Heavy-Quark Symmetry Implies Stable Heavy Tetraquark Mesons Q_{i}Q_{j}q[over ¯]_{k}q[over ¯]_{l}

Estia J. Eichten and Chris Quigg

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Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_iQ_j\bar{q}_k\bar{q}_l$

Estia J. Eichten* and Chris Quigg†
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510 USA
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For very heavy quarks Q, relations derived from heavy-quark symmetry predict the existence of novel narrow doubly heavy tetraquark states of the form $Q_iQ_j\bar{q}_k\bar{q}_l$ (subscripts label flavors), where q designates a light quark. By evaluating finite-mass corrections, we predict that double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will be stable against strong decays, whereas the double-charm states $cc\bar{q}_k\bar{q}_l$, mixed beauty+charm states $bc\bar{q}_k\bar{q}_l$, and heavier $bb\bar{q}_k\bar{q}_l$ states will dissociate into pairs of heavy-light mesons. Observation of a new double-beauty state through its weak decays would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.

Following the discovery of the charmonium-associated state $X(3872)$ by the BELLE collaboration [1], experiments have led a renaissance in hadron spectroscopy [2].

Many of the newly observed states invite identification with compositions less spare than the traditional quark–antiquark meson and three-quark baryon schemes [3]. Tetraquark states composed of a heavy quark and antiquark plus a light quark and antiquark have attracted much attention. The observed candidates all fit the form $c\bar{c}q_k\bar{q}_l$, where the light quarks q may be $u, d,$ or s. No such states are observed significantly below threshold for strong decays into two heavy-light meson states $\bar{c}q_l + c\bar{q}_k$; all have strong decays to $c\bar{c}$ charmonium + light mesons.

In this Letter we examine the possibility of tetraquark configurations for which all strong decays are kinematically forbidden. We show that, in the heavy-quark limit, stable—hence exceedingly narrow—$Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist. To apply this insight, we take into account corrections for finite heavy-quark masses to deduce which tetraquark states containing b or c quarks should be stable. The most promising example is a $J^P = 1^+$ isoscalar double-b meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}$.

In the heavy-quark limit, the lowest-lying tetraquark configurations resemble the helium atom, a factorized system with separate dynamics for the compact heavy color-$\bar{\mathbf{3}}$ Q_iQ_j "nucleus" and for the light quarks bound to the stationary color charge. [We recall that the one-gluon-exchange interaction is attractive for two quarks forming a color antitriplet, with half the strength of the attraction between a quark and antiquark bound in a color singlet.] At large $Q_i - Q_j$ separations, which become increasingly important as the heavy-quark masses decrease, the light $\bar{q}_k\bar{q}_l$ cloud screens the Q_iQ_j interaction, so that the $Q_iQ_j\bar{q}_k\bar{q}_l$ complex may rearrange into a pair of heavy-light mesons [4]. For heavy quarks Q_iQ_j bound in a color $\bar{\mathbf{3}}$ by an effective potential of the "Cornell" Coulomb + linear form at half strength for both components [5], the rms core radii are $\langle r^2\rangle^{1/2} = 0.28\text{ fm}\,(cc); 0.24\text{ fm}\,(bc); 0.19\text{ fm}\,(bb)$, all considerably smaller than the size of the associated tetraquark states. Hence the core-plus-light (anti)quarks idealization should be a reliable guide to the masses of ground-state tetraquarks containing charms and bottoms.

The ground state of the attractive $\bar{\mathbf{3}}$ Q_iQ_j configuration may have total spin $S_{Q_iQ_j} = 1$ for identical quarks $(i = j)$ or for quarks of different flavors $(i \neq j)$ in a symmetric flavor configuration $\{Q_iQ_j\}$ or total spin $S_{Q_iQ_j} = 0$ for quarks of different flavors $(i \neq j)$ in an antisymmetric flavor configuration $[Q_iQ_j]$. To construct a color-singlet $Q_iQ_j\bar{q}_k\bar{q}_l$ state, the light $\bar{q}_k\bar{q}_l$ must be in a color-$\mathbf{3}$. For the tetraquark ground state, both the heavy Q_iQ_j and light $\bar{q}_k\bar{q}_l$ pairs must be in $(\ell = 0)$ s-waves. To satisfy the Pauli principle, the flavor-symmetric $\{\bar{q}_k\bar{q}_l\}$ state must have total (light-quark) spin $j_\ell = 1$, whereas the flavor-antisymmetric $[\bar{q}_k\bar{q}_l]$ must have $j_\ell = 0$.

Stability in the heavy-quark limit. For very heavy quarks, a hadron mass receives negligible contributions from the motion of the heavy quarks and spin interactions. Accordingly, the following relations hold among the masses of heavy-light and doubly-heavy-light mesons and baryons [6]:

$$\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y)\\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y)\\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\;.
\end{aligned} \tag{1}$$

(In the limit, a heavy core is a heavy core!)

It is easy to see that the dissociation of $Q_iQ_j\bar{q}_k\bar{q}_l$ into two heavy-light mesons is kinematically forbidden, for sufficiently heavy quarks. The $\mathcal{Q}$-value for the decay is

$$\begin{aligned}\mathcal{Q} \equiv m(Q_iQ_j\bar{q}_k\bar{q}_l) - [m(Q_i\bar{q}_k) + m(Q_j\bar{q}_l)] = \\ \Delta(q_k, q_l) - \tfrac{1}{2}\left(\tfrac{2}{3}\alpha_{\mathrm{s}}\right)^2(1 + O(v^2))\overline{M} + O(1/\overline{M}) \ ,\end{aligned} \tag{2}$$

where $\Delta(q_k, q_l)$, the contribution due to light dynamics, becomes independent of the heavy-quark masses, $\overline{M} \equiv (1/m_{Q_i} + 1/m_{Q_j})^{-1}$ is the reduced mass of Q_i and Q_j, and α_{s} is the strong coupling. The velocity-dependent hyperfine corrections, here negligible, are calculable in the NRQCD formalism [7]. For large enough values of $\overline{M}$, the middle term dominates, so the tetraquark is stable against decay into two heavy-light mesons.

The other possible decay channel is to a doubly heavy baryon and a light antibaryon,

$$(Q_iQ_j\bar{q}_k\bar{q}_l) \to (Q_iQ_jq_m) + (\bar{q}_k\bar{q}_l\bar{q}_m) \ . \tag{3}$$

By Eq. 1, we have

$$m(Q_iQ_j\bar{q}_k\bar{q}_l) - m(Q_iQ_jq_m) = m(Q_xq_kq_l) - m(Q_x\bar{q}_m) \ . \tag{4}$$

In the heavy-quark regime, the flavored-baryon–flavored-meson mass difference on the right-hand side of Eq. 4 has the generic form $\Delta_0 + \Delta_1/M_{Q_x}$. Using the observed mass differences, $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV, and choosing effective quark masses $m_c \equiv m(J/\psi)/2 = 1.55$ GeV, $m_b \equiv m(\Upsilon)/2 = 4.73$ GeV, we find $\Delta_1 = 176.6\ \mathrm{MeV}^2$ and $\Delta_0 = 303$ MeV, hence the mass difference in the heavy-quark limit is 303 MeV. All of these mass differences are smaller than the mass of the lightest antibaryon, $m(\bar{p}) = 938.27$ MeV, so we conclude that no decay to a doubly heavy baryon and a light antibaryon is kinematically allowed. *This completes the demonstration that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist.*

Beyond the heavy-quark limit. To ascertain whether stable tetraquark mesons might be observed, we must estimate masses of the candidate configurations. Numerous model calculations exist in the literature [8], but it is informative to make estimates in the spirit of heavy-quark symmetry.

The leading-order corrections for finite heavy-quark mass correspond to hyperfine spin-dependent terms and a kinetic energy shift that depends only on the light degrees of freedom,

$$\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}} \ , \tag{5}$$

where $\mathcal{M} = m_{Q_i}$ or $m_{Q_i} + m_{Q_j}$ denotes the mass of the heavy-quark core for hadrons containing one or two heavy quarks and the coefficients $\mathcal{S}$ and $\mathcal{K}$ are to be determined from experimental data summarized in Table I. The spin splittings lead directly to the coefficients $\mathcal{S}$ tabulated in the last column. The pattern of the spin coefficients is entirely consistent with the expectations of heavy-quark symmetry.

The kinetic energy shift due to light quarks will be different in $Q\bar{q}$ mesons and Qqq baryons. By comparing the centroid (or center-of-gravity, cog) masses for the charm and bottom systems we can extract the difference of the kinetic-energy coefficients $\mathcal{K}$ for states that contain one or two light quarks, viz. $\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$. For example,

$$\begin{aligned}&[m((cud)_{\bar{\mathbf{3}}}) - m(c\bar{d})] - [m((bud)_{\bar{\mathbf{3}}}) - m(b\bar{d})] \\ &\qquad = \delta\mathcal{K}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) = 5.11\ \mathrm{MeV} \ ,\end{aligned} \tag{6}$$

from which we extract $\delta\mathcal{K} = 0.0235\ \mathrm{GeV}^2$. The resulting mass shifts are

$$\begin{aligned} m(\{cc\}(\bar{u}\bar{d})) - m(\{cc\}d)&: \ \frac{\delta\mathcal{K}}{4m_c} = 2.80\ \mathrm{MeV} \\ m((bc)(\bar{u}\bar{d})) - m(\{bc\}d)&: \ \frac{\delta\mathcal{K}}{2(m_c + m_b)} = 1.87\ \mathrm{MeV} \\ m(\{bb\}(\bar{u}\bar{d})) - m(\{bb\}d)&: \ \frac{\delta\mathcal{K}}{4m_b} = 1.24\ \mathrm{MeV}\end{aligned} \tag{7}$$

These values are small—only slightly larger than the isospin breaking effects that we neglect as too small to affect the question of stability [12].

Combining the heavy-quark-symmetry relations of Eq. 1 with the leading-order corrections we obtain the masses of ground-state $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks summarized in Table II [15]. As inputs for the doubly heavy baryons not yet experimentally measured, we use the model calculations of Karliner and Rosner [13].

Narrow Tetraquark States. As we explained in the discussion surrounding Eq. 4, strong decays of $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to a doubly heavy baryon and a light antibaryon are kinematically forbidden for all the ground states. Strong decay to a pair of heavy-light mesons will occur if the tetraquark state lies above threshold. For $J^P = 0^+$ or 2^+, a $Q_iQ_j\bar{q}_k\bar{q}_l$ meson might decay to a pair of heavy-light pseudoscalar mesons while for $J^P = 1^+$ the allowed decay channel would be a pseudoscalar plus a vector meson. According to our mass estimates, the only tetraquark mesons below threshold are the axial vector $\{bb\}[\bar{u}\bar{d}]$ meson, $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]}$, that is bound by 121 MeV and the axial vector $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons bound by 48 MeV. We expect all the other $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks to lie at least 82 MeV above the corresponding thresholds for strong decay [16]. Promising final states include $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{d}]} \to \Xi^0_{bc}\bar{p}$, $B^-D^+\pi^-$, and $B^-D^+\ell^-\bar{\nu}$ (which establishes a weak decay), $\mathcal{T}^{\{bb\}-}_{[\bar{u}\bar{s}]} \to \Xi^0_{bc}\bar{\Sigma}^-$, $\mathcal{T}^{\{bb\}0}_{[\bar{d}\bar{s}]} \to \Xi^0_{bc}(\bar{\Lambda}, \bar{\Sigma}^0)$, and so on.

As others have noted [8, 17], unstable doubly heavy tetraquarks might be reconstructed as resonances in the "wrong-sign" combinations of DD, DB, and BB. The

TABLE I. Representative masses [9], in MeV, and derived quantities for ground-state hadrons containing heavy quarks.

State[a]	j_ℓ	Mass ($j_\ell + \frac{1}{2}$)	Mass ($j_\ell - \frac{1}{2}$)	Centroid	Spin Splitting	$\mathcal{S}$ [GeV^2]
$D^{(*)}$ ($c\bar{d}$)	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7	0.436
$D_s^{(*)}$ ($c\bar{s}$)	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8	0.446
Λ_c $(cud)_{\mathbf{\bar{3}}}$	0	2286.46	–	–		–
Σ_c $(cud)_{\mathbf{6}}$	1	2518.41	2453.97	2496.93	64.44	0.132
Ξ_c $(cus)_{\mathbf{\bar{3}}}$	0	2467.87	–	–		–
Ξ_c' $(cus)_{\mathbf{6}}$	1	2645.53	2577.4	2622.82	68.13	0.141
Ω_c $(css)_{\mathbf{6}}$	1	2765.9	2695.2	2742.33	70.7	0.146
Ω_{cc} $(ccu)_{\mathbf{\bar{3}}}$	0	3621.40[b]	–		–	
$B^{(*)}$ ($b\bar{d}$)	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33	0.427
$B_s^{(*)}$ ($b\bar{s}$)	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5	0.459
Λ_b $(bud)_{\mathbf{\bar{3}}}$	0	5619.58	–		–	
Σ_b $(bud)_{\mathbf{6}}$	1	5832.1	5811.3	5825.2	20.8	0.131
Ξ_b $(bds)_{\mathbf{\bar{3}}}$	0	5794.5	–		–	
Ξ_b' $(bds)_{\mathbf{6}}$	1	5955.33	5935.02	5948.56	20.31	0.128
Ω_b $(bss)_{\mathbf{6}}$	1		6046.1			
B_c ($b\bar{c}$)	$\frac{1}{2}$	*6329*[c]	6274.9	*6315.4*[c]	*54*[c]	*0.340*[c]

[a] Subscripts denote flavor-SU(3) representations for heavy baryons.

[b] From the LHC*b* observation, Ref. [10].

[c] Inferred from the lattice QCD calculation of Ref. [11].

TABLE II. Expectations for ground-state tetraquark masses, in MeV.[a] The column labeled HQS Relation gives the sum of the right-hand-side of Eq. 1 and the kinetic-energy mass shifts of Eq. 8. Here q denotes an up or down quark.

State	J^P	j_ℓ	$m(Q_iQ_jq_m)$ (cog)	HQS relation	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	0	3663[b]	$m(\{cc\}u) + 315$	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	0	3764[c]	$m(\{cc\}s) + 392$	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	3663	$m(\{cc\}u) + 526$	4146, 4167, 4210	D^+D^0, D^+D^{*0} 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	0	6914	$m([bc]u) + 315$	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	0	7010[d]	$m([bc]s) + 392$	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	1	6914	$m([bc]u) + 526$	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	0	6957	$m(\{bc\}u) + 315$	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	0	7053[d]	$m(\{bc\}s) + 392$	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	6957	$m(\{bc\}u) + 526$	7461, 7472, 7493	BD/B^*D 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	0	10176	$m(\{bb\}u) + 306$	10482	$B^-\bar{B}^{*0}$ 10603	$\boxed{-121}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	0	10252[c]	$m(\{bb\}s) + 391$	10643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10695/10691	$\boxed{-48}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	1	10176	$m(\{bb\}u) + 512$	10674, 10681, 10695	B^-B^0, B^-B^{*0} 10559, 10603	115, 78, 136

[a] Masses of the unobserved doubly heavy baryons are taken from Ref. [13]; for lattice evaluations of b-baryon masses, see Ref. [14]

[b] Based on the mass of the LHC*b* Ξ_{cc}^{++} candidate, 3621.40 MeV, Ref. [10].

[c] Using the s/d mass differences of the corresponding heavy-light mesons.

[d] Evaluated as $\frac{1}{2}[m(c\bar{s}) - m(c\bar{d}) + m(b\bar{s}) - m(b\bar{d})] + m(bcd)$.

doubly charged $\mathcal{T}_{[\bar{d}\bar{s}]}^{\{cc\}++} \to D^+D_s^+$, etc. would stand out as *prima facie* evidence for a non-$q\bar{q}$ level.

While the production of $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons is undoubtedly a rare event, we draw some encouragement for near-term searches from the large yield of B_c mesons recorded in the LHC*b* experiment [18] and the not inconsiderable rate of Double-Υ production observed in 8-TeV pp collisions by the CMS experiment, $\sigma(pp \to \Upsilon\Upsilon +$ anything$) = 68 \pm 15$ pb [19]. The ultimate search instrument might be a future electron–positron Tera-Z factory, for which the branching fractions [9] $Z \to b\bar{b} = 15.12 \pm 0.05\%$ and $Z \to b\bar{b}b\bar{b} = (3.6 \pm 1.3) \times 10^{-4}$ offer hope of many events containing multiple heavy quarks.

Concluding remarks. We have shown that, in the heavy-quark limit, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks must exist. Our estimates of tetraquark masses lead us to expect that strong decays of the $J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$, $\{bb\}[\bar{u}\bar{s}]$, and $\{bb\}[\bar{d}\bar{s}]$ states are kinematically forbidden, so that these states should be exceedingly narrow, decaying only through the charged-current weak interaction. Observation of any of these states would signal the existence of a new form of stable matter, in which the doubly heavy color-$\bar{\mathbf{3}}$ Q_iQ_j diquark is a basic building block. The unstable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquarks—particularly those with small $\mathcal{Q}$-values—may be observable as resonances decaying into pairs of heavy-light mesons, if they are not too broad to stand out above backgrounds.

Note added. After completing this article, we learned of interesting calculations of tetraquark masses that also highlight the likelihood of a stable doubly heavy tetraquark [20]. These papers do not, however, show that a stable $Q_iQ_j\bar{q}_k\bar{q}_l$ tetraquark is compulsory in the heavy-quark limit.

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[*] Email: eichten@fnal.gov; ORCID: 0000-0003-0532-2300

[†] Email: quigg@fnal.gov; ORCID: 0000-0002-2728-2445

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