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A solvable family of driven-dissipative many-body systems

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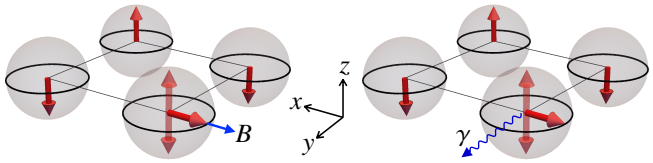
Exactly solvable models have played an important role in establishing the sophisticated modern understanding of equilibrium many-body physics. And conversely, the relative scarcity of solutions for *non-equilibrium* models greatly limits our understanding of systems away from thermal equilibrium. We study a family of non-equilibrium models, some of which can be viewed as dissipative analogues of the transverse-field Ising model, in that an effectively classical Hamiltonian is frustrated by dissipative processes that drive the system toward states that do not commute with the Hamiltonian. Surprisingly, a broad and experimentally relevant subset of these models can be solved efficiently. We leverage these solutions to compute the effects of decoherence on a canonical trapped-ion-based quantum computation architecture, and to prove a no-go theorem on steady-state phase transitions in a many-body model that can be realized naturally with Rydberg atoms or trapped ions.

The understanding of equilibrium many-body physics in both classical and quantum systems has relied heavily on exact solutions of simplified models. For example, solutions of the classical and quantum (transverse-field) Ising models have elucidated the structure of classical and quantum phase transitions, respectively [1, 2], and the connections between them. For quantum systems that are *not* in thermal equilibrium such solutions are comparatively scarce, though important progress has been made in numerous specialized models that either: (a) are isolated and one-dimensional or harmonic [3–7], (b) impose nontrivial dissipation on otherwise free bosons or fermions [8–17], (c) impose highly fine-tuned dissipation [14], or (d) are coupled to an environment only at a boundary [18–21]. On the other hand, interacting quantum spin systems that are dissipative in the bulk and driven so that they do not thermalize are expected to exhibit a variety of behaviors not found in equilibrium, including unusual multi-critical points [22], new critical exponents [23], and the existence of zero-entropy entangled steady states [24–27]. And, naturally, such systems play a central role in the theory of quantum computation in the presence of decoherence. While recent experimental advances have enabled the controlled study of such physics in systems ranging from exciton-polariton fluids [28–32], to trapped ions [33, 34], to Rydberg gases [35–37], the minimal microscopic models expected to capture the essential qualitative physics—many-body quantum master equations [23]—continue to pose severe challenges to existing theoretical techniques.

In this manuscript we investigate a broad and experimentally relevant class of driven-dissipative many-body spin models, some of which can be viewed as dissipative analogues of the transverse-field Ising model (TFIM), and show that they can be solved efficiently. The TFIM captures a characteristic feature of low-temperature quantum systems more generally: Even at zero temperature, the ordering associated with a classical Hamiltonian (the Ising model) can be frustrated by the persistence of quantum fluctuations induced by the transverse field (Fig. 1, left panel). In particular, the transverse field favors a zero-temperature density matrix that does not commute with the Ising Hamiltonian. As a result, energy minimization forces the system to develop quantum correlations, and—except in certain very special circumstances, e.g. nearest-neighbor interactions in 1D—one must resort to approximate methods or sophisticated numerics [38–40] to calculate system properties. The models we consider realize a non-equilibrium analogue of this scenario, in which a classical Hamiltonian is frustrated by the presence of *dissipative* fluctuations (Fig. 1, right panel) [25, 41, 42]. Strikingly, we find that the inclusion of a broad class of dissipative processes favoring a *steady-state* density matrix that does not commute with the Hamiltonian is relatively benign. In particular, operators in the Heisenberg picture remain dynamically localized for finite-ranged Hamiltonians, enabling the time-dependence of correlation functions to be efficiently obtained (even for an infinite system) by solving a finite-dimensional system of



Transverse field: $B\sum_j \hat{\sigma}^x_j$	Dissipation: $\sqrt{\gamma}(\hat{\sigma}^y_j - i\hat{\sigma}^z_j)$
Incompatible ground state: $\|\downarrow_x\rangle$	Incompatible steady-state: $\|\downarrow_x\rangle$

FIG. 1. In the transverse-field Ising model (left panel) ordering with respect to a "classical" Hamiltonian $\hat{H} = \sum_{j\neq k} h_{j,k}\hat{\sigma}_j\hat{\sigma}_k$ is frustrated, even at $T = 0$, because the ground state of the transverse field (expressed as a density matrix) does not commute with $\hat{H}$. The models solved here generalize this scenario to the case where order is frustrated by the inclusion of a *dissipative process* whose *steady state* does not commute with $\hat{H}$. In the example shown (right panel), Markovian dissipation implemented by jump operators $\hat{\sigma}^y_j - i\hat{\sigma}^z_j$ drives each spin towards the ground-state of a transverse field.

equations. This structure exists, for example, even when dissipation alone drives the system towards a dark state that is the ground state of a transverse field. It also exists when the dissipation is explicitly derived from a fluctuating transverse field; evidently, even though the Ising model is not generally expected to be solvable in the presence of a *particular* (static or time-varying) transverse field, the dynamics of the transverse-field Ising model averaged over *all* such fields can be computed exactly.

To illustrate the utility of these solutions we exploit them to prove that an experimentally relevant subset of the models considered possess a finite dissipative gap, and therefore cannot undergo dissipative phase transitions. We then apply this result to a model of interacting Rydberg atoms studied in Refs. [43, 44], thereby confirming the structure of the phase diagram inferred from recently developed approximate techniques [45]. From a more applied perspective, we emphasize that Ising-like Hamiltonians, despite being "classical", induce truly quantum dynamics and play a central role in the production of states sought for quantum information tasks [46–48]. Their solvability in the presence of dissipation thus affords numerous exciting opportunities to investigate the effect of decoherence on the generation of useful entangled states. As an illustrative example, we compute the effects of a fluctuating transverse field (dephasing) on Ising dynamics generated by Mølmer-Sørensen gates [48–50], which can impose important technical limitations in trapped-ion based approaches to quantum computation [51].

Dissipative models.—The models we consider can be constructed in close analogy to the spin-1/2 transverse-field Ising model. There, one starts with an Ising Hamiltonian

$$\hat{H} = \sum_{j\neq k} h_{j,k}\hat{\sigma}^z_j\hat{\sigma}^z_k, \tag{1}$$

which is "classical" in the sense that it can be diagonalized by a choice of local basis. Therefore, its eigenstates (e.g., $|\uparrow_z,\downarrow_z,\ldots,\uparrow_z,\downarrow_z\rangle$) are in one-to-one correspondence with the configurations of a classical Ising model; they have well defined local values of the z-component of spin, and in this sense there are no quantum fluctuations [52]. In equilibrium, a natural question to ask is how one can modify $\hat{H}$ such that, in the low temperature limit where thermal fluctuations vanish, quantum fluctuations remain. One simple strategy is to add single-body terms to the Hamiltonian that are not diagonalized by the eigenstates of $\hat{H}$. The usual culprit is a transverse field, resulting in the tranverse-field Ising Hamiltonian $\hat{H}_{\mathrm{TFIM}} = \hat{H} + B\sum_j \hat{\sigma}^x_j$. For $B \neq 0$, even at zero temperature (i.e. in the quantum ground state of H_{TFIM}) there will be fluctuations between the classical eigenstates of $\hat{H}$.

In a driven-dissipative setting, we are no longer interested in properties of the ground state, which control the low-temperature equilibrium physics, but rather the properties of the steady state, which control the long-time, non-equilibrium physics. A natural dissipative generalization of the procedure used above to frustrate the zero-temperature ordering associated with $\hat{H}$ is to introduce single-body dissipative processes that drive the system toward a steady state that does not commute with $\hat{H}$. Assuming that dissipation can be treated in the Born-Markov approximation, the dynamics of an open quantum system with Hamiltonian $\hat{H}$ is governed by a Markovian master equation of the form [53–55]

$$\dot{\hat{\rho}} = \mathcal{L}(\hat{\rho}) \equiv -i[\hat{H},\hat{\rho}] + \mathcal{D}(\hat{\rho}), \tag{2}$$

where $\mathcal{D}(\star) \equiv \sum_{j,\alpha}\frac{\gamma_{j\alpha}}{2}(2\hat{J}_{j\alpha}\star\hat{J}^\dagger_{j\alpha} - \{\hat{J}^\dagger_{j\alpha}\hat{J}_{j\alpha},\star\})$. The dissipation is induced by jump operators $\hat{J}_{j\alpha}$, each of which we assume to be supported on a single site j. The index α may take on multiple values in order to describe multiple dissipative channels on a given site, though in what follows we will generally consider only one jump operator on each site, dropping the index α. Consider the dissipative dynamics in the absence of the Hamiltonian, described by $\dot{\hat{\rho}} = \mathcal{D}(\hat{\rho})$. The steady-state solution of the purely dissipative dynamics is determined implicitly by solving $\mathcal{D}(\hat{\rho}_{\mathrm{dis}}) = 0$. If $\hat{\rho}_{\mathrm{dis}}$ commutes with the Hamiltonian, then it is automatically also a solution of $\mathcal{L}(\hat{\rho}) = 0$ and thus is a proper steady state of the complete dynamics including both coherent evolution and dissipation. If it does not, then in close analogy to the ground state of the TFIM we expect the steady state to possess fluctuations in the sense of admixing of classical states [25, 41, 42].

Surprisingly, a large class of dissipators that frustrate the classical Hamiltonian, i.e. for which $[\hat{\rho}_{\mathrm{dis}},\hat{H}] \neq 0$, nevertheless admit exact solutions for the dynamics of observables. In particular, suppose that

$$\mathrm{Tr}[\hat{\sigma}^z_j\mathcal{D}(\hat{\sigma}^\pm_k)] = 0 \quad (\text{for all } j,k). \tag{3}$$

Note that this condition is automatically guaranteed for $j \neq k$ because we assume single-site jump operators, and therefore $\mathcal{D}$ does not change the support of an operator. Thus only the case $j = k$ imposes an additional constraint on the *form* of the jump operators. For $\hat{H}$ of the form in Eq. (1), we also have $\mathrm{Tr}[\hat{\sigma}^z_j[\hat{H},\hat{\sigma}^\pm_k]] = 0$, and therefore whenever the jump operators are chosen to satisfy Eq. (3), the complete Liouvillian also obeys

$$\mathrm{Tr}[\hat{\sigma}^z_j\mathcal{L}(\hat{\sigma}^\pm_k)] = 0 \quad (\text{for all } j,k). \tag{4}$$

Taken together with the equality $\mathrm{Tr}[\hat{\mathbb{1}}_j\mathcal{L}(\hat{\sigma}^\pm_k)] = 0$, which follows trivially from the definition of $\mathcal{L}$ (and ensures the conservation of probability) regardless of the form of the jump operators, Eq. (4) can be understood colloquially as a statement that $\mathcal{L}$, when applied to the density matrix in the z-basis, does not map coherences (off-diagonal elements) onto populations (diagonal elements), as illustrated in Fig. 2a. In what follows, we will show that finite-range Hamiltonians of the form in Eq. (1), subjected to dissipation obeying Eq. (3), can be solved efficiently in the thermodynamic limit.

There are many natural jump operators satisfying Eq. (3). For example, dephasing, spontaneous emission, and incoherent pumping along the z axis all do, and the models studied here therefore subsume finite-range versions of the models studied in [56, 57] (and realized experimentally in [33]) as

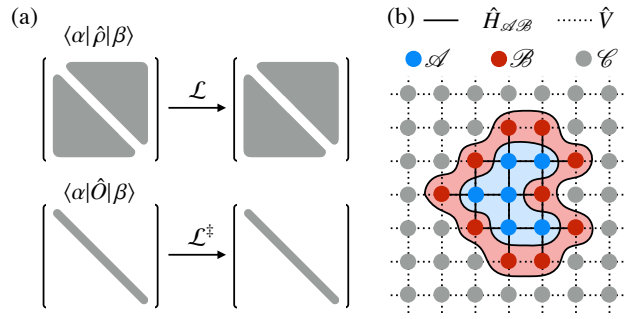


FIG. 2. (a) Schematic representation of Eq. (4): In the z basis, $\mathcal{L}$ does not map coherences to populations. Equivalently, in the Heisenberg picture, $\mathcal{L}^\ddagger$ does not map populations onto coherences. (b) When the conditions described in (a) are met, the dynamics of any observable supported on a subsystem $\mathscr{A}$ can be computed by identifying the nearest neighbors of the set $\mathscr{A}$, denoted $\mathscr{B}$, tracing out the part of the density matrix supported on $\mathscr{C}$, and then solving the master equation projected into the remaining finite-dimensional system.

special cases. However, all the examples of dissipation just mentioned lead to steady states $\hat{\rho}_{\rm dis}$ that commute with $\hat{H}$, and thus the steady states in the presence of $\hat{H}$ remain trivial. Examples of jump operators satisfying Eq. (3) but producing steady states that *do not* commute with $\hat{H}$ include dephasing along any direction in the xy plane; $\hat{J} = \cos(\theta)\hat{\sigma}^x + \sin(\theta)\hat{\sigma}^y$, or spontaneous emission along any axis in the xy plane; $\hat{J} = \hat{\sigma}^z + i(\cos(\theta)\hat{\sigma}^x + \sin(\theta)\hat{\sigma}^y)$. Note that in the latter example, $\hat{\rho}_{\rm dis}$ is the ground state of a transverse field.

Localization of correlations.—The time-dependent expectation value of an arbitrary operator $\hat{O}$, initially supported on a set of sites $\mathscr{A}$, can be written as

$$\underset{O(t) = \mathrm{Tr}[\hat{O}\exp(t\mathcal{L})\hat{\rho}_0],}{\text{Schrödinger picture:}} \quad \underset{O(t) = \mathrm{Tr}[\hat{\rho}_0\exp(t\mathcal{L}^\ddagger)\hat{O}].}{\text{Heisenberg picture:}} \tag{5}$$

The Heisenberg-picture expression utilizes the adjoint Liouvillian $\mathcal{L}^\ddagger(\hat{O}) = i[\hat{H},\hat{O}] + \mathcal{D}^\ddagger(\hat{O})$, with the adjoint dissipator given by $\mathcal{D}^\ddagger(\star) = \sum_{j,\alpha}\frac{\gamma_{j\alpha}}{2}(2\hat{J}^\dagger_{j\alpha}\star\hat{J}_{j\alpha} - \{\hat{J}^\dagger_{j\alpha}\hat{J}_{j\alpha},\star\})$. $\hat{O}$ could be, e.g., a product of two spin operators on different sites, in which case $O(t)$ is an equal-time correlation function. Equation (4) can be recast in terms of the Heisenberg-picture Liouvillian as $\mathrm{Tr}[\hat{\sigma}^\pm_j\mathcal{L}^\ddagger(\hat{\sigma}^z_k)] = 0$; thus, $\mathcal{L}^\ddagger$ does not map populations onto coherences (Fig. 2a). This condition can be restated in the following useful way: Referring to an operator as "diagonal on the set $\mathscr{S}$" if it commutes with all operators $\hat{\sigma}^z_j$ for $j\in\mathscr{S}$, one can show that the set of operators that are diagonal on $\mathscr{S}$ is closed under the action of $\mathcal{L}^\ddagger$. With this structure in mind, we refer to $\mathcal{L}^\ddagger$ as *diagonality preserving* (in the z basis), a property which plays a key role in the solution of Eq. (2).

To understand the solvability of the models under consideration, it is helpful to remember why computing $O(t)$ for a many-body master equation is generally a difficult task. Consider the expansion $O(t) = \sum_{n=0}^\infty(t^n/n!)\mathrm{Tr}[\hat{\rho}_0\hat{O}_n]$, where $\hat{O}_n = (\mathcal{L}^\ddagger)^n\hat{O}$, and take $\hat{O}_0 = \hat{O}$ to be a single-spin operator on some site of the lattice. If $\hat{H}$ contains only nearest-neighbor interactions, then $\hat{O}_1$ will contain products of two spin operators, supported on both the initial site and its nearest neighbors. In general, we expect that $\hat{O}_n$ contains terms with products between n (or at least of order n) spin operators supported on a set of radius $\sim n$, and the number of such operators grows at least exponentially with n. Therefore, barring some simplifying structure, computing the dynamics to order n in time is exponentially difficult in n.

To see how this situation is avoided in the present context, we take $\hat{O}$ to be initially supported on the set $\mathscr{A}$, and define the set of sites $\mathscr{B}$ that are nearest neighbors of $\mathscr{A}$ as all sites outside of $\mathscr{A}$ for which a term in the Hamiltonian has simultaneous support on $\mathscr{A}$ and $\mathscr{B}$. We further define $\mathscr{C}$ to be the complement of $\mathscr{A}\bigcup\mathscr{B}$ (Fig. 2b). Because $\hat{H}$ does not contain terms supported on both $\mathscr{A}$ and $\mathscr{C}$, and $\mathcal{D}$ is composed of local jump operators, we see that $\hat{O}_1 \equiv \mathcal{L}^\ddagger(\hat{O})$ is supported entirely on $\mathscr{A}\bigcup\mathscr{B}$. Importantly, because $\mathcal{L}^\ddagger$ is *diagonality preserving*, $\hat{O}_1$ is diagonal on $\mathscr{B}$. Next consider $\hat{O}_2 = \mathcal{L}^\ddagger(\hat{O}_1)$: Because $\hat{O}_1$ is diagonal on $\mathscr{B}$, commutation with the diagonal operator $\hat{H}$ (which does not connect $\mathscr{A}$ to $\mathscr{C}$) cannot enlarge the support beyond $\mathscr{A}\bigcup\mathscr{B}$. And, as was the case for $\hat{O}_1$, the diagonality-preserving nature of $\mathcal{L}^\ddagger$ ensures that $\hat{O}_2$ is diagonal on $\mathscr{B}$. Iterating this argument through a formal inductive proof [58], it follows that $\hat{O}_n$ is supported on $\mathscr{A}\bigcup\mathscr{B}$ for all n. As a result, we can write $O(t) = \mathrm{Tr}_{\mathscr{A}\cup\mathscr{B}}\big[\hat{\rho}_{\mathscr{A}\mathscr{B}}\sum_{n=0}^\infty(t^n/n!)(\mathcal{L}^\ddagger_{\mathscr{A}\mathscr{B}})^n\hat{O}\big]$. Here, $\hat{\rho}_{\mathscr{A}\mathscr{B}} = \mathrm{Tr}_{\mathscr{C}}[\hat{\rho}_0]$ is the initial reduced density matrix obtained by tracing over $\mathscr{C}$, and $\mathcal{L}_{\mathscr{A}\mathscr{B}}$ is obtained from $\mathcal{L}$ by eliminating all terms in $\hat{H}$ and $\mathcal{D}$ with support on $\mathscr{C}$. Returning to the Schrödinger picture we find our primary result,

$$O(t) = \mathrm{Tr}_{\mathscr{A}\cup\mathscr{B}}\big[\hat{O}\exp(t\mathcal{L}_{\mathscr{A}\mathscr{B}})\hat{\rho}_{\mathscr{A}\mathscr{B}}\big]. \tag{6}$$

Thus $O(t)$ can be computed efficiently (i.e. from an effective problem defined within a finite system) whenever the Hamiltonian is of finite range, such that $\mathscr{B}$ is finite. It follows immediately that correlations are localized for finite-ranged Hamiltonians. For example, consider a connected correlation function $C^{\mu\nu}_{jk} \equiv \langle\hat{\sigma}^\mu_j\hat{\sigma}^\nu_k\rangle - \langle\hat{\sigma}^\mu_j\rangle\langle\hat{\sigma}^\nu_k\rangle$ ($\mu,\nu\in\{x,y,z\}$), and suppose the system starts in a product state. If $\hat{H}$ is of finite range r (meaning that $h_{j,k} = 0$ whenever $|\boldsymbol{r}_j - \boldsymbol{r}_k| > r$), and sites j and k are separated by a distance $d_{jk} > 2r$ (so that sites j and k do not share any neighbors, and $\mathscr{A}\bigcup\mathscr{B}$ decomposes into two disjoint sets), then it follows from Eq. (6) that $\langle\hat{\sigma}^\mu_j\hat{\sigma}^\nu_k\rangle = \langle\hat{\sigma}^\mu_j\rangle\langle\hat{\sigma}^\nu_k\rangle$ at all times [59]. Therefore, $C^{\mu\nu}_{jk}$ vanishes identically unless $d_{jk} < 2r$.

Applications.—Just as many-body ground states can suddenly change in character as a parameter in the Hamiltonian is continuously adjusted, signaling a quantum phase transition, steady states can exhibit a sudden change when a parameter in the Liouvillian is continuously adjusted, signaling a dissipative phase transition. While a quantum phase transition is associated with the closing of an energy gap in the Hamiltonian's spectrum, a dissipative phase transition is associated with the closing of a *dissipative gap* in the Liouvillian's spectrum [60]. Consider the dissipative transverse-field Ising

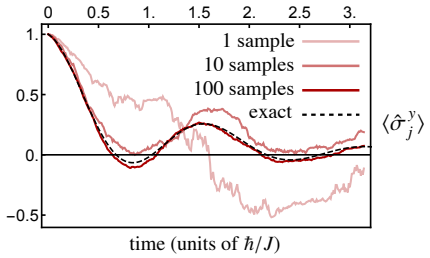


FIG. 3. Quench in the fluctuating-transverse-field Ising model starting with all spins polarized along $+y$ (10 spins in 1D with periodic boundary conditions, $\gamma = J/4$).

model studied in Refs. [43, 44],

$$\hat{H}_{\mathrm{TFIM}} = J\sum_{\langle j,k\rangle}\hat{\sigma}^z_j\hat{\sigma}^z_k + \Delta\sum_j\hat{\sigma}^x_j, \qquad \hat{J}_j = \frac{1}{2}(\hat{\sigma}^y_j - i\hat{\sigma}^z_j), \tag{7}$$

in which fluctuations due to both dissipation (strength γ) *and* a transverse field (strength Δ) are considered. Note that in the absence of the Ising term the dissipation drives the system into a dark state $|{\downarrow_x},\ldots,{\downarrow_x}\rangle$. Thus either dissipation or energy minimization with respect to the transverse field (for $\Delta > 0$) drives the system towards the same state. If $\Delta = 0$ (for arbitrary J, γ), the model is of the form assumed in Eq. (1) and the dissipator satisfies Eq. (3), so our solutions can be applied. In Ref. [44], a careful analysis of the variational techniques developed in [45] suggests that the system is disordered for any value of γ at $\Delta = 0$ in high dimensions. Here, we can make this conclusion rigorous in *any* dimension: By exploiting the solvability of the model, it can be proven [58] that at $\Delta = 0$ there must be a finite dissipative gap $\geq \gamma$. Since the model is disordered at $\gamma = \infty$ and a gap exists at $\Delta = 0$ for any $\gamma > 0$, it must be disordered along the entire $\Delta = 0$ axis. Given the Liouvillian stability results of Ref. [61], a dissipative gap at $\Delta = 0$ may actually imply something much stronger: a gap persists for small $\Delta > 0$, ruling out a phase transition for $\Delta \lesssim \gamma$ and implying that the system is disordered for sufficiently small Δ at any $\gamma > 0$.

The solutions developed here can also be used to calculate dynamics of the Ising model in the presence of a fluctuating transverse field [62, 63],

$$\hat{H}_{\mathrm{fTFIM}}(t) = J\sum_{\langle j,k\rangle}\hat{\sigma}^z_j\hat{\sigma}^z_k + \sum_j\Delta_j(t)\,\hat{\sigma}^x_j. \tag{8}$$

Here the transverse fields $\Delta_j(t)$ are Gaussian random variables with white-noise spectrum, $\overline{\Delta_j(t_1)\Delta_k(t_2)} = \gamma\delta_{j,k}\delta(t_1 - t_2)$, and observables are to be computed with respect the stochastic Schrödinger equation $\partial_t|\psi(t)\rangle = -i\hat{H}_{\mathrm{fTFIM}}|\psi(t)\rangle$. This model arises naturally in ion trap experiments, where it captures the important effects of qubit dephasing along the quantization axis during Ising dynamics induced by Mølmer-Sørensen gates [48–51], and hence it plays a prominent role in the description of decoherence effects on trapped-ion based approaches to quantum computation. Note that the brute-force solution of this model requires averaging over the dynamics of an Ising model with a time-dependent transverse field, which cannot (in general) be done efficiently. Nevertheless, it is well known that this model can be mapped exactly onto the master equation [64, 65]

$$\dot{\hat{\rho}} = -i[J\sum_{\langle j,k\rangle}\hat{\sigma}^z_j\hat{\sigma}^z_k, \hat{\rho}] + \gamma\sum_j(\hat{\sigma}^x_j\,\hat{\rho}\,\hat{\sigma}^x_j - \hat{\rho}), \tag{9}$$

which obeys Eq. (3) and thus can be efficiently solved by the methods developed here [see also Refs. [66, 67] for a constructive approach to Eq. (9) starting from Eq. (8)].

In Fig. 3 we show numerical simulations of the stochastic Schrödinger equation starting from an initial state polarized along the y direction, $|\psi(0)\rangle = |{\uparrow_y}\ldots{\uparrow_y}\rangle$. These simulations are computationally very expensive as they require a large sampling of the solutions to an exponentially large set of coupled differential equations with random transverse field values, and they are not feasible for more than ~ 10 spins (giving in this case a system of 2^{10} ordinary differential equations to sample from). However, one can clearly see that the average over sufficiently many samples converges to the results obtained from the exact solution of Eq. (9) (black-dashed line).

Outlook.—The underlying algebraic structure exploited here persists in a much more general class of Hamiltonians that are solvable in the absence of dissipation. For example, our results can be generalized to systems with arbitrary finite-dimensional Hilbert spaces on each site and time-dependent Hamiltonians. Long-range interactions spoil some aspects of the solution, but the absence of dissipative phase transitions can still be proven. Hamiltonians that are not diagonalized by a local choice of basis but can still be written as a sum of local commuting terms, such as the toric code [27, 68], are amenable to similar solution techniques and will be explored in future work.

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