This is the accepted manuscript made available via CHORUS. The article has been published as:

Long-Lived Inverse Chirp Signals from Core-Collapse in Massive Scalar-Tensor Gravity
Ulrich Sperhake, Christopher J. Moore, Roxana Rosca, Michalis Agathos, Davide Gerosa, and Christian D. Ott
DOI: [10.1103/PhysRevLett.119.201103](https://doi.org/10.1103/PhysRevLett.119.201103)
Long-lived inverse chirp signals from core collapse in massive scalar-tensor gravity

Ulrich Sperhake,1,2,∗ Christopher J. Moore,1,3 Roxana Rosca,3
Michalis Agathos,1 Davide Gerosa,2,† and Christian D. Ott2

1 DAMTP, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK
2 TAPIR 350-17, Caltech, 1200 E. California Boulevard, Pasadena, California 91125, USA
3 IST-CENTRA, Departamento de Física, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal

(Dated: October 2, 2017)

This letter considers stellar core collapse in massive scalar–tensor theories of gravity. The presence of a mass term for the scalar field allows for dramatic increases in the radiated gravitational wave signal. There are several potential smoking gun signatures of a departure from general relativity associated with this process. These signatures could show up within existing LIGO–Virgo searches.

Introduction – General relativity (GR) has successfully passed numerous tests [1, 2] and, in the words of [3], “occupies a well-earned place next to the standard model as one of the two pillars of modern physics”. And yet, the enigmatic nature of dark energy and dark matter evoked in the explanation of cosmological and astrophysical observations [4], as well as theoretical considerations regarding the renormalization of the theory in a quantum theory sense, indicate that GR may ultimately need modifications in the low and/or high-energy regime [5].

Tests of GR have so far been almost exclusively limited to relatively weak fields. But the recent breakthrough detection of gravitational waves (GWs) by LIGO [6] has opened a new observational channel towards strong-field gravity, and tests of Einstein’s theory are a key goal of the new field of GW physics [7, 8]. Most GW-based tests either (i) construct a phenomenological parameterization of possible deviations from the expected physics and seek to constrain the different parameters, or (ii) model the physical system in the framework of a chosen alternative theory to see if it can better explain the observed data.

The latter approach faces significant challenges; the candidate theory must agree with GR in the well-tested weak-field regime and yet lead to measurable strong-gravity effects. Furthermore a mathematical understanding of the theory, in particular its well-posedness, is necessary for fully non-linear simulations. One of the most popular candidate extensions of GR are scalar tensor (ST) theories of gravity [9, 10], adding a scalar sector to the vector and tensor fields of Maxwell-GR. Scalar fields naturally arise in higher-dimensional theories including string theory, feature prominently in cosmology, and ST theories have a well-posed Cauchy formulation. ST theories also give rise to the most concrete example of a strong deviation from GR known to date: the spontaneous scalarization of neutron stars [11]. The magnitude of this effect facilitates strong constraints on the parameter space of ST theory through binary pulsar observations [12–14]. These bounds, as well as the impressive constraints obtained from the Cassini mission [15], however, are all based on observations of widely separated objects and, therefore, apply only to massless ST theory (or theories with a scalar mass μ ≤ 10−19 eV yielding a Compton wavelength, λc = (2πℏ)/(μc), greater than or comparable to the objects’ separation [3, 16]).

Deviations of black-hole spacetimes from GR are limited in ST gravity due to the no-hair theorems [17, 18], although we note that scalar radiation has been observed in black-hole binary simulations for non-trivial scalar potentials [19] or boundary conditions [20]. Nevertheless, the most straightforward way to bypass the no-hair theorems is to depart from vacuum. Neutron stars and stellar core collapse thus appear to be the most promising systems to search for characteristic signatures; cf. [21–23] and references therein.

Here, we perform the first study of dynamic strong-field systems in massive ST theory through exploring GW generation in core collapse. As we will see below, the GW signal is dominated by the rapid phase transition from weak to strong scalarization and the ensuing dispersion of the signal. We therefore focus in this study on spherically symmetric models which capture the key features of the collapse responsible for spontaneous scalarization.

The most promising range of the scalar field mass μ for generating strong scalarization and satisfying existing binary pulsar constraints has been identified as μ ≥ 10−15 eV [16, 24]. In massive ST theory, low frequency modes with f < f∗ = μ/(2πℏ) decay exponentially with distance rather than radiate towards infinity. For masses μ > 10−13 eV (f∗ > 24.2 Hz), the GW power detectable inside the LIGO sensitivity window 10 Hz ≤ f ≤ 103 Hz would be considerably reduced due to this effect. We therefore study in this work the range 10−15 eV ≤ μ ≤ 10−13 eV.

Formalism – The starting point of our formulation is the generic action for a scalar-tensor theory of gravity that (i) involves a single scalar field non-minimally coupled to the metric, (ii) obeys the covariance principle, (iii) results in field equations of at most second differential order, and (iv) satisfies the weak equivalence principle. In the Einstein frame, the action can be written in the form (using natural units G = c = 1) [5, 10]

\[ S = \frac{1}{16\pi} \int dx^4 \sqrt{-g} \left[ R - 2\phi^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 4V(\phi) \right] + S_m, \quad (1) \]
where $\varphi$ is the scalar field, $F(\varphi)$ the coupling function, $V(\varphi)$ the potential, and $R$ and $\bar{g}$ are the Ricci scalar and determinant constructed from the conformal metric $\bar{g}_{\mu\nu}$. $S_m$ denotes the contribution due to matter fields, that couple to the physical or Jordan-Fierz metric $g_{\mu\nu}=\bar{g}_{\mu\nu}/F(\varphi)$ and the physical energy momentum tensor is $T^{\mu\nu}=2(-g)^{-1/2}\delta S_m/\delta g_{\mu\nu}$; assumed here to describe a perfect fluid with baryon density $\rho$, pressure $P$, internal energy $\epsilon$, enthalpy $H$ and 4-velocity $u^\alpha$,

$$T_{\alpha\beta}=\rho Hu_{\alpha}u_{\beta}+Pg_{\alpha\beta}, \quad H=1+\epsilon+P/\rho. \quad (2)$$

The equations of motion are given by

$$\ddot{\bar{g}}_{\alpha\beta}=2\partial_{\alpha}\varphi\partial_{\beta}\varphi-\bar{g}_{\alpha\beta}\partial_{\mu}\varphi\partial_{\mu}\varphi+8\pi\bar{g}_{\alpha\beta}-2V\bar{g}_{\alpha\beta},$$

$$\nabla^\mu\nabla_{\mu}\varphi=2\pi(F\varphi/F)\bar{T}+V\varphi,$$

$$\nabla_{\mu}\bar{T}^{\mu\alpha}=-\frac{1}{2}\frac{F}{F^2}\bar{T}\bar{g}^{\alpha\beta}\partial_{\beta}\varphi, \quad \nabla_{\mu}(\rho u^\mu)=0, \quad (3)$$

where the conformal energy momentum tensor is $\bar{T}_{\alpha\beta}=T_{\alpha\beta}/F$, $\nabla$ is the covariant derivative constructed from $\bar{g}_{\mu\nu}$, the subscript $,\varphi$ denotes $d/d\varphi$ and the last equation arises from conservation of the matter current density in the physical frame.

Henceforth, we assume spherical symmetry, writing

$$ds^2=\bar{g}_{\mu\nu}dx^\mu dx^\nu=-F\alpha^2 dt^2+FX^2 dr^2+r^2 d\Omega^2, \quad (4)$$

where $\alpha=\alpha(t,r), X=X(t,r)$ and we also define for convenience $\Phi=\ln(\sqrt{F}\alpha)$ and the gravitational mass $\mathcal{m}=r[1-(FX^2)^{-1}]^{1/2}$. In spherical symmetry, the 4-velocity in the Jordan frame is $u^\mu=(1-v^2)^{-1/2}[\alpha^{-1}, v X^{-1}, 0, 0]$, where the velocity field $v$ as well as the other matter variables $\rho, P, \epsilon$ and $H$ are also functions of $(t, r)$. High-resolution shock capturing requires flux a conservative formulation of the matter equations which is achieved by (cf. [23]) changing from variables $(\rho, v, H)$ to

$$D=\frac{\rho X F^{-3/2}}{\sqrt{1-v^2}}, \quad S^\nu=\frac{\rho H v F^{-2}}{(1-v^2)}, \quad \tau=\frac{S^\nu}{v} \frac{v}{F^2} - D. \quad (5)$$

Finally, we introduce $\eta=X^{-1}\partial_t \varphi$ and $\psi=\alpha^{-1}\partial_t \varphi$. The resulting system of equations is identical to Eqs. (2.21), (2.22), (2.27), (2.28), (2.33)-(2.39) in [23] except for the following additional potential terms (bracketed numbers denote right-hand-sides in Ref. [23])

$$\partial_t \Phi=\left[2.21\right]-rFX^2 V,$$

$$\partial_t \mu=\left[2.22\right]+r^2 V, \quad \partial_t \psi=\left[2.28\right]-\alpha F V\varphi,$$

$$s_{S^\nu}=\left[2.38\right]-rV\alpha X F (S^\nu v - \tau - D + F^{-2} P), \quad (6)$$

where $s_{S^\nu}$ is the source term in the evolution of $S^\nu$. All other equations in the above list remain unaltered.

We have implemented these equations by adding the potential terms to the GR1D code originally developed in [25] and extended to massless ST theory in [23]. As in [23], we use a phenomenological hybrid equation of state (EOS) $P=P_c+P_{th}$, $\epsilon=\epsilon_c+\epsilon_{th}$ with the cold part

$$\rho<\rho_{\text{th}}: P_c=K_1\rho^{r_1}, \quad \epsilon_c=\frac{K_1}{\Gamma_1-1}\rho^{r_1-1},$$

$$\rho>\rho_{\text{th}}: P_c=K_2\rho^{r_2}, \quad \epsilon_c=\frac{K_2}{\Gamma_2-1}\rho^{r_2-1}+E_3, \quad (7)$$

where $\rho_{\text{th}}=2 \times 10^{14} \text{g cm}^{-3}$, $K_1=4.9345 \times 10^{14} \text{[cgs]}$, $K_2$ and $E_3$ follow from continuity; $\epsilon_{th}$ measures the departure of the evolved internal energy $\epsilon$ from the cold contribution and generates a thermal pressure component $P_{th}=(\Gamma_{th}-1)\rho\epsilon_{th}$. We thus have three parameters to specify the EOS. As in [23], we consider $\Gamma_1=\{1.28, 1.3, 1.32\}$ for the subnuclear, $\Gamma_2=\{2.5, 3\}$ for the supernuclear EOS and $\Gamma_{th}=\{1.35, 1.5\}$ for the thermal description combining a mixture of relativistic and non-relativistic gases. For the conformal factor, we use the quadratic Taylor expansion commonly employed in the literature [11, 26] and the potential endows the scalar field with a mass $\mu$,

$$F=\exp(-2\alpha_0 \varphi - \beta_0 \varphi^2), \quad V=h^{-2}\mu^2 \varphi^2/2. \quad (8)$$

The discretization, grid and boundary treatment are identical to those described in detail in Sec. 3 of [23].

Simulations – For the simulations reported here, we employ a uniform grid with $\Delta r=166$ m inside $r=40$ km and logarithmically increasing grid spacing up to the outer boundary at $9 \times 10^9$ km. As detailed in the Supplemental Material, we observe convergence between first and second order, in agreement with the use of first and second order accurate discretization techniques in the code, resulting in a numerical uncertainty of about 4% in the wave signals reported below.

All simulations start with the WH12 model of the catalog of realistic pre-SN models [27] with initially vanishing scalar field. The evolution is then characterized by six parameters: the above mentioned EOS parameters $\Gamma_1, \Gamma_2$ and $\Gamma_{th}$ as well as mass $\mu$ of the scalar field and $\alpha_0, \beta_0$ in the conformal function which we vary in the ranges $0 \leq \mu \leq 10^{-13} \text{eV}$, $10^{-4} \leq \alpha_0 \leq 1, \quad -25 \leq \beta_0 \leq -5$. Our observations in these simulations are summarized as follows. (i) The collapse dynamics are similar to the scenario displayed in the left panels of Fig. 4 in [23] as conjectured therein, the baryonic matter strongly affects the scalar radiation but itself is less sensitive to the scalar field. (ii) For sufficiently negative $\beta_0$ the scalar field reaches amplitudes of order unity, independent of the EOS. Even in the massless case $\mu=0$, we observe this strong scalarization; the key impact of the massive field therefore lies in the weaker constraints on $\alpha_0, \beta_0$ rather than a direct effect of terms involving $\mu$. For illustration, we plot in Fig. 1 the wave signal $r\varphi$ extracted at $5 \times 10^4$ km for various parameter combinations. These waveforms are to be compared with those obtained for present observational bounds in the core collapse in massless ST theory as shown in Fig. 6 of [23]. The amplitudes
modes decay with radius. These conditions determine the Fourier transform of the signal at large radii in terms of the signal on the extraction sphere (note the $\omega$ ranges),

$$\tilde{\sigma}(\omega;r) = \tilde{\sigma}(\omega;r_{\text{ex}}) \begin{cases} e^{-i\sqrt{\omega^2 - \omega_s^2}(r - r_{\text{ex}})} & \text{for} \omega < -\omega_s, \\ e^{i\sqrt{\omega^2 - \omega_s^2}(r - r_{\text{ex}})} & \text{for} \omega > -\omega_s. \end{cases} \tag{9}$$

Note that the power spectrum, $|\tilde{\sigma}(\omega;r)|^2$, is unchanged during propagation except for the exponential suppression of frequencies $|\omega| < \omega_s$.

As signals propagate, they spread out in time, but the frequency content above the critical frequency $\omega_s$ remains unchanged. Consequently, the number of wave cycles in the signal increases with propagation distance; cf. Fig. 2. In the limit of large distances (relevant for LIGO observations of galactic supernovae) the signals are highly oscillatory, i.e. the phase varies much more rapidly than the frequency, and the inverse Fourier transform of Eq. (9) may be evaluated in the stationary phase approximation (SPA [28]). At each instant the signal is quasi-monochromatic with frequency

$$\Omega(t) = \omega_s/\sqrt{1 - [(r - r_{\text{ex}})/t]^2} \quad \text{for} \quad t > r - r_{\text{ex}}. \tag{10}$$

This time–frequency structure sounds like an inverse chirp, with high frequencies arriving before low ones. The origin of this structure can be understood by noting that each frequency component arrives after the travel time of the associated group velocity. Using the SPA the time domain signal is given by

$$\phi(t, r) = {\sqrt{\Omega^2 - \omega_s^2(r - r_{\text{ex}})} - \Omega t} - \frac{\pi}{4} + \text{Arg}[\tilde{\sigma}(\Omega, r_{\text{ex}})],$$

and the SPA frequency, $\Omega(t)$, is given by Eq. (10).

The Jordan frame metric perturbation is determined by the scalar field $\varphi$ (the tensorial GW degrees of freedom vanish in spherical symmetry). Any GW detector, small compared to the GW wavelength $\lambda = 2\pi/\omega$, measures the electric components of the Riemann tensor: $R_{00ij}$. In massless ST theory this 3-tensor is transverse to the GW wavevector, $R_{00ij} \propto \delta_{ij} - k_i k_j$, with strain amplitude $h_B = 2h_0 \varphi$ (this is called a breathing mode). In massive ST theory there is an additional longitudinal mode, $R_{00ij} \propto k_i k_j$, with suppressed amplitude $h_L = (\omega_s/\omega)^2 h_B$. A GW interferometer responds identically (up to a sign) to both of these polarizations meaning they cannot be distinguished [2]; henceforth we refer to the overall measurable scalar signal with amplitude $h_S = h_B - h_L = 2a_0[1 - (\omega_s/\omega)^2] \varphi$. In practice this factor reduces the strain only by at most a few % at $t \lesssim 10^{10}\text{s}$.

**LIGO observations** – GW signals from stellar collapse in ST theory may show up in several ways in existing LIGO–Virgo searches. In each case there is, in principle, a smoking gun which allows the signal to be distinguished from other types of sources. Here, it is argued that a new

![Waveforms extracted at 5 $\times$ 10$^4$ km. The legend lists deviations from the fiducial parameters $\mu = 10^{-14}$ eV, $\alpha_0 = 10^{-2}$, $\beta_0 = -20$, $\Gamma_1 = 1.3$, $\Gamma_2 = 2.5$, $\Gamma_h = 1.35$. Observed here are larger by $\sim 10^4$ for neutron star formation from less massive progenitors and even exceed the strong signals in black hole formation from more massive progenitors by $\sim 100$. This hyper-scalariulation of the collapsing stars in massive ST theory (as compared with the more strongly constrained massless case) and the resulting substantially larger GW signals are one of the key results of this work. Translating this increase into improved observational signatures for GW detectors, however, requires careful consideration of the signal’s dispersion as it propagates from source to detector; this is the subject of the remainder of this letter.**  

**Wave extraction and propagation** – At large distances from the source, the dynamics of the scalar field are well approximated by the flat-space equation, $\partial_t^2 \varphi - \nabla^2 \varphi + \hbar^{-2} \mu^2 \varphi = 0$, which, in spherical symmetry, reduces to a 1D wave equation for $\sigma \equiv r \varphi$. Plane-wave solutions propagate with phase and group velocities $v_{\sigma/n} = [1 - (\omega_s^2/\omega^2)]^{1/2}$ for angular frequencies above $\omega_s \equiv \mu/\hbar$, but are exponentially damped for lower frequencies.

In the massless case ($\mu = 0$) the general solution for $\sigma$ is the sum of an ingoing and an outgoing pulse propagating at the speed of light. This makes interpreting the output of core collapse simulations particularly simple; one extracts the scalar field $\sigma(t; r_{\text{ex}})$ at a sufficiently large extraction radius $r_{\text{ex}}$, and after imposing outgoing boundary conditions the signal at $r > r_{\text{ex}}$ is $\sigma(t; r) = \sigma(t - (r - r_{\text{ex}}); r_{\text{ex}})$.

In the massive case, the situation is complicated by the dispersive nature of wave propagation. However, an analytic solution for the field at large radii can still be written down, albeit in the frequency domain; $\tilde{\sigma}(\omega; r) \equiv \int dt \sigma(t; r) e^{i\omega t}$. The boundary conditions need to be modified for the massive case; frequencies $|\omega| > \omega_s$ propagate and we continue to impose the outgoing condition for these, however frequencies $|\omega| < \omega_s$ are exponential (giving or damped) and we impose that these...
Monochromatic searches — The highly dispersed signal (described by Eq. (11), see right-hand panels of Fig. 2) at large distances can last for the 1s of propagation the signal becomes increasingly oscillatory, and the long-lived memory effect is exponentially suppressed. Right-hand panels: The amplitude (top) and frequency (bottom) as functions of time for the scalar field \(\varphi\) from the same simulation as the other panels but at a distance of 10 kpc (it is not practical to plot the long, highly oscillatory time-domain signals at large distances). Also shown by the dotted and dashed curves are the amplitude profiles from other simulations using \(\alpha_0 = 10^{-2}\) and \(\alpha_0 = 10^0\); the amplitude of the scalar field depends relatively weakly on \(\alpha_0\). For the simulations shown here, the energy radiated in scalar GWs is \(\sim 10^{-13} M_\odot\).

Stochastic searches — As shown above, stellar core collapse in massive ST theory can generate large amplitude signals, allowing them to be detected at greater distances. However, the signals propagate dispersively, spreading out in time and developing a sharp spectral cut-off at the frequency of the scalar mass. The long duration signals from distant sources can overlap to form a stochastic background of scalar GWs with a characteristic spectral shape around this frequency. A detailed analysis of this stochastic signal covering a wider range of ST parameters and progenitor models will be presented in [32].

FIG. 2. Left-hand panel: the frequency–domain power spectrum of the scalar field \(\sigma \equiv r \varphi\) at the extraction sphere and 1 light second further out; the exponential decay of frequencies \(f < \omega_*/(2\pi)\) can be clearly seen. This simulation was performed for a 12 \(M_\odot\) star with \(\mu = 10^{-14}\) eV, \(\alpha_0 = 10^{-4}\); \(\beta_0 = -20\). Centre panel: the time–domain scalar field profiles for the two curves shown in the left-hand panel; during the 1s of propagation the signal becomes increasingly oscillatory, and the long-lived memory effect is exponentially suppressed. Right–hand panels: The amplitude (top) and frequency (bottom) as functions of time for the scalar field \(\varphi\) from the same simulation as the other panels but at a distance of 10 kpc (it is not practical to plot the long, highly oscillatory time–domain signals at large distances). Also shown by the dotted and dashed curves are the amplitude profiles from other simulations using \(\alpha_0 = 10^{-2}\) and \(\alpha_0 = 10^0\); the amplitude of the scalar field depends relatively weakly on \(\alpha_0\). For the simulations shown here, the energy radiated in scalar GWs is \(\sim 10^{-13} M_\odot\).

For \(\mu = 10^{-14}\) eV, for example, we obtain for SN1987A a frequency \(\Omega/(2\pi) \approx 128\) Hz and rate of change \(\dot{\Omega}/(2\pi) \approx 2\) Hz/yr, using distance \(D := r - r_{cs} = 51.2\) kpc and time \(t - D = 30\) yr.
The scalar-tensor theory of gravity with massive degrees of freedom (e.g. \cite{33}), but the results reported here already demonstrate the qualitatively new range of opportunities offered in this regard by the dawn of GW astronomy.

\begin{thebibliography}{99}


\bibitem{5} E. Berti \textit{et al.}, \textit{CQG} \textbf{32}, 243001 (2015).


\bibitem{12} P. C. C. Freire \textit{et al.}, \textit{MNRAS} \textbf{423}, 3328 (2012).

\bibitem{13} J. Antoniadis \textit{et al.}, \textit{Science} \textbf{340}, 6131 (2013).


\bibitem{19} J. Healy \textit{et al.}, \textit{Class. Quantum Grav.} \textbf{29}, 232002 (2011).


\bibitem{24} S. Morisaki and T. Suyama, (2017), 1707.02809.


\bibitem{28} C. Bender and S. Orszag, \textit{Advanced Mathematical Methods for Scientists and Engineers} (McGraw-Hill, 1978).


\end{thebibliography}