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Phys. Rev. Lett. **119**, 140501 — Published 2 October 2017

DOI: [10.1103/PhysRevLett.119.140501](https://doi.org/10.1103/PhysRevLett.119.140501)

# Universal Limit on Communication

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I derive a universal upper bound on the capacity of any communication channel between two distant systems. The Holevo quantity, and hence the mutual information, is at most of order  $E\Delta t/\hbar$ , where  $E$  is the average energy of the signal, and  $\Delta t$  is the amount of time for which detectors operate. The bound does not depend on the size or mass of the emitting and receiving systems, nor on the nature of the signal. No restrictions on preparing and processing the signal are imposed. As an example, I consider the encoding of information in the transverse or angular position of a signal emitted and received by systems of arbitrarily large cross-section. In the limit of a large message space, quantum effects become important even if individual signals are classical, and the bound is upheld.

In communication theory, one typically studies problems such as signal optimization, compression, or error correction, given a particular channel. Here, I will consider instead whether a channel with a desired capacity is realizable by any means, given the laws of physics.

It will be assumed that the detection of the signal can be described using quantum field theory. A universal bound on the von Neumann entropy of quantum fields [1] will be combined with the Holevo theorem [2]. This will yield a simple, robust, and surprisingly strong bound on the information that can be conveyed between two arbitrarily large systems with arbitrary resources. The bound depends only on the energy of the signal and the length of time over which the signal can be examined.

*Communication Between Distant Large Systems* Suppose that Alice controls an arbitrarily large, bounded region of space, with arbitrary matter and energy content. For concreteness we can consider a “planet”—an approximately spherical system of radius  $R_A$ —but this will not be important. Alice would like to send a message to Bob, who resides in a distant region, outside of some much vaster sphere of radius  $R_B$  (see Figures).

Bob has already surrounded Alice with detectors. For example, the entire sphere at  $R_B$  could be densely tiled with detectors. We require  $R_B \gg R_A$  but we impose no upper limit on either  $R_A$  or  $R_B$ . We need not assume that gravity is weak at Alice’s location (though this can always be arranged by increasing  $R_A$  and diluting her system). We do assume that gravity is weak at  $R_B$ , as would be the case for large  $R_B$  in an asymptotically flat spacetime.

Let  $E$  be the energy of the signal Bob receives from Alice. ( $E$  includes the rest mass, if any.) We suppose that the time period during which Bob’s detectors will be operating is known to Alice, and that it has duration  $\Delta t \ll R_B/c$ . (We set the speed of light  $c = 1$  below.)

We do not restrict the amount of time that Alice is given to prepare her signal: she gets an arbitrarily early start. Nor do we restrict the amount of time for which Bob can process his detector output, nor the energy re-

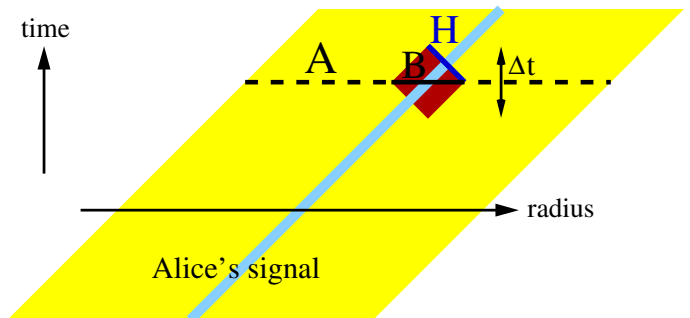


FIG. 1. Alice (not shown) has sent a signal (light blue) to a distant sphere controlled by Bob. In this diagram, each point represents a sphere, and a  $45^\circ$  slope corresponds to the speed of light. Bob’s detectors are on for a time  $\Delta t$ , allowing them to probe a spacetime region (red diamond) that contains a spatial shell  $B$  of width  $c\Delta t$ , assumed here to be wider than the signal. Alice had unlimited space and time when preparing the signal; her power is equivalent to accessing a much larger region  $A \supset B$  (dashed) at the time of detection. Recently discovered universal entropy bounds apply to the null hypersurface  $H$  (dark blue). They are shown here to constrain the capacity of this communication channel, independently of how it is implemented, in terms of the signal energy and  $\Delta t$ . This is surprising: no limit is placed on the radius of the sphere, so one might have expected that Bob can receive infinitely many distinct classical signals of a fixed energy at different angles. It can be shown, however, that the classical description breaks down before the bound is violated.

sources available to Alice and Bob for generating and processing the signal. With these minimal restrictions, *how much information can Alice send to Bob?*

Alice and Bob have agreed on a set of  $N$  possible messages, from which Alice will select message  $a$ , with probability  $p(a)$ , to be sent to Bob as a physical signal. If Bob can distinguish reliably between all  $N$  signals, and thus determine which of the  $N$  possible signals actually arrived, then Bob gains an amount of information equal

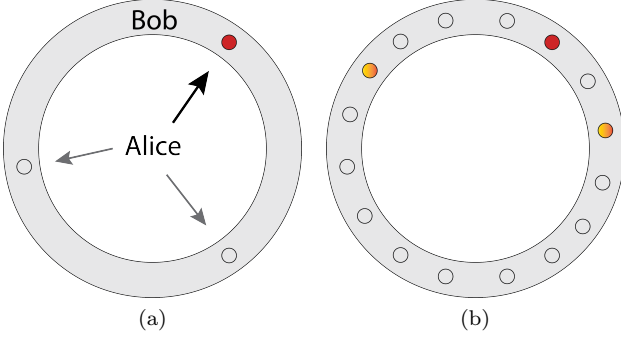


FIG. 2. (a) Small message space. Alice sends one of three previously agreed-upon classical signals (red dot) to Bob. Bob resides at a great distance  $R_B$  and operates three detectors at the potential signal sites. By examining which detector responded, he learns an amount  $H(A:B) = \log 3$  of information. (b) Large message space. Alice sends one of  $N$  classical signals, where  $N \gg E\Delta t/\hbar$ . Bob's detectors can only explore a finite region (grey shell) of width  $\Delta t$ . Irreducible quantum noise creates false detections (yellow), which preclude Bob from identifying Alice's message.

to the Shannon entropy of the message set,

$$H(A) \equiv - \sum_a p(a) \log p(a) \leq \log N. \quad (1)$$

This can be justified as follows. Consider Bob after his detectors have received Alice's signal, but before he inspects them. At this point he may describe the message by a classical probabilistic ensemble, with Shannon entropy  $H(A)$ . This quantifies Bob's initial ignorance. After finding the message to be  $b$ , Bob updates the probability distribution to  $p'(a) = \delta_{ab}$ , with vanishing Shannon entropy. Thus Bob's ignorance has decreased by  $H(A)$ , i.e., he has gained an amount  $H(A)$  of information.

With no restrictions on  $N$ , how much information can Bob gain, if the average signal energy is  $E$ , and Bob's detectors operate for a time period of duration  $\Delta t$ ?

*Unbounded Classical Message Space* It would appear that Bob can gain an unbounded amount of information, because there is no limit on the number of distinct signals, at fixed average energy, such that each signal is well-localized in time to much better than  $\Delta t$ .

For example, if  $E\Delta t \gg \hbar$ , then Alice can send a classical flash of light to Bob. Its duration, and hence its spatial support, will be short compared to  $\Delta t$ . Alice sends only one such signal, encoding the message in the angular direction of the signal. Bob has detectors at all angles on his distant sphere. From the solid angle at which Bob receives the signal, he will learn what Alice's message is.

Now, at *fixed* system size  $R_A$ , there would be a quantum limit on the angular resolution Alice can achieve. With energy  $E$ , she can resolve distances  $d \gtrsim \hbar/E$ . The number of distinct pixels that can light up on the surface

of her system is

$$N \lesssim \frac{R_A^2}{d^2} \lesssim \frac{R_A^2 E^2}{\hbar^2} \quad (2)$$

But the system size is not fixed. We see from Eq. (2) that Alice can make the message space as large as she likes, simply by emitting from a large enough sphere.

At any fixed  $N$ , suppose that each message to be equally likely,  $p(a) = 1/N$ , so  $H(A) = \log N$ . Since there is no upper bound on  $N$ , Alice can make  $H(A)$  as large as she likes, at fixed  $E, \Delta t$ .

Indeed, the problem is not with Alice. Rather, it lies with Bob's ability to distinguish an arbitrary number of classical signals. This is a quantum effect, and before we can understand it, we must reformulate our classical protocol in quantum language.

*Quantum Description* We will need the following standard definitions. The von Neumann entropy of a quantum state is  $S(\rho) \equiv -\text{tr} \rho \log \rho$ . The relative entropy of  $\rho$  with respect to  $\sigma$  is  $S(\rho||\sigma) \equiv \text{tr} \rho \log \rho - \text{tr} \rho \log \sigma$ . Both are non-negative.

Recall that Alice had an arbitrarily large time to prepare a signal. After radial propagation, she could have affected the quantum state in a wide shell  $A$ —far wider than the shell accessed by Bob. This shell is far from Alice, who is no longer relevant herself. We take  $A$  to be the region of space occupied by the state prepared by Alice, at the time when it is being measured by Bob; see Fig. 1. (Their communication will be less efficient if  $A$  is polluted by signals from other parties, or by noise. This would only make our upper bound more comfortably satisfied.)

Alice prepared the signal state  $\rho_a^A$  with probability  $p(a)$ . The state of ignorance is the average state,

$$\rho_{\text{av}}^A = \sum_{a=1}^N p(a) \rho_a^A, \quad (3)$$

Since the signal states correspond to distinct classical states, they are mutually orthogonal as quantum states:  $\rho_a \rho_{a'} = 0$  for  $a \neq a'$ . This implies that the von Neumann entropy of the average state is at least the Shannon entropy:

$$S(\rho_{\text{av}}^A) = H(A) + \sum p(a) S(\rho_a^A) \geq H(A). \quad (4)$$

Bob's detectors are only operating for a time  $\Delta t$ . This means that he has access only to a shell  $B$  of thickness  $\Delta t$  (see Fig. 1).  $B$  is a subsystem (or more generally, a subalgebra) of the region  $A$  which is controlled by Alice.  $A$  will have finite width in practice, as shown in Fig. 1. But since it can be much larger than  $\Delta t$ , we may idealize  $A$  as all of space for calculational purposes. With finite  $A$ , our bound could only get tighter.

The state in the subregion  $B$  accessed by Bob is fully described by the reduced density operator

$$\rho^B \equiv \text{tr}_{A-B} \rho^A. \quad (5)$$

The trace is over the complement of  $B$ , the region not probed by Bob's detectors. We may regard this trace as a quantum channel by which classical information is communicated [3–5]. If Alice arranged for the signal state  $\rho_a^A$  to be present in  $A$ , the channel output will be  $\rho_a^B$ . Bob attempts to decode the message by performing a measurement on the system  $B$ . The most general measurement is described by a set of positive operators  $E_i$  that sum to the identity,  $\sum_i E_i = \mathbf{1}$ . The conditional probability that Bob obtains outcome  $b$  is given by  $p(b|a) = \text{tr}_B(\rho_a^B E_b)$ .

*Bounds on the Channel Capacity* In general,  $p(b|a) \neq \delta_{ab}$ , which means that Bob is unable to distinguish Alice's signals perfectly. The information he gains is quantified by the classical mutual information,

$$H(A:B) \equiv H(A) + H(B) - H(A, B), \quad (6)$$

which satisfies  $0 \leq H(A:B) \leq \min\{H(A), H(B)\}$ . Here,  $H(A, B)$  is the Shannon entropy of  $p(a, b) = p(a)p(b|a)$ , the joint probability that Alice sends  $a$  and Bob finds  $b$ ; and  $p(b) = \sum_a p(a, b)$  is the marginal (i.e., total) probability that Bob finds  $b$ . For example, if Bob's result is completely uncorrelated with Alice's message then  $H(A:B) = 0$ , and Bob gains no information. If it is perfectly correlated, then  $H(A:B) = H(A)$ , and Bob gains all of the information about Alice's message.

For classical information sent over a quantum channel,  $p(b|a)$  depends on the choice of operators  $E_i$ . However,  $H(A:B)$  is bounded from above by the Holevo quantity [2],  $\chi$ , which depends only on the channel output:

$$H(A:B) \leq \chi \equiv S(\rho_{\text{av}}^B) - \sum_a p(a) S(\rho_a^B). \quad (7)$$

The von Neumann entropy of a bounded region  $B$ ,  $S(\rho^B)$ , diverges in quantum field theory, because of short-distance entanglement between excitations localized to either side of the boundaries of  $B$  [1, 6, 7]. Since  $\sum_a p(a) = 1$ , these divergences cancel in Eq. (7).

Still, it is instructive to write Eq. (7) in a form where only finite quantities appear. Consider the *reduced vacuum*,  $\sigma^B = \text{tr}_{A-B} \sigma^A$ , where  $\sigma^A$  is the global vacuum state (empty Minkowski space). The *vacuum-subtracted entropy* [8] for any quantum state in Bob's subregion is defined as

$$\Delta S(\rho^B) \equiv S(\rho^B) - S(\sigma^B). \quad (8)$$

(Vacuum subtraction is well-defined only in weakly gravitating regions such as  $B$ . If gravity was strong, the shape of space would depend on the quantum state. Then it would not be clear how to reduce two different states to the “same” region.) We now find

$$H(A:B) \leq \chi = \Delta S(\rho_{\text{av}}^B) - \sum_a p(a) \Delta S(\rho_a^B). \quad (9)$$

For individual signals, which are well-localized to  $B$ , the vacuum-entanglement contributions cancel out in

Eq. (8), so  $\Delta S(\rho_a^B) = S(\rho_a^A)$ . If this remained true after averaging, we would have  $\Delta S(\rho_{\text{av}}^B) = S(\rho_{\text{av}}^A)$  for the signal ensemble. We could then use Eq. (4) to recover the maximum classical channel capacity,  $H(A)$ , from Eq. (9).

However, I will now derive an upper bound on  $\Delta S(\rho_{\text{av}}^B)$  that does not increase with the number of distinct classical signals, at fixed average signal energy. This means that for large enough  $N$ ,  $\Delta S(\rho_{\text{av}}^B) \ll S(\rho_{\text{av}}^A)$ .

The log of  $\sigma^B$  defines a *modular Hamiltonian* operator  $\hat{K}$ , via

$$\sigma^B = \frac{e^{-\hat{K}}}{\text{tr}_B e^{-\hat{K}}}. \quad (10)$$

See e.g. Sec. V.2 in Ref. [9] for a detailed treatment. The *modular energy* of a reduced state  $\rho^B$  is defined as

$$\Delta K(\rho^B) \equiv \text{tr}_B \hat{K} \rho^B - \text{tr}_B \hat{K} \sigma^B. \quad (11)$$

This quantity is useful because it allows us to trade the information theoretic quantities appearing in the Holevo bound for physical quantities. This is a central point, and it relies crucially on recent, new results in quantum field theory [1, 7, 10].

The modular Hamiltonian for general spatial regions is a formal, highly nonlocal operator which does not admit a simple physical interpretation. However, for a finite portion of a null plane (such as the hypersurface  $t = x$  in Minkowski space),  $\hat{K}$  can be expressed as a weighted integral over the local stress tensor; see Eq. (4.4) in [1] and Eq. (2.7) in [7]:

$$\hat{K} = \frac{2\pi}{\hbar} \int d^2y \int_0^{\Delta t} d\lambda g(\lambda) \hat{T}_{ab} k^a k^b. \quad (12)$$

Here  $\lambda = t - z$  in Cartesian coordinates, and  $k^a = (\frac{d}{d\lambda})^a$  is the null vector tangent to the null generators of the hypersurface. Eq. (12) extends (nontrivially) to a null hypersurface  $H$  orthogonal to a large sphere in asymptotically Minkowski space [10], with  $\lambda = t - r$  in spherical (Bondi) coordinates. The portion of Alice's signal accessible to Bob passes through  $H$ ; see Fig. 1.

Eq. (12) provides an interpretation of  $\Delta K$  in terms of the physical stress tensor expectation value,  $\langle T_{ab} \rangle$ . Note that Eq. (12) is relativistically invariant; it does not depend on the state of motion of Bob. Since the integral only accesses region  $B$ , the expectation value  $\langle T_{ab} \rangle$  can be taken in the state  $\rho^B$  or  $\rho^A$ . One can prove [7] that  $g(\lambda)$  satisfies

$$g(0) = g(\Delta t) = 0, \quad \left. \frac{dg}{d\lambda} \right|_0 = - \left. \frac{dg}{d\lambda} \right|_{\Delta t} = 1, \quad (13)$$

$g \geq 0$ , and  $|dg/d\lambda| \leq 1$ . (E.g., for free bosonic fields,  $g(\lambda) = \lambda(1 - \lambda/\Delta t)$ .) This implies an upper bound on  $\Delta K$  that isolates an integral of the energy density:

$$\Delta K \leq \frac{2\pi}{\hbar} \frac{\Delta t}{2} \int d^2y \int_0^{\Delta t} d\lambda T_{ab} k^a k^b. \quad (14)$$

To gain further intuition, we note that this upper bound is well-approximated by  $E\Delta t/\hbar$ . To see this, consider a signal localized near the  $x$ -axis. We have  $T_{ab}k^ak^b = T_{tt} - 2T_{tx} + T_{xx}$ , where  $T_{tt}$  is the energy density,  $T_{tx}$  the momentum density, and  $T_{xx}$  is the pressure. First, suppose that the signal is a nonrelativistic object of width  $\Delta x(0)$  and mass  $E(0)$  in its rest frame. If Bob and the object move with relative velocity  $\beta$  in the radial direction, then the energy and width in Bob's frame are  $E(\beta) = \gamma E(0)$  and  $\Delta x(\beta) = \Delta x(0)/\gamma$ , where  $\gamma = (1 - \beta^2)^{-1/2}$ . The time Bob needs to examine the object is  $\Delta t(\beta) = \Delta x(\beta)$ . Thus  $E(\beta)\Delta t(\beta)$  is frame-independent, like Eq. (12), and it suffices to verify that approximate equality holds in the rest frame. Since  $T_{tx}$  integrates to zero and  $T_{xx} \ll T_{tt}$  for nonrelativistic objects, the integral in Eq. (14) reduces to  $\int d^2y d\lambda T_{tt} \equiv E$ . At the opposite extreme, consider instead an ultrarelativistic signal, such as an outgoing photon wavepacket of size and characteristic wavelength  $\Delta t$ . Since the photon's energy is  $E \sim \hbar/\Delta t$ ,  $E\Delta t$  is again invariant. For an outgoing photon,  $T_{tt} = T_{xx} = -T_{tx}$ , so the integral in Eq. (14) yields  $E$  up to an  $O(1)$  factor.

Positivity of the relative entropy  $S(\rho^B || \sigma^B)$  implies [8]

$$\Delta S(\rho^B) \leq \Delta K(\rho^B), \quad (15)$$

In the Holevo bound (9), this allows us to trade the vacuum subtracted entropy of the signal average for physical energy and time:

$$H(A:B) \leq \chi \leq \Delta K(\rho_{\text{av}}^B) - \sum_a p(a) \Delta S(\rho_a^B). \quad (16)$$

This result implies that the channel capacity cannot be made arbitrarily large by enlarging the message space, at fixed average signal energy and fixed average vacuum-subtracted entropy of the individual signals,  $\Delta S(\rho_a^A)$ .

If all signals are classical and well-localized to  $B$ , then  $\Delta S(\rho_a^B) \geq 0$ ,<sup>1</sup> and the bound on the channel capacity simplifies to

$$H(A:B) \leq \chi \leq \Delta K(\rho_{\text{av}}^B) \lesssim E_{\text{av}}\Delta t/\hbar, \quad (17)$$

where the final inequality follows from the discussion after Eq. (14). Here  $E_{\text{av}}$  denotes the energy expectation value of in the signal average state  $\rho_{\text{av}}^{(B)}$ .

Eqs. (16) and (17) are the main result of this paper.<sup>2</sup> I will turn next to its physical interpretation: for a sufficiently large message space, quantum effects become important, even if each individual signal is classical.

<sup>1</sup> If the individual signals are quantum, then  $\Delta S(\rho_a^B)$  can be negative. It would be interesting to constrain this regime further.

<sup>2</sup> Bekenstein's pioneering constraints on channel capacity [11, 12] used an entropy bound weaker than Eq. (15), involving the largest dimension of the problem instead of  $\Delta t$ . That upper bound would be of order  $E_{\text{av}}R_B$  and thus would not constrain the channel capacity of arbitrarily large systems ( $R_B \rightarrow \infty$ ). See also Ref. [13]. Bounds involving Newton's constant [14] become trivial in the weakly gravitating setting considered here.

*Reduced Vacuum and Irreducible Noise* In order to understand the bound on channel capacity at an intuitive level, let us revisit our earlier example: the signal space consists of  $N$  distinct classical signals of identical energy, and  $p(a) = 1/N$ . Then Eq. (1) implies  $H(A) \rightarrow \infty$  as  $N \rightarrow \infty$ . But  $H(A:B)$  remains bounded by Eq. (17). What prevents Bob from simply observing Alice's blatant classical signal?

By Eq. (10), the reduced vacuum is a thermal state with (arbitrary) temperature  $\beta^{-1}$ , with respect to the Hamiltonian  $\beta\hat{K}$ .<sup>3</sup> Choosing  $\beta = \Delta t/\hbar$  for definiteness, we see that any particular signal state has nonvanishing, Boltzmann-suppressed probability of being observed in the reduced vacuum:  $\log P_i \sim -\Delta K \sim -E\Delta t/\hbar$ . For  $E\Delta t \gg \hbar$ ,  $P_i$  is exponentially small for all  $i$ , consistent with our intuition.

But if Alice's sphere has many pixels, the enormous number of possible signal states can overcome the suppression, so that some false signals will appear in Bob's detectors [17, 18] (Fig. 2). The expected number of false detections is

$$N_{\text{false}} \sim NP_i. \quad (18)$$

This becomes greater than unity for  $\log N \gg E\Delta t/\hbar$ , that is, precisely in the regime where the bound would otherwise be violated. In this regime, Bob will detect signals that Alice did not send. Since Bob cannot determine which signal is the "real" one, the protocol we have devised is not obviously useful for communicating information. Therefore, it does not provide a counterexample to Eq. (17).

(We might ask where the energy of the false flashes is coming from,  $E_{\text{false}} = N_{\text{false}}E$ . The answer is that it comes from Bob, who expends an average energy at least of order  $\hbar/\Delta t$ , per detector pixel, just to localize the detector operation to the time interval  $\Delta t$ . The total energy put in by Bob is  $N\hbar/\Delta t$ , which is much greater than  $E_{\text{false}}$  in the regime  $E\Delta t \gg \hbar$ .)

<sup>3</sup> Note that  $\hat{K}$  is distinct from the standard Hamiltonian that generates time translations for static observers in Minkowski space. The definition (10) of a modular Hamiltonian does not generally give rise to a geometric interpretation in terms of time translations. An exception is the familiar example of the reduced state obtained by restricting the global Minkowski vacuum to the Rindler wedge  $0 < |t| < x$ . That state is thermal with respect to a modular Hamiltonian that generates the time evolution of a family of accelerated observers [15, 16]. This perspective affords some intuition regarding the properties of  $\hat{K}$  quoted above, and by Eq. (10), the properties of the reduced vacuum  $\sigma^B$ . The boundary of any region agrees with the boundary of Rindler space at sufficiently short distances, and the entanglement structure of the vacuum at short distances must reflect this. This explains, for example, why the function  $g(\lambda)$  limits to the Rindler form (13) near the two boundaries of the null interval we consider.

Alice and Bob can eliminate false signals by pruning the message space. For example, they may take only a small subset of Alice's pixels to correspond to actual messages. Then Bob does not need to operate such a large number of detectors. If  $\log N \ll E\Delta t/\hbar$ , then it is very unlikely that even one of his  $N$  detectors will produce false signals. In this regime, Eq. (17) is consistent with perfect communication,  $H(A) \approx H(A:B)$ .

*Acknowledgments* It is a pleasure to thank S. Aaronson, H. Casini, N. Engelhardt, I. Halpern, J. Maldacena, A. Strominger, A. Wall, and especially Z. Fisher for discussions. This work was supported in part by the Berkeley Center for Theoretical Physics, by the National Science Foundation (award numbers 1521446 and 1316783), by FQXi, and by the US Department of Energy under contract DE-AC02-05CH11231.

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- [1] R. Bousso, H. Casini, Z. Fisher, and J. Maldacena, "Proof of a Quantum Bousso Bound," *Phys.Rev.* **D90** no. 4, (2014) 044002, [arXiv:1404.5635 \[hep-th\]](#).
- [2] A. S. Holevo, "Bounds for the quantity of information transmitted by a quantum communication channel," *Problems of Information Transmission* **9 (3)** (1973) 177183.
- [3] J. Preskill, "Lecture notes on quantum computation." Chapter 5, <http://www.theory.caltech.edu/~preskill/ph229/#lecture>, 1998.
- [4] M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*. Cambridge University Press, Cambridge, New York, 2000.
- [5] B. Schumacher and M. D. Westmoreland, "Relative entropy in quantum information theory," [arXiv:quant-ph/0004045](#).
- [6] M. Srednicki, "Entropy and area," *Phys. Rev. Lett.* **71** (1993) 666–669, [hep-th/9303048](#).
- [7] R. Bousso, H. Casini, Z. Fisher, and J. Maldacena, "Entropy on a null surface for interacting quantum field theories and the Bousso bound," *Phys.Rev.* **D91** no. 8, (2015) 084030, [arXiv:1406.4545 \[hep-th\]](#).
- [8] H. Casini, "Relative entropy and the Bekenstein bound," *Class.Quant.Grav.* **25** (2008) 205021, [arXiv:0804.2182 \[hep-th\]](#).
- [9] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras*. Springer Verlag, Berlin, 1992.
- [10] R. Bousso, "Asymptotic Entropy Bounds," [arXiv:1606.02297 \[hep-th\]](#).
- [11] J. D. Bekenstein, "Energy Cost of Information Transfer," *Phys. Rev. Lett.* **46** (1981) 623–626.
- [12] J. D. Bekenstein, "Entropy Content and Information Flow in Systems with Limited Energy," *Phys. Rev. D* **30** (1984) 1669.
- [13] S. G. Avery and M. F. Paulos, "Universal Bounds on the Time Evolution of Entanglement Entropy," *Phys. Rev. Lett.* **113** no. 23, (2014) 231604, [arXiv:1407.0705 \[hep-th\]](#).
- [14] S. Lloyd, V. Giovannetti, and L. Maccone, "Physical Limits to Communication," *Phys. Rev. Lett.* **93** no. 10, (2004) 100501.
- [15] J. Bisognano and E. Wichmann, "On the Duality Condition for Quantum Fields," *J.Math.Phys.* **17** (1976) 303–321.
- [16] W. G. Unruh, "Notes on black hole evaporation," *Phys. Rev. D* **14** (1976) 870.
- [17] D. Marolf and R. D. Sorkin, "On the status of highly entropic objects," *Phys. Rev.* **D69** (2004) 024014, [arXiv:hep-th/0309218 \[hep-th\]](#).
- [18] D. Marolf, D. Minic, and S. F. Ross, "Notes on space-time thermodynamics and the observer dependence of entropy," *Phys.Rev.* **D69** (2004) 064006, [arXiv:hep-th/0310022 \[hep-th\]](#).