-be Goldstone bosons of the broken $\mathrm{SU}(5)$. We checked that there exists a large region of parameter space for which all components of the 24 and 75 develop large positive squared masses, apart from the $(3,2)_{-\frac{5}{6}}$ and $(\bar{3},2)_{\frac{5}{6}}$ for which the mass-squared matrix is given by

$$\mathcal{M}^2_{(3,2)} = -\tfrac{1}{18}(g_2 + 11\, g_3 + 15\, g_3')\begin{pmatrix} \frac{v_{75}^2}{5} & \frac{v_{24}v_{75}}{2\sqrt{10}} \\ \frac{v_{24}v_{75}}{2\sqrt{10}} & \frac{v_{24}^2}{8} \end{pmatrix}. \tag{17}$$

We have used relations between parameters satisfied at the stationary point of the potential. The constant of proportionality is a combination of coupling constants, defined in Eq. (5), and can take either sign. The matrix (17) has a vanishing determinant so that one of the linear combinations of the fields is massless while the other is heavy.

The representation 45 does not take part in $\mathrm{SU}(5)$ breaking and its $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ components generically have masses at the GUT scale. Since one of those fields is the SM Higgs, a cancellation between some of the parameters of the potential is required. To show that such an arrangement is possible, it is sufficient to consider only the explicit mass term for the 45 along with the terms mixing it with the 24 in Eq. (5). A small SM Higgs mass contribution is obtained for:

$$M_{45}^2 + \left(h_1 - \tfrac{67}{240} h_2 + \tfrac{31}{120} h_2' - \tfrac{13}{60} h_3 - \tfrac{5}{16} h_3'\right) v_{24}^2 \simeq 0\,. \quad (18)$$

We verified that there exists a wide range of parameters for which the GUT-scale masses of all other components of the 45 are positive. The fine-tuning in Eq. (18) is equivalent to the standard $\mathrm{SU}(5)$ doublet-triplet splitting problem and perhaps may be solved by introducing additional $\mathrm{SU}(5)$ representations along the lines of [19, 20].

Quark and lepton masses.—The SM electron Yukawa emerges from the term:

$$Y_l\, 5^c 10\, 45_H^* \supset y_l\, l\, H^* e^c\,. \quad (19)$$

The terms contributing to the SM down quark mass are:

$$Y_d^{ij}\, 40_i\, 50_j^c\, 45_H^* \supset y_d\, q\, H^* d^c\,, \quad (20)$$

and for the SM up quark we have:

$$Y_u^{ij}\, 40_i\, 40_j\, 45_H \supset y_u\, q\, H\, u^c\,. \quad (21)$$

There is no need to correct the typical $\mathrm{SU}(5)$ relation between the electron and down quark Yukawas, since they are not directly related in our model.

Proton stability at loop level.—So far, we have shown that the model proposed in this letter is completely free from any tree-level proton decay. As it turns out, it is also possible to forbid proton decay at any order in perturbation theory.

First we note that the model has no proton decay at any loop order mediated by vector gauge bosons or scalars from the 45 irrep. This can be argued on symmetry grounds. All the Lagrangian terms in Eqs. (3) and (4), apart from $\lambda_3^i\, 24_H 10\, \overline{40}_i$, $\lambda_5^i\, 75_H 10\, \overline{40}_i$ and $\lambda_8^i\, 75_H 5^c\, \overline{50_i^c}$, are invariant under:

$$5^c \to -5^c\,,\ \ 10 \to -10\,. \quad (22)$$

Under this transformation, the SM leptons are odd while the SM quarks are even. For proton decay one must have an odd number of leptons in the final state and none in the initial state, and there must be no heavy particles in either the initial or final states. This is odd under the transformation (22), and hence forbidden.

The only remaining loop-level proton decay channels are those mediated by the scalars from the 24 and 75 irreps. To forbid these, we assume that the spontaneous breaking of $\mathrm{SU}(5)$ is nonlinearly realized [21] and we can replace the 24 and 75 irreps by nondynamical condensates [11]. The 24 and 75 scalar sector of the theory is then described by a nonlinear sigma model [22, 23]. This concludes the proof that in our model the proton is stable.

Conclusions.—We have constructed a grand unified model in four dimensions based on the gauge group $\mathrm{SU}(5)$ which does not exhibit any proton decay. This was accomplished by assigning the quarks and leptons to different irreps of $\mathrm{SU}(5)$. In order to forbid proton decay at tree level, three relations between the model parameters have to hold. In addition, for proton stability at any loop order, the $\mathrm{SU}(5)$ breaking has to be nonlinearly realized. Abandoning the requirement of absolute proton stability removes the necessity of any tuning or the nonlinear symmetry breaking, and the model is consistent with experiments for a large range of natural parameter values.

The model has additional desirable features. Upon adding one [24] or several [25, 26] extra scalar representations it allows for gauge coupling unification if some of the scalar fields from the 45 irrep are at the TeV scale. It also contains no problematic relation between the electron and down quark Yukawa plaguing the standard $\mathrm{SU}(5)$ models. However, the usual doublet-triplet splitting problem still persists and requires further model building, perhaps along the lines of a non-supersymmetric version of [19].

Let us stress again that our goal was just to show through an explicit construction that, contrary to common belief, four-dimensional grand unified theories with a stable proton do exist. We hope that this may inspire new directions in model building and revive the interest in grand unification, which perhaps deserves more attention in spite of negative results from proton decay experiments.

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