Casimir-Lifshitz Torque Enhancement by Retardation and Intervening Dielectrics

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Casimir-Lifshitz torque enhancement by retardation and intervening dielectrics

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We investigate two effects that lead to a surprising increase in the calculated Casimir-Lifshitz torque between anisotropic, planar, semi-infinite slabs. Retardation effects, which account for the finite speed of light, are generally assumed to decrease the strength of Casimir-Lifshitz interactions. However, the nonretarded approximation underestimates the Casimir-Lifshitz torque at small separations by as much as an order of magnitude. Also, Casimir-Lifshitz forces are typically weakened with the insertion of an intervening dielectric. However, a dielectric medium can increase the short-range Casimir-Lifshitz torque by as much as a factor of two. The combined effects of retardation and an intervening dielectric dramatically enhance the Casimir-Lifshitz torque in the experimentally accessible regime, and should not be neglected in calculation or experimental design.

Quantum and thermal fluctuations of electromagnetic fields cause a force between uncharged, macroscopic objects. In 1948, Casimir calculated the attractive force between two parallel, semi-infinite conductors [1]. This result was generalized by Lifshitz to include dielectrics, then by Parsegian and Barash to include anisotropic materials [2–4]. The distance dependence of the free energy of the confined electromagnetic modes causes a force, and the angular dependence of the confined modes due to geometric or dielectric anisotropy causes a torque. Although some efforts are in progress, the Casimir-Lifshitz torque has yet to be verified experimentally [5–10].

At small separations, the Casimir-Lifshitz effect is equivalent to a van der Waals interaction. The connection between the Casimir-Lifshitz and van der Waals formulations is summarized in a recent review by Woods [11]. The van der Waals free energy per unit area between two optically isotropic, planar dielectrics is often written in terms of a Hamaker constant A_0, as $\Omega(d) = -\frac{A_0}{12\pi d^2}$. To account for the finite speed of light, the Hamaker constant becomes a distance-dependent Hamaker coefficient in the Casimir-Lifshitz formulation, resulting in a free energy per unit area given by $\Omega(d) = -\frac{A(d)}{12\pi d^2}$. The Hamaker coefficient reduces to the Hamaker constant at short ranges [12, 13]. This distance dependence encodes the effect of retardation, or the finite speed of light. Retardation weakens Casimir-Lifshitz interactions between isotropic slabs, and $A(d)$ decays from A_0 to 0 as d increases. For two birefringent plates, the Hamaker coefficient depends on the relative angle between the plates, resulting in an angular dependence of the free energy per unit area:

$$\Omega(d,\theta) = -\frac{A(d,\theta)}{12\pi d^2}, \tag{1}$$

and hence a torque per unit area $M(d,\theta)$ arises between the two materials:

$$M(d,\theta) = -\frac{\partial \Omega(d,\theta)}{\partial \theta}. \tag{2}$$

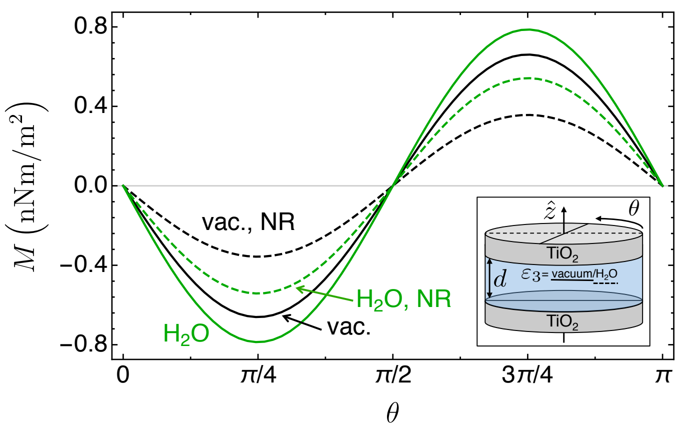

FIG. 1. Casimir-Lifshitz torque per unit area between two TiO_2 slabs separated by 30 nanometers of vacuum (black) or water (green). In the nonretarded approximation (dashed), the Casimir-Lifshitz torque reduces to the van der Waals torque. The calculated torque is increased by the intervening dielectric as well as by retardation effects. Inset: geometry of the system in question.

For small separations, the anisotropic van der Waals torque is thought to act as an alignment mechanism for nematic liquid crystals [14]. While there have been some qualitative experiments to explore this phenomenon[15–18], there have been no quantitative measurements and little theoretical exploration of the effects of retardation.

In this letter, we explore the effect of retardation on the Casimir-Lifshitz torque and find two surprising situations that lead to an enhancement of the torque. The geometry of the system is shown in the inset of Fig. 1, and the two effects in question are apparent in the plotted torques per unit area between parallel TiO_2 slabs at a separation of 30 nm. First, we demonstrate that the anisotropic part of the Hamaker coefficient (corresponding to the Casimir-

Lifshitz torque) between semi-infinite dielectric slabs is generally increased by retardation at small separations. For common birefringent crystals such as TiO_2, the enhancement is most prominent when the slabs are separated by tens of nanometers. As a result, the calculated torque is significantly greater when retardation effects are included than when they are neglected. Furthermore, we demonstrate that the insertion of a dielectric medium can increase the Casimir-Lifshitz torque at small separations. We demonstrate that common dielectric materials (such as liquids with optical refractive indices near 1.5), can increase the calculated Casimir-Lifshitz torque between TiO_2 slabs by as much as a factor of two. This effect persists throughout the experimentally accessible regime of separations on the order of 1–100 nm.

Heuristically, we can describe this effect by noting that the Casimir-Lifshitz torque depends on the relative orientation of the two plates. Even if the strength of the Casimir-Lifshitz interaction $\Omega(d,\theta)$ is decreased for all θ by inserting a dielectric medium, it may be decreased more for $\theta = \pi/2$ than for $\theta = 0$. As a result, the energy difference between the two orientations is increased, which corresponds to an enhancement of the torque.

As a result of these two effects, the calculated Casimir-Lifshitz torque can be greatly increased when retardation effects and intervening dielectric media are included. The enhancement of the torque by retardation implies that calculations that neglect retardation should be re-examined, as they may significantly underestimate the torque. A similar retardation effect was predicted for the Casimir-Lifshitz force in special cases involving systems with thin metallic films [19] or high anisotropy [20, 21]. The enhancement of the torque by insertion of a dielectric implies that an intermediate dielectric may be helpful in experiments designed to measure the Casimir-Lifshitz torque. To our knowledge, there is no analogous effect for the Casimir-Lifshitz force. Together, these results open new venues for manipulation of fluctuation forces at the nanoscale and have major implications for the design of Casimir-Lifshitz torque experiments.

We consider a system of two parallel, semi-infinite, half-spaces of birefringent materials separated by distance d, as shown in the inset of Fig. 1. The materials have their optic axes in the xy plane but are rotated by angle θ relative to each other. The Hamaker coefficient $A(d,\theta)$, can be split into isotropic and anisotropic parts $A^{(0)}(d)$ and $A^{(2)}(d)$, respectively:

$$A(d,\theta) \approx A^{(0)}(d) + A^{(2)}(d)\cos(2\theta)\,. \tag{3}$$

The cosine-like dependence in Eq. 3 is valid for materials with small birefringence. We define the isotropic and anisotropic parts of the Hamaker coefficient as in [20]: $A^{(0)}(d) = A(d,\pi/4)$, $A^{(2)}(d) = A(d,\pi/2) - A(d,0)$. The Casimir-Lifshitz torque is then approximated by:

$$M(d,\theta) \approx -\frac{A^{(2)}(d)\sin(2\theta)}{6\pi d^2}. \tag{4}$$

The effects of retardation on the Casimir-Lifshitz torque are encoded in $A^{(2)}(d)$.

At finite temperature, the Casimir-Lifshitz free energy is a sum over Matsubara frequencies $\xi_n = n * 2\pi k_{\mathrm{B}}T/\hbar$ (where T is the temperature of the system, 298 K in this work):

$$\Omega(d,\theta) = \frac{k_{\mathrm{B}}T}{4\pi^2}\sum_{n=0}^{\infty}{}' \int_0^{\infty} r\mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \ln D_n(r,\varphi)\,, \tag{5}$$

where r and φ are the radial and azimuthal components of a wave vector, and $D_n(r,\phi) = 0$ represents the dispersion condition for surface modes between the slabs [4, 22]. With substitution of the dimensionless $\chi = rd$, the retarded Hamaker coefficient can be written as a sum of contributions at the Matsubara frequencies:

$$A(d,\theta) = \sum_{n=0}^{\infty}{}' A_n(d,\theta) \tag{6a}$$

$$A_n(d,\theta) = -\frac{3k_{\mathrm{B}}T}{\pi}\int_0^{\infty} \chi\mathrm{d}\chi \int_0^{2\pi} \mathrm{d}\varphi \ln D_n(\chi,\varphi)\,. \tag{6b}$$

To examine the effects of retardation, we consider the dependence of a single Matsubara term $A_n(d,\theta)$ on the dimensionless $r_n = 2\sqrt{\varepsilon_3}\xi_n d/c$. Physically, r_n is the ratio of round-trip travel time for light between the plates ($2\sqrt{\varepsilon_3}d/c$) to the characteristic decay time of the Matsubara frequency ($1/\xi_n$) [12]. Therefore, r_n is a measure of retardance: as $d/c \to 0$, $r_n \to 0$. The contribution from a Matsubara term depends only on r_n and the dielectric properties at the corresponding imaginary frequency ξ_n. Each Matsubara term is split into isotropic and anisotropic terms as above: $A_n^{(0)}(r_n) = A_n(r_n,0)$, $A_n^{(2)}(r_n) = A_n(r_n,\pi/2) - A_n(r_n,0)$. By isolating $A_n^{(2)}(r_n)$, we can examine the effect of retardation on individual Matsubara terms that contribute to the Casimir-Lifshitz torque.

Following the notation of [20, 21], we define the anisotropy of the ith material as:

$$\delta_{i\perp} = \frac{\varepsilon_{i\perp} - \varepsilon_3}{\varepsilon_3}, \quad \delta_{i\parallel} = \frac{\varepsilon_{i\parallel} - \varepsilon_3}{\varepsilon_3}, \tag{7}$$

where ε is the real part of the dielectric function evaluated at imaginary frequency $\mathrm{i}\xi$. As in [20], we expand the integrand of Eq. 6b for small $\delta_{i\perp}$ and $\delta_{i\parallel}$ to second order (there is no zeroth or first order contribution). The integral over wavevectors is carried out analytically with the use of the exponential integral $\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t}\mathrm{d}t$:

$$A_n(r_n,\theta) = A_n^{(0)}(r_n) + A_n^{(2)}(r_n)\cos(2\theta) + ... \tag{8a}$$

$$A_n^{(2)}(r_n) = \frac{3k_{\mathrm{B}}T}{256}\left(\delta_{1\parallel} - \delta_{1\perp}\right)\left(\delta_{2\parallel} - \delta_{2\perp}\right)\zeta(r_n) \tag{8b}$$

$$\zeta(r_n) = \frac{1}{2}\Big[e^{-r_n}\left(-r_n^3 + r_n^2 + 2r_n + 2\right) - \mathrm{Ei}(-r_n)\left(r_n^4 + 4r_n^2\right)\Big]. \tag{9}$$

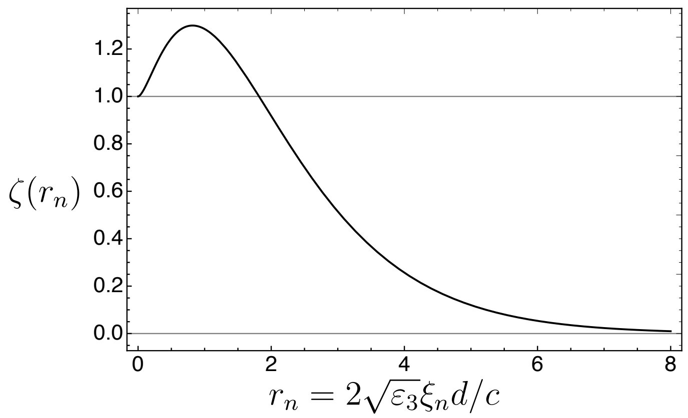

FIG. 2. For small birefringence, contributions of Matsubara terms to the anisotropic part of the Hamaker coefficient (and therefore the Casimir-Lifshitz torque), which is proportional to $\zeta(r_n)$, are increased by retardation at small separations.

To second order in $\delta_{i\perp}$ and $\delta_{i\|}$, the $\cos(2\theta)$ dependence is exact. The $A_n^{(0)}(r_n)$ term is independent of θ and does not contribute to the torque. In the nonretarded limit, $\zeta(r_n \to 0) = 1$, Eq. 8b reduces to:

$$A_{n,\mathrm{NR}}^{(2)} = \frac{3k_\mathrm{B}T}{256}\left(\delta_{1\|} - \delta_{1\perp}\right)\left(\delta_{2\|} - \delta_{2\perp}\right). \qquad (10)$$

Now we examine the dependence of $A_n^{(2)}(r_n)$ on r_n, which is wholly contained within $\zeta(r_n)$ (Eq. 9). This function is plotted in Fig. 2. The nonmonotonicity of $\zeta(r_n)$ is surprising—it implies that for small values of r_n, the contribution of a single Matsubara term to the Casimir-Lifshitz torque is increased by retardation. We can make an even stronger claim: for small d, the total Casimir-Lifshitz torque is also increased by retardation. This behavior is because for real materials, a finite number of Matsubara terms contribute to Casimir-Lifshitz interactions (all materials become optically transparent as $\xi \to \infty$). As the distance between two materials approaches 0, the set of r_n's corresponding to this finite set of Matsubara terms will fall in the retardation-enhancement region where $r_n \lesssim 1.82$ and $\zeta(r_n) > 1$. This means that the sum of Matsubara terms will also be enhanced by retardation in this limit. Therefore, for small separations and small birefringence, retardation will generally cause an increase in the Casimir-Lifshitz torque.

This calculation is to second order in $\delta_{i,\perp}$ and $\delta_{i,\|}$, but for materials with higher anisotropy (such as those chosen to maximize the Casimir-Lifshitz torque) the approximation is less accurate. However, this nonmonotonicity persists in the analytic expansion to third order in $\delta_{i\perp}$ and $\delta_{i\|}$ as well. Furthermore, numerical exploration of the parameter space shows that all combinations of dielectric constants produce a nonmonotonic dependence on r_n of the anisotropic part of the Hamaker coefficient.

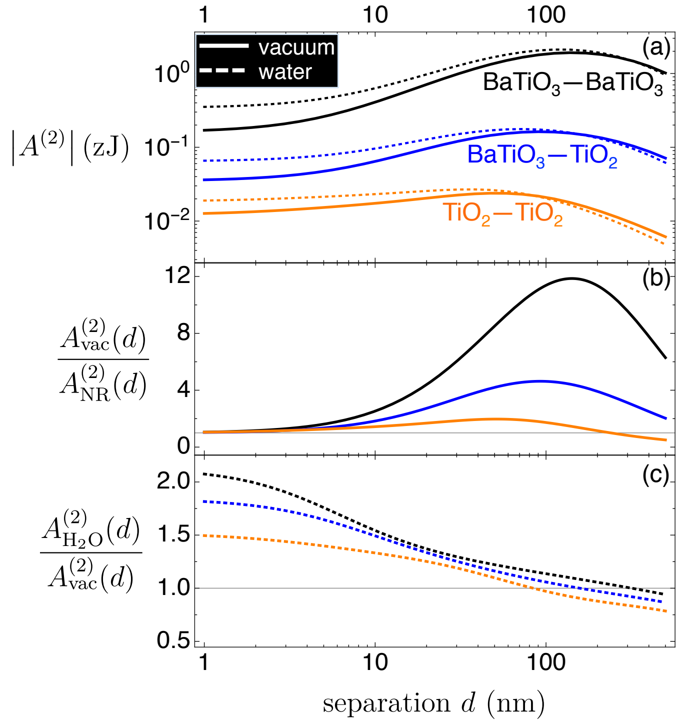

FIG. 3. The effects of retardation and insertion of a water layer on the anisotropic part of the Casimir-Lifshitz energy. (a) Anisotropic part of the Hamaker coefficient for three material combinations ($BaTiO_3$–$BaTiO_3$, $BaTiO_3$–TiO_2, and TiO_2–TiO_2 in black, blue, and orange) when the materials are separated by vacuum (solid) and water (dashed). (b) Ratio of the anisotropic part of Hamaker coefficients with retardation effects to the nonretarded approximation. (c) Ratio of the anisotropic part of Hamaker coefficients for materials separated by water to those separated by vacuum. For $d \lesssim 90$ nm, the torque is enhanced by retardation and by the intervening water for all three material combinations.

To demonstrate the generality of this effect, we calculate the anisotropic part of the Casimir-Lifshitz interaction for two $BaTiO_3$ slabs (strong birefringence), two TiO_2 slabs, and one $BaTiO_3$ and one TiO_2 slab in Fig. 3. The material dispersions in this paper are modeled using the parameters from [23] and [24]. The anisotropic part of the Hamaker coefficient $A_n^{(2)}(d)$ is plotted for these material combinations in Fig. 3a. The nonmonotonicity of $A_n^{(2)}(d)$ and the increase due to the inclusion of water is clear. The effect of retardation is even clearer in Fig. 3b, which plots the ratio of the full calculation to the nonretarded approximation: the full calculation yields a torque several times stronger than the nonretarded calculation (by nearly a factor of 12). When the plates are separated by ≈ 30 nm, the calculated torque is typically $\gtrsim 50\%$ stronger when retardation effects are included. However, the magnitude of the enhancement is highly dependent on the choice of birefringent material.

Retardation effects are always an experimental reality,

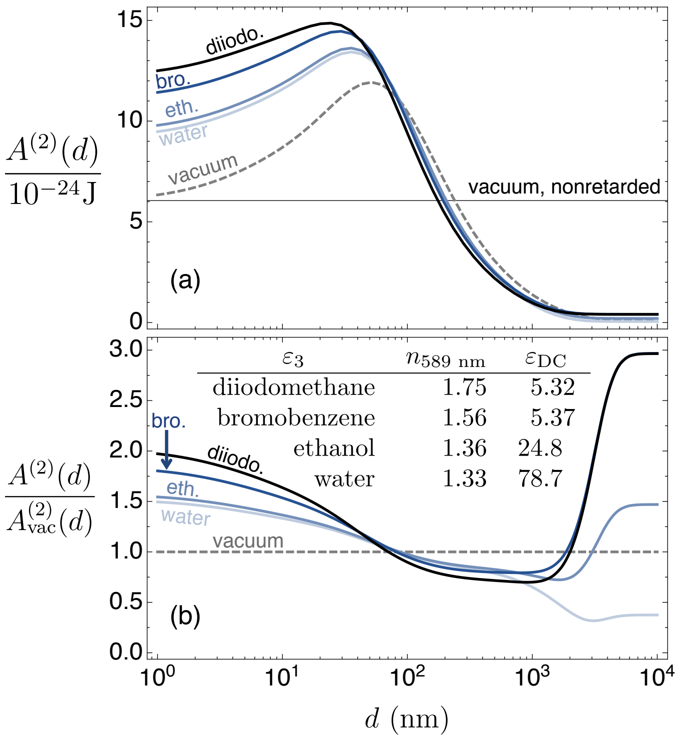

FIG. 4. (a) The combined effects of retardation and an intervening dielectric media on anisotropy of Casimir-Lifshitz free energy between two parallel slabs of TiO_2. The anisotropic part of the Hamaker coefficient is increased by these two effects for $d \lesssim 100$ nm. (b) The effect of the intervening dielectric media are isolated by scaling $A^{(2)}(d)$ to the value when the plates are separated by vacuum. At small separations ($d < 50$ nm), the liquids with higher optical refractive indices cause the greatest increase in the torque. At very large separations ($d > 50$ μm), the $n = 0$ Matsubara term dominates, so only the DC dielectric constant is relevant.

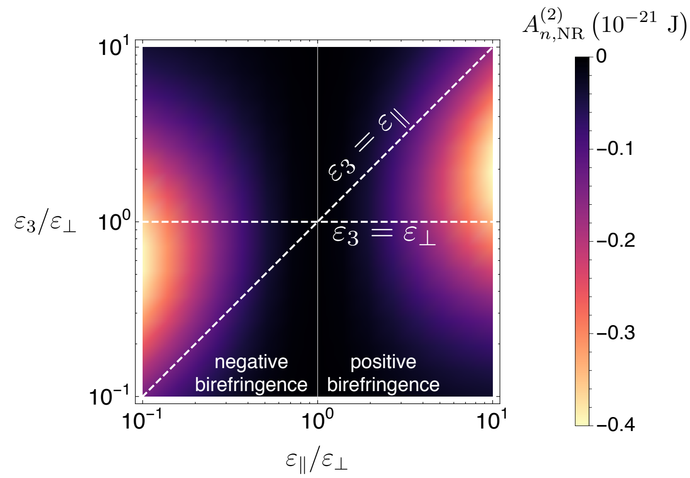

FIG. 5. Each Matsubara term contributes $A^{(2)}_{n,\mathrm{NR}}$ to the anisotropy of the Casimir-Lifshitz interaction. For two identical birefringent slabs, the ε_3 that maximizes $A^{(2)}_{n,\mathrm{NR}}$ (and therefore the Casimir-Lifshitz torque) is between $\varepsilon_\perp$ and $\varepsilon_\parallel$.

so the difference between the nonretarded and full calculations cannot be measured. However, one could choose to include an intervening dielectric medium between two birefringent materials. This addition modifies the dispersion relation $D_n(r, \varphi)$ in Eq. 5, which can significantly increase the Casimir-Lifshitz torque. Figures 3(a) and 3(c) show the effect of filling the vacuum gap with water, which causes a significant increase in the torque over a broad range of separations.

We examine the effect of the intervening dielectric in more detail by comparing $A^{(2)}(d)$ for two TiO_2 slabs separated by vacuum and several distinct fluids in Fig. 4. The inclusion of dielectrics with higher refractive indices at optical frequencies (which dominate short-range Casimir-Lifshitz interactions) results in higher Casimir-Lifshitz torques at small separations. In fact, torques are enhanced for most experimentally accessible separations (≈ 75 nm for these materials—beyond this separation, the torques are extremely weak). For $d \lesssim 100$ nm, the torque is also enhanced by retardation effects. Beyond this point, retardation effects weaken the torque (affecting the systems with dielectric media the most). At very large separations at room temperature, only the $n = 0$ Matsubara term contributes—this term is dominated by thermal fluctuations and is not affected by retardation [12, 13]. Therefore, the same torque enhancement seen at short ranges reappears for three of the materials for $d > 2$ μm. The DC dielectric constant for water and ethanol, however, are so high that the long-range torque is reduced instead of enhanced. We note that the Casimir-Lifshitz torque at separations greater than a micron may be too small to measure in currently proposed experiments when a dielectric medium is introduced. At a separation of $d = 30$ nm, the maximum nonretarded torque between the plates across vacuum is 3.6×10^{-10} Nm/m^2. This is increased by a factor of 1.9 when retardation effects are included, and by a factor of 2.2 when retardation effects and an intervening diiodomethane medium are included. Although intervening media can cause other experimental difficulties, a large increase in the torque may represent a worthwhile trade-off.

In Fig. 5, we calculate how the nonretarded torque is affected, more generally, by a dielectric medium. The torque is increased by inserting a medium with a dielectric function somewhere between $\varepsilon_\perp$ and $\varepsilon_\parallel$ of the birefringent material. For a given choice of $\varepsilon_\perp$ and $\varepsilon_\parallel$, $\varepsilon_3 = 1$ does not typically maximize the torque. For example, with $\varepsilon_\perp = 5.81$ and $\varepsilon_\parallel = 6.62$ (as for the DC dielectric terms of TiO_2), the optimal ε_3 is about 6. In this case, the contribution to the Casimir-Lifshitz torque is nearly tripled by the insertion of such a dielectric (compared to vacuum).

The distance dependence of Casimir-Lifshitz interactions is further complicated by retardation screening of high frequency contributions at larger separations. The interplay of dielectric functions can lead to a rich variety of unusual effects, as demonstrated in [25, 26]. However, we emphasize that the effects we demonstrate here are distinct from those that rely on particular combinations of dielectric materials. The torque enhancement by retardation is independent of dielectric functions, and the enhancement by the inclusion of a dielectric medium is quite general and appears even in the nonretarded calculation.

In this Letter we have shown that, at short distances, the Casimir-Lifshitz torque between parallel slabs is increased by retardation. The nonretarded approximation can underestimate the torque by as much as an order of magnitude. This is the case even at separations on the order of 10 nm, a regime in which retardation effects are often ignored. Furthermore, an intervening dielectric medium often increases the Casimir-Lifshitz torque by a significant amount. A carefully selected dielectric liquid can make the torque stronger and more experimentally accessible. We encourage researchers to include the effects of retardation and an intervening medium, as they may make measurements realizable in surprising conditions.

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