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Quantum depletion of a homogeneous Bose–Einstein condensate

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We have measured the quantum depletion of an interacting homogeneous Bose–Einstein condensate, and confirmed the 70-year old theory of N. N. Bogoliubov. The observed condensate depletion is reversibly tuneable by changing the strength of the interparticle interactions. Our atomic homogeneous condensate is produced in an optical-box trap, the interactions are tuned via a magnetic Feshbach resonance, and the condensed fraction determined by momentum-selective two-photon Bragg scattering.

After superfluidity of liquid ^{4}He was discovered in 1937 [1, 2], its connection to Bose–Einstein condensation was posited by F. London [3] and L. Tisza [4]. However, while at zero temperature liquid helium is 100% superfluid, less than 10% of the atoms are actually in the Bose–Einstein condensate (BEC) [5, 6]; most of the particles are coherently expelled from the condensate by strong interactions, and spread over a wide range of momenta. In 1947 N. N. Bogoliubov developed a theory that explains the microscopic origin of such interaction-driven, quantum depletion of a BEC [7]. This theory has become a cornerstone of our conceptual understanding of quantum fluids, but is quantitatively valid only for relatively weak interactions, and could not be tested with liquid helium. The connection between condensation and superfluidity, as well as superconductivity, is still a topic of active discussion; for a modern perspective see [8, 9].

Nowadays, gaseous atomic BECs provide a flexible setting for exploring the rich physics of interacting Bose fluids [10–12], and many liquid-helium-inspired theories can now be directly confronted with experiments. According to the Bogoliubov theory, for a homogeneous Bose gas of particle density n and interactions characterised by the scattering length a, and assuming $\sqrt{na^3} \ll 1$, the condensed fraction at zero temperature is [13]

$$n_{\rm BEC}/n = 1 - \gamma\sqrt{na^3}\,, \tag{1}$$

where $\gamma = 8/(3\sqrt{\pi}) \approx 1.5$. This prediction was tested using diffusion Monte Carlo simulations, and found to be quantitatively valid for $na^3 \lesssim 10^{-3}$ [14], but an experimental confirmation has been lacking. Effects of quantum depletion have been observed in harmonically trapped atomic gases, both by enhancing the role of interactions in optical lattices [15] (see also [16, 17]) and in high-resolution studies of the expansion of a weakly-interacting gas [18]. However, only semiquantitative comparison with theory has been possible, due to complications associated with the addition of the lattice, the inhomogeneity of the clouds, and/or the interpretation of the expansion measurements [19].

In this Letter, we test and verify the Bogoliubov theory of quantum depletion in a textbook setting, using a homogeneous ^{39}K BEC [20]. We produce our clouds in a cylindrical optical-box trap (see Fig. 1), of radius $R = 32$ μm and length

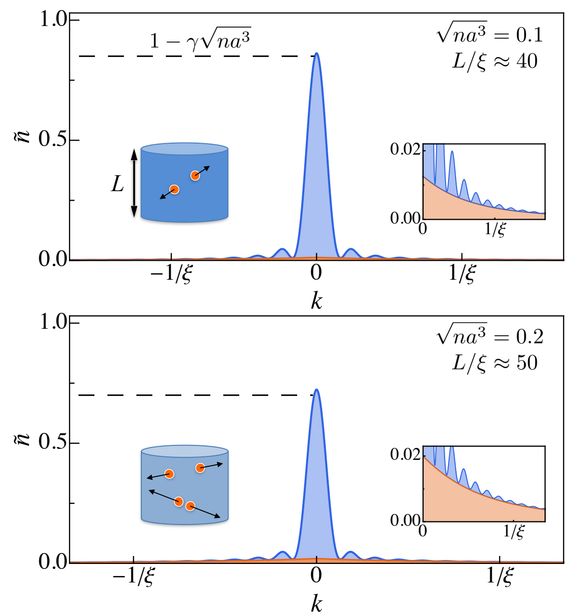

FIG. 1. Momentum distribution of a zero-temperature homogeneous Bose gas. We consider a gas of density n and size L, and two different values of the scattering length a. We show the expected 1D momentum distribution $\tilde{n}(k)$ (see text), normalised so that $\tilde{n}(0) = 1$ would correspond to no quantum depletion (setting γ in Eq. (1) to 0). The total $\tilde{n}(k)$ consists of the BEC peak (blue), with a Heisenberg-limited width $\propto 1/L$, and a broad quantum-depletion pedestal (orange) of characteristic width $1/\xi$, where ξ is the healing length. To a good approximation, the low-k distribution is the same as for a pure BEC, just scaled by a factor $1 - \gamma\sqrt{na^3}$, indicated by the dashed lines. For this illustration we use experimentally relevant values of L/ξ, but exaggerated values of $\sqrt{na^3}$, to make the orange shading visible in the main panels. Also note that we assume that the very broad $\tilde{n}_{\rm QD}(k)$ is not affected by finite-size effects. The cartoons on the left depict the coherent excitations out of the (blue) condensate, which occur as pairs of atoms with opposite momenta. The right insets highlight the fact that $\tilde{n}_{\rm QD}(k) \gg \tilde{n}_{\rm BEC}(k)$ at large k.

$L = 50$ μm [21], tune the interaction strength via a magnetic Feshbach resonance [22], and measure the condensed fraction by spectroscopic 'BEC filtering' [23] - using Doppler-sensitive two-photon Bragg scattering [24, 25] we *spatially* separate the BEC from the high-momentum components of the gas.

Bragg spectroscopy of ultracold atomic gases [24, 25] gives access to the dynamic structure factor $S(q,\omega)$ in conceptually the same way as inelastic neutron scattering does for liquid helium [5, 6]; here $\hbar q$ and $\hbar\omega$ are the momentum and energy of an excitation. We briefly highlight some differences between our measurements and those performed on liquid helium. First, after preparing a strongly interacting gas, and just before probing it, we suddenly turn off the interactions. This eliminates final-state interaction effects, and allows clean and direct probing of the suddenly frozen momentum distribution. Second, in our experiments the momentum $\hbar\mathbf{q}$ is imparted to an atom via a stimulated (coherent) two-photon process, and $\mathbf{q}$ and ω are defined by the differences in the momenta and frequencies of the photons from two intersecting laser beams. In an equivalent picture, the atom's energy changes through *elastic* scattering off a *moving* optical-lattice potential, formed by the interference of the two laser beams, which has period $2\pi/q$ and speed ω/q. For an atom with initial momentum $\hbar\mathbf{k}$, the scattering resonance is given by $\omega = \hbar q^2/(2m) + \hbar\mathbf{k}\cdot\mathbf{q}/m$, where m is the atom mass and the $\mathbf{k}\cdot\mathbf{q}$ term arises due to the Doppler effect. This $\mathbf{k}$ dependence of the scattering resonance allows a spectroscopic measurement of the momentum distribution and, hence, the condensed fraction. Finally, note that since the atomic states $|\mathbf{k}\rangle$ and $|\mathbf{k}+\mathbf{q}\rangle$ are coherently coupled by the Bragg beams, an atom undergoes Rabi oscillations between the two states as a function of the duration of the Bragg light pulse, with a period set by the two-photon Rabi frequency Ω [see Fig. 2(a)].

In our setup [26], $\mathbf{q}$ is aligned with the axis of the cylindrical box trap (z) and $q = 1.7\times 2\pi/\lambda$, where $\lambda = 767$ nm. The Bragg resonance condition thus depends only on an atom's initial momentum along z, and by counting the diffracted atoms we effectively probe the one-dimensional (1D) momentum distribution of the cloud, $\tilde{n}(k)$, given by the integral of the 3D distribution along the two transverse directions. We aim to diffract only the condensed atoms, so we tune ω to $\hbar q^2/(2m)$. In frequency space our spectroscopic resolution is set by Ω, which corresponds to a momentum resolution of $\Omega m/q$.

More specifically, we want to *spatially* separate the BEC from the quantum depletion (QD), which relies on a separation of three momentum scales, $1/L \ll 1/\xi \ll q$, where $\xi = 1/\sqrt{8\pi n a}$ is the healing length. In Fig. 1 we illustrate the expected $\tilde{n}(k)$ for a zero-temperature gas: $\tilde{n}(k) = \tilde{n}_{\rm BEC}(k) + \tilde{n}_{\rm QD}(k)$, where $\tilde{n}_{\rm BEC}$ has a Heisenberg-limited width $\propto 1/L$ [27] and exponentially suppressed high-k tails, while $\tilde{n}_{\rm QD}(k)$ has a width $\propto 1/\xi$ and long polynomial tails [18, 28–30] (see [31] for details). The inequality $L/\xi \gg 1$ thus ensures that $\tilde{n}_{\rm QD}(k)$ extends over a much wider range of momenta than $\tilde{n}_{\rm BEC}(k)$, so Ω can be chosen such that a Bragg pulse diffracts essentially the whole BEC and almost none of the QD. The inequality $q\xi \gg 1$ ensures that the momentum kick received by a diffracted atom, $\hbar q$, is much larger than the QD momentum spread, so that after the Bragg pulse, and a sufficiently long subsequent time-of-flight, the diffracted and the non-diffracted portions of the cloud clearly separate in real space [see Fig. 2(a)]. For all our measurements $L/\xi > 30$ and $q\xi > 12$.

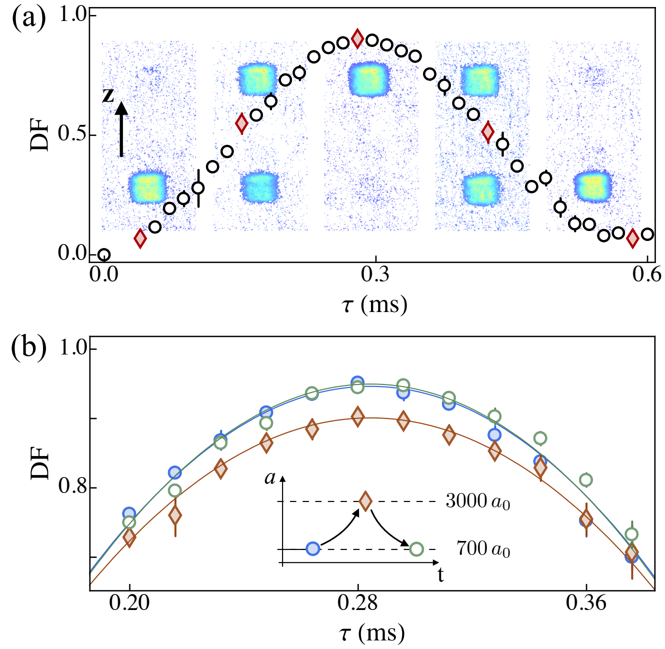

FIG. 2. Bragg filtering and reversible interaction-tuning of the condensed fraction. (a) Diffracted fraction (DF) as a function of the Bragg pulse duration, τ, for $\Omega = 2\pi\times 1.8$ kHz and $a \approx 3000\ a_0$. Absorption images in the background show the stationary (bottom) and diffracted (top) clouds, for the data points indicated by the red diamonds. (b) Diffracted fraction for τ close to π/Ω, for three different preparations of the cloud (see inset): at 700 a_0 (filled blue circles), after raising a from 700 a_0 to 3000 a_0 in 80 ms (orange diamonds), and after reducing it back to 700 a_0 in another 80 ms (open green circles). We see that increasing a reversibly reduces the maximal diffracted fraction. All error bars show standard statistical errors in the mean.

We start by producing a quasi-pure weakly-interacting BEC of density $n \approx 3.5\times 10^{11}$ cm^{-3} in the lowest ^{39}K hyperfine state, $|F=1, m_F=1\rangle$ in the low-field basis, which features a Feshbach resonance centred at 402.70(3) G [32]. We prepare the BEC at $a = 200\ a_0$, where a_0 is the Bohr radius, so $\sqrt{na^3} < 10^{-3}$, and in time-of-flight expansion we do not discern any thermal fraction. We then (in $150-250$ ms) increase a to a value in the range $700-3000\ a_0$, and measure the condensed fraction. To prepare the initial quasi-pure BEC we lower the trap depth U_0 to $\approx k_{\rm B}\times 20$ nK, but before increasing a we adiabatically raise U_0 by a factor of 5, to ensure that $U_0 \gg \hbar^2/(2m\xi^2)$. The largest a that we explore here is limited by imposing requirements that: (i) during the whole experiment the atom loss due to three-body recombination is $< 10\%$, and (ii) if we reduce a back to 200 a_0 we do not observe any signs of heating; for a discussion of additional measurements at even larger a (with larger particle loss) see [31].

Just before turning off the trap and applying the Bragg pulse, we rapidly (in 60 μs) turn off the interactions, using a radio-frequency pulse to transfer the atoms to the

$|F=1, m_F=0\rangle$ state, in which $a \approx 0$ [32]. This freezes the momentum distribution before we probe it, and allows the diffracted and non-diffracted components of the gas to separate in space without collisions.

After the Bragg pulse, we wait for 10 ms and then take an absorption image along a direction perpendicular to z [see Fig. 2(a)]. In 10 ms the diffracted and non-diffracted portions of the gas separate by $\approx 220\ \mu$m, while neither expands significantly beyond the original size of the box-trapped cloud.

In Fig. 2(a) we show a typical variation of the diffracted fraction of the gas with the duration of the Bragg pulse, τ, for our chosen $\Omega = 2\pi \times 1.8$ kHz (see [31]). In the background we show representative absorption images of the stationary (bottom) and diffracted (top) clouds.

Assuming that we perfectly filter out the condensate from the high-k components of the gas, the condensed fraction of the cloud is given by the maximal diffracted fraction, η, observed for $\tau = \pi/\Omega \approx 0.28$ ms. We see that η is slightly below unity, which is expected due to quantum depletion, but can in practice also be observed for other reasons, including experimental imperfections and the inevitably nonzero temperature of the cloud. It is therefore important that our measurements are differential - we study the variation of η with a, while keeping other experimental parameters the same. It is also crucial to verify that the tuning of η with a is adiabatically reversible, which excludes the possibility that the condensed fraction is reduced due to non-adiabatic heating or losses.

In Fig. 2(b) we focus on $\tau \approx \pi/\Omega$, and show measurements for three different experimental protocols: for a cloud prepared at 700 a_0, after increasing a to 3000 a_0, and after reducing it back to 700 a_0 (see inset). We see that η is indeed reduced when a is increased, and also that this effect is fully reversible (within experimental errors); we have verified such reversibility for our whole experimental range of a values.

In Fig. 3 we summarise our measurements of the variation of η with the interaction parameter $\sqrt{na^3}$. We observe the expected linear dependence, with $\eta(0)$ close to unity. Fitting the data with $\eta(0)(1-\gamma\sqrt{na^3})$ gives $\gamma = 1.5(2)$, in agreement with Eq. (1).

Finally, we numerically assess the systematic effects on γ due to non-infinite L/ξ and a small nonzero temperature T, which are both $\lesssim 20\%$, and partially cancel. The results of this analysis are shown in the inset of Fig. 3; for details see [31]. The dashed line shows the simulated η for $T = 0$ and our values of n, L and Ω. For any non-infinite Ω, the tails of the BEC momentum distribution are not fully captured by the Bragg pulse, which slightly reduces $\eta(0)$. More importantly, we diffract some of the quantum-depletion atoms, which reduces the apparent γ. A linear fit (omitted for clarity) gives that for $T = 0$ we actually expect $\gamma \approx 1.2$. The small systematic differences between our data and this simulation can be explained by a small nonzero temperature. A nonzero temperature generally reduces η due to thermal depletion, the momentum tails of which are not diffracted by the Bragg pulse. Moreover, if the gas is initially prepared (at 200 a_0) at a small $T > 0$, this does not merely reduce η by a constant offset

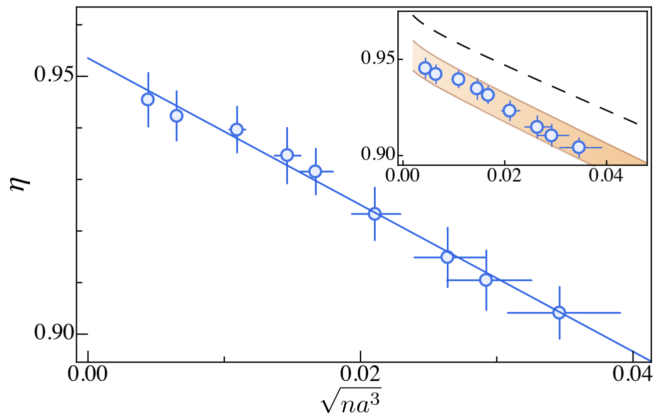

FIG. 3. Measurement of the quantum depletion. We plot the maximal diffracted fraction η versus the interaction parameter $\sqrt{na^3}$. A linear fit (solid line) gives $\eta(0) = 0.954(5)$ and $\gamma = 1.5(2)$. Vertical error bars show fitting errors, while horizontal ones reflect the uncertainty in the position of the Feshbach resonance and a 10% uncertainty in n. Inset: Analysis of systematic effects. We show numerical simulations for $T = 0$ (dashed line) and for initial temperatures (at $a = 200\ a_0$) between 3.5 and 5 nK (orange shading, from top to bottom); see text and [31] for more details.

(independent of $\sqrt{na^3}$), but slightly increases the apparent γ; even adiabatically increasing a increases the thermal depletion, because it modifies both the dispersion relation and the particle content of the thermally populated low-k excitations [28, 31]. As indicated by the orange shaded region, our data are consistent with an initial T between 3.5 and 5 nK; this is compatible with the fact that we do not discern the corresponding thermal fractions of $\lesssim 10\%$ in time-of-flight expansion at 200 a_0, and is reasonable for our trap depth of ≈ 20 nK. Due to these effects the expected dependence of η on $\sqrt{na^3}$ is also not perfectly linear, but this effect is negligible on the scale of the experimental errors.

In conclusion, within a 15% statistical error and 20% systematic effects, we have quantitatively confirmed the Bogoliubov theory of quantum depletion of a Bose–Einstein condensate, which is one of the cornerstones of our understanding of interacting quantum fluids. The largest interaction strength that we could reliably explore is already at the limit of agreement between Bogoliubov's analytical theory and Monte Carlo simulations; adiabatically increasing $\sqrt{na^3}$ by another factor of two should allow quantitative studies of the regime where the two theories disagree at the level of our demonstrated experimental precision (see [31]). The methods employed here could be extended to study the momentum distribution of the quantum depletion, and could also be useful for sensitive thermometry of homogeneous ultracold Bose gases.

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