Skyrme Insulators: Insulators at the Brink of Superconductivity

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Skyrme insulators: insulators at the brink of superconductivity

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Current theories of superfluidity are based on the idea of a coherent quantum state with topologically protected, quantized circulation. When this topological protection is absent, as in the case of ^{3}He-A, the coherent quantum state no longer supports persistent superflow. Here we argue that the loss of topological protection in a superconductor gives rise to an insulating ground state. We specifically introduce the concept of a *Skyrme insulator* to describe the coherent dielectric state that results from the topological failure of superflow carried by a complex vector order parameter. We apply this idea to the case of SmB_6, arguing that the observation of a diamagnetic Fermi surface within an insulating bulk can be understood as a realization of this state. Our theory enables us to understand the linear specific heat of SmB_6 in terms of a neutral Majorana Fermi sea and leads us to predict that in low fields of order a Gauss, SmB_6 will develop a Meissner effect.

While it is widely understood that superfluids and superconductors carry persistent "supercurrents" associated with the rigidity of the broken symmetry condensate[1], it is less commonly appreciated that the remarkable persistence of supercurrents has its origins in topology. The order parameter of a conventional superfluid or superconductor lies on a circular manifold (S^1) and the topologically stable winding number of the order parameter, like a string wrapped multiple times around a rod, protects a circulating superflow. However, if the order parameter lies on a higher dimensional manifold, such as the surface of a sphere (S^2), then the winding has no topological protection and putative supercurrents relax their energy through a continuous reduction of the winding number, leading to dissipation [see Fig. 1]. This topological failure of superfluidity is observed in the A phase of ^{3}He, which exhibits dissipation [2–5]. Similar behavior has also been observed in spinor Bose gases, where the decay of Rabi oscillations between two condensates reveals the unravelling superflow[6].

Here we propose an extension of this concept to superconductors, arguing that when a charge condensate fails to support a topologically stable circulation, the resulting medium forms a novel dielectric. Though our arguments enjoy general application, they are specifically motivated by the Kondo insulator, SmB_6. While transport [7–9] and photoemission [10–14] measurements demonstrate that SmB_6 is an insulator with topological surface states, the observation of bulk quantum oscillations [15, 16], linear specific heat, anomalous thermal and ac optical conductivity[17–20] have raised the fascinating possibility of a "neutral" Fermi surface in the bulk, which paradoxically, exhibits Landau quantization. Landau quantization is normally understood as a semi-classical quantization of cyclotron motion[21]. Rather general arguments tell us that gauge invariance makes the Coulomb and Lorentz forces inseparable: particles interact with the vector potential $\mathbf{A}$ via the gauge invariant kinetic momentum $\pi = (\mathbf{p} - e\mathbf{A})$; the corresponding equation of motion $d\pi/dt = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ necessarily contains both $\mathbf{E}$ and $\mathbf{B}$ as respective temporal and spatial gradients of the underlying vector potential. Thus quasiparticles which develop a Landau quantization in response to

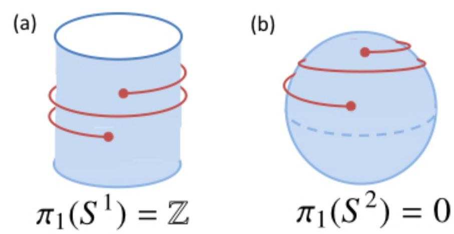

FIG. 1. Illustration of topological stability. The stability of a supercurrent is analogous to topological stability of a string wrapped around a surface. (a) The winding number of a string wrapped around a rod is topologically stable and it can not be unravelled (b) A string wrapped around the equator of a sphere unravels due to a lack of topological stability.

the vector potential should also respond to its time-derivative, the electric field $\mathbf{E} \equiv -\partial\mathbf{A}/\partial t$, forming a metal. In other words, unless the bulk somehow breaks gauge invariance, quantized cyclotron motion is incompatible with insulating behavior. This reasoning motivates the hypothesis that SmB_6 is a failed superconductor, formed from a topological breakdown of an underlying condensate.

General arguments tell us that the condition for the stability of a superfluid is determined by the order parameter manifold G/H formed between the symmetry group G of the Hamiltonian and the invariant subgroup H of the order parameter. The absence of coherent bulk superflow requires that the first homotopy class $\pi_1(G/H) \neq \mathbb{Z}$ is sparse, lacking the infinite set of integers which protect macroscopic winding of the phase. This means that G/H is a higher dimensional non-Abelian manifold, most naturally formed through the condensation of bosons or Cooper pairs with angular momentum. Thus in spinor Bose gases, an atomic spinor condensate lives on an $SU(2)$ manifold with $\pi_1(SU(2)) = 0$: in this case the observed decay of vorticity gives rise to Rabi oscillations[6]. Similarly, in superfluid ^{3}He-A, an $SO(3)$ manifold associated with a dipole-locked triplet paired state[4, 5], for which

$\pi_1(SO(3)) = Z_2$ allows a single vortex, but no macroscopic circulation in the bulk

In the solid state, the conditions for a topological failure of superconductivity are complicated by crystal anisotropy. If the condensate carries orbital angular momentum, it will tend to lock to the lattice, collapsing the manifold back to $U(1)$. On the other hand, if the order parameter has s-wave symmetry, its $U(1)$ manifold allows stable vortices.

There are two ways around this no-go argument. The first, is if there is an additional "isospin" symmetry of the order parameter. For example, the half-filled attractive Hubbard model[22], which forms a "supersolid" ground-state with a perfect spherical (S^2) manifold of degenerate charge density and superconducting states, with pure superconductivity along the equator and a pure density wave at the pole. In this special case, supercurrents can always decay into a density wave.

A second route is suggested by crystal field theory, which allows the restoration of crystalline isotropy for low spin objects, such as a spin 1/2 ferromagnet in a cubic crystal. Were an analogous s-wave spin-triplet condensate to form, isotropy would be assured. Rather general arguments suggest that the way to achieve an s-wave spin triplet, is through the development of odd-frequency pairing. The Gorkov function of a triplet condensate has the form

$$\mathbf{d}(1-2) = \langle \psi_\alpha(1)(i\sigma_2\vec{\sigma})_{\alpha\beta}\psi_\beta(2)\rangle. \tag{1}$$

where $i \equiv (\vec{x}_i, t_i), (i = 1, 2)$ are the space-time co-ordinates of the electrons. Exchange statistics enforce the pair wavefunction $\mathbf{d}(X) = -\mathbf{d}(-X)$ to be odd under particle exchange. Conventionally, $\mathbf{d}(\vec{x}, t) = -\mathbf{d}(-\vec{x}, t)$ is an odd function of *position*, leading to odd-angular momentum pairs. By contrast, an *s-wave* triplet is even in space and must therefore be odd in time, $\mathbf{d}(|x|, t) = -\mathbf{d}(|x|, -t)$, as first proposed by Berezinsky [23–28]. Odd-frequency triplet pairing has been experimentally-established as a proximity effect in hybrid superconductor-ferromagnetic tunnel junctions[27, 28], but for *spontaneous* odd-frequency pairing, we need to identify an equal-time order parameter. Following [26], we can do this by writing the time derivative of the Gorkov function using the Heisenberg equation of motion:

$$\mathbf{\Psi}(1) = \left.\frac{\partial \mathbf{d}(1-2)}{\partial t_1}\right|_{1=2} = \langle [\psi_\alpha(1), H](\sigma_2\vec{\sigma})_{\alpha\beta}\psi_\beta(1)\rangle. \tag{2}$$

The specific form of this composite operator depends on the microscopic physics, but the important point to notice is that it is an equal-time expectation value which defines a complex vector order parameter $\mathbf{\Psi} = \mathbf{\Psi}_1 + i\mathbf{\Psi}_2$.

The case of SmB_6 motivates us to examine a concrete example of this idea. We consider a Kondo lattice of local moments ($\mathbf{S}_j$) interacting with electrons via an exchange interaction of form $H = J\sum_j \mathbf{S}_j \cdot \psi^\dagger(x_j)\vec{\sigma}\psi(x_j)$, for which $[\psi_\alpha(x), H] = J(\mathbf{S}(x)\cdot\vec{\sigma})_{\alpha\gamma}\psi_\gamma(x)$, giving rise to an equal-time, composite pair order parameter between local moments and s-wave pairs [26, 29]

$$\mathbf{\Psi}(x) \propto \langle \psi_\uparrow(x)\psi_\downarrow(x)\mathbf{S}(x)\rangle. \tag{3}$$

In microscopic theory, it is actually more natural to consider an antiferromagnetic version of composite order, formed between the staggered magnetization and the pair density, $\mathbf{\Psi}(x) = (-1)^{i+j+k}\langle \psi_\uparrow(x)\psi_\downarrow(x)\mathbf{S}(x)\rangle$ [25, 29–31]. These details do not however affect the phenomenology.

We now consider a general Ginzburg Landau free energy for an s-wave triplet condensate. Unlike a p-wave triplet, the absence of orbital components to the order parameter considerably simpifies the Ginzburg Landau free energy density[32],

$$f = \frac{1}{2m}|(-i\hbar\nabla - 2e\vec{A})\mathbf{\Psi}|^2 + a|\mathbf{\Psi}|^2 + b|\mathbf{\Psi}^*\cdot\mathbf{\Psi}|^2 + d|\mathbf{\Psi}\cdot\mathbf{\Psi}|^2, \tag{4}$$

where $\vec{A}$ is the vector potential, minimally coupled to the order parameter. Provided $d > 0$, the condensate energy is minimized when $\mathbf{\Psi}\cdot\mathbf{\Psi} = 0$ and the real and imaginary parts of the order parameter are orthogonal $\mathbf{\Psi} = |\Psi|(\hat{\mathbf{l}} + i\hat{\mathbf{m}})$. The triplet odd-frequency order parameter thus defines a triad $(\hat{\mathbf{l}}, \hat{\mathbf{m}}, \hat{\mathbf{n}})$ of orthogonal vectors with principal axis $\hat{\mathbf{n}} = \hat{\mathbf{l}} \times \hat{\mathbf{m}}$.

Eliminating the amplitude degrees of freedom [25, 32, 33], the long-wavelength action has the following form

$$\mathcal{F} = \int d^4x \left[\frac{\rho_\perp}{2}(\partial_\mu\hat{\mathbf{n}})^2 + \frac{\rho_s}{2}(\omega_\mu - qA_\mu)^2 + \frac{F_{\mu\nu}^2}{16\pi}\right]. \tag{5}$$

Here $q = 2e/\hbar$, and we have adopted the relativistic limit of the action to succinctly include both electric and magnetic fields[34], using the Minkowski signature ($x_\mu^2 \equiv \vec{x}^2 - x_0^2$ with $c = 1$) and denoting $A_\mu = (-V, \vec{A})$ as the four-component vector potential. The first two terms describe the condensate action, where $\omega_\mu = \hat{\mathbf{m}}\cdot\partial_\mu\hat{\mathbf{l}}$ is the rate of precession of the order parameter about the $\hat{\mathbf{n}}$ axis. ρ_s is the nominal superfluid stiffness, while $\rho_\perp$ determines the magnetic rigidity. The last term is the field energy, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor. The stiffness coefficients $\rho_\perp, \rho_s$ are temperature dependent and are obtained by integrating out the thermal and quantum fluctuations of the microscopic degrees of freedom. Under the gauge transformation $(\hat{\mathbf{l}} + i\hat{\mathbf{m}}) \to e^{i\phi}(\hat{\mathbf{l}} + i\hat{\mathbf{m}})$ and $qA_\mu \to qA_\mu + \partial_\mu\phi$, the vectors $\hat{\mathbf{l}}$ and $\hat{\mathbf{m}}$ rotate through an angle ϕ about the $\hat{\mathbf{n}}$ axis, so the angular gradient transforms as $\omega_\mu \to \omega_\mu + \partial_\mu\phi$, and thus the currents $J^\mu = q\rho_s(\omega^\mu - qA^\mu)$ and free energy are gauge invariant. The equivalence of electron gauge transformations and spin-rotation means that gauge transformations are entirely contained within the $SO(3)$ manifold of the order parameter.

To analyze how the superflow is destablized, we examine the screening of electromagnetic fields. From Ampères equation $4\pi J^\mu = \partial_\nu F^{\mu\nu}$, we observe if $\partial_\nu F^{\mu\nu} = 0$, corresponding to uniform internal fields, then the supercurrent vanishes $J^\mu = q\rho_s(\omega^\mu - qA^\mu) = 0$. In a superconductor, this condition is only be achieved by the complete exclusion of fields, but here the texture of the composite order parameter is able to continually adjust with the vector potential so that $\omega^\mu = qA^\mu$, enabling the current to vanish in the presence of internal fields. To examine this further, we take the curl of Ampères equation,

$$(1 - \lambda_L^2\partial^2)F^{\mu\nu} = q^{-1}\Omega^{\mu\nu}, \tag{6}$$

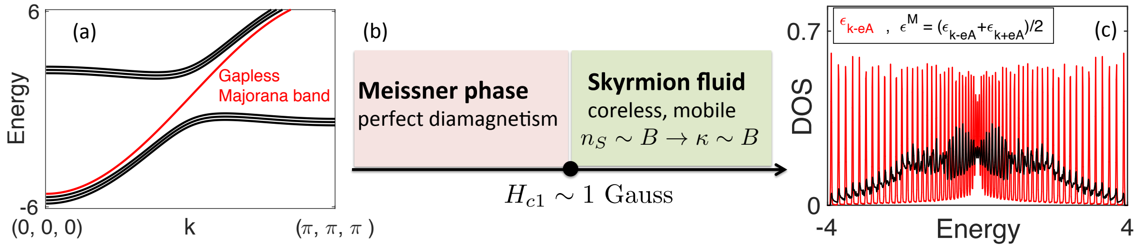

FIG. 2. (a) Hybridization of 3 localized Majorana fermions per spin with 4 Majorana fermions of the conduction band leads to one gapless Majorana Fermi surface. (b) Magnetic field phase diagram of a Skyrme insulator. (c) Landau quantization of the projected Majorana Fermi surface.

where $\lambda_L = (4\pi q^2 \rho_s)^{-1/2}$ is the London penetration depth. This modified London equation contains the additional term $\Omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$, which is the curl of the gradient of the order parameter. In a conventional superconductor, $\omega^\mu = \partial^\mu \phi$ is the gradient of the superconducting phase so $\Omega^{\mu\nu} = 0$ vanishes causing fields to be expelled. However, the quantity $\Omega^{\mu\nu}$ is finite, and can be written in the form $\Omega^{\mu\nu} = \hat{\mathbf{n}} \cdot (\partial^\nu \hat{\mathbf{n}} \times \partial^\mu \hat{\mathbf{n}})$, which is the Mermin-Ho relation[33] for the skyrmion density of the $\hat{\mathbf{n}}$ field. From (6), we see that on scales long compared with the penetration depth, where gradients of the field can be neglected, the average skyrmion density locks to the average external field, $\overline{\Omega^{\mu\nu}} = q\overline{F^{\mu\nu}}$, where the lines denote a coarse-grained average. This relation expresses the screening of supercurrents by charged skyrmions; it also holds in non-relativistic versions of this theory[34]. Moreover, phase rotations around the $\hat{\mathbf{n}}$ axis are now absorbed into the electromagnetic field (Anderson Higg's effect), leaving behind a residual order parameter manifold with $SO(3)/U(1) \equiv S^2$ symmetry. While the homotopy analysis yields no stable vortices $\pi_1(S^2) = 0$, it does allow for the topologically stable skyrmion solutions $\pi_2(S^2) = \mathbb{Z}$ that screen the superflow and allow penetration of electric and magnetic fields. We shall actually consider lines of skyrmion , formed by stacking two dimensional skyrmion configurations, similar to vortex lines in three dimensional superconductors. We call the corresponding dielectric a "Skyrme insulator".

Written in non-relativistic language, the equations relating the skyrmion density to the penetrating fields are

$$\begin{aligned} \frac{1}{2\pi}\overline{\hat{\mathbf{n}} \cdot (\partial_i \hat{\mathbf{n}} \times \partial_j \hat{\mathbf{n}})} &= -\epsilon_{ijk}\left(\frac{B_k}{\Phi_0}\right) \\ \frac{1}{2\pi}\overline{\hat{\mathbf{n}} \cdot (\partial_i \hat{\mathbf{n}} \times \partial_t \hat{\mathbf{n}})} &= \frac{2e}{h}E_i \end{aligned} \tag{7}$$

where $\Phi_0 = 2\pi/q = h/2e$ is the flux quantum, and the overline denotes a coarse-grained average over space or time. The first term in (7) relates the areal density of skyrmions to the magnetic field, allowing a magnetic field to penetrate with a density of one flux quantum per half-skyrmion. The second term in (7) describes the unravelling of supercurrents due to phase slippage[2] created by domain wall or instanton configurations of the order parameter. The integral of this term over a time t and length L of the wire, counts the number of domain-walls $N = -\frac{2e}{h}(V_2 - V_1)t$ crossing the wire in time t, in the presence of a finite voltage drop $V_2 - V_1$. This voltage generation mechanism is similar to the development of insulating behavior in disordered two-dimensional superconductors[35]. We conclude that the failure of the superconductivity does not reinstate a metal, which would screen out electric fields, but transforms it into a dielectric into which both electric and magnetic fields freely penetrate.

Unlike vortices, skyrmions are coreless, with short-range interactions, so we expect them to form an unpinned liquid, analogous to the vortex liquid of type II superconductors, which restores the broken $U(1)$ symmetry on macroscopic scales. How then would we distinguish a Skyrme insulator from a more conventional dielectric? Since the density of (half) skyrmions, $n_s = B/\Phi_0$ is proportional to a magnetic field, one signature of a skyrmion liquid is a thermal conductivity $\kappa \propto H$ proportional to the applied field H. In a Drude model, the drift velocity $v_d = \mu(-\nabla T)$ is proportional to the temperature gradient and the skyrmion mobility μ. If $\mathcal{Q}$ is the heat content per unit length, then $\kappa = \mathcal{Q}\mu n_S$, so that

$$\kappa = \left(\frac{\mu\mathcal{Q}}{\Phi_0}\right) H. \tag{8}$$

is proportional to the applied field.

A further consequence is the development of a low field Meissner phase. In a fixed external magnetic field $\mathbf{H}$, we consider the Gibb's free energy $\mathcal{G} = \mathcal{F} - \int d^3x \mathbf{H} \cdot \mathbf{B}(x)/(4\pi)$. Taking the field $B_z = n_S(x)\Phi_0$ to lie in the z-direction, where $n_S = \frac{1}{2\pi}\Omega^{12}$ is the areal skyrmion density,

$$\mathcal{G} = \int d^3x \left[\frac{\rho_\perp}{2}(\partial_\mu \mathbf{n})^2 + \frac{(H - \Phi_0 n_S(x))^2}{8\pi} - \frac{H^2}{8\pi}\right], \tag{9}$$

This corresponds to an $O(3)$ sigma model in which the skyrmions have a finite chemical potential $\mu_S = \Phi_0 H/4\pi$, per unit length. Suppose the corresponding energy of a skyrmion is ϵ_S/a per unit length, where a is the lattice spacing, then providing $H < H_c = 4\pi\epsilon_S/\Phi_0 a$, the skyrmion energy will exceed the chemical potential, and they will be excluded from the fluid. In SI notation,

$$\mu_0 H_c = \frac{4}{137}\left(\frac{V_S}{ac}\right), \tag{10}$$

where we have replaced $\frac{e^2}{\hbar c} = 1/137$ and $\epsilon_S = eV_S$. Below this field, skyrmions and field lines will be expelled, so the material will exhibit a Meissner effect (Fig. 2(b)).

We now discuss the possible microscopic origin of this order, and its possible application to SmB_6. Various anomalous aspects of insulating SmB_6 can be speculatively associated with the properties of a Skyrme insulator. The recent observation of an unusual thermal conductivity in insulating SmB_6, that is linear in field, $\kappa \propto H$[19] is most naturally interpreted as a kind of flux liquid expected in such a phase, a hypothesis that could be checked by confirming if the anomalous thermal conductivity lies perpendicular to the field direction.

A second test of this hypothesis, is the magnetic susceptibility. In a heavy fermion compound, the order parameter stiffness ρ is set by the Kondo temperature T_K, $\rho \sim k_B T_K/a$[25], where a is the lattice spacing, so the the energy of a skyrmion is approximately $k_B T_K$ per unit lattice spacing a and $eV_K \sim k_B T_K$. For SmB_6 we estimate $V_K = 1meV$, and with $a = 10^{-9}m$ we obtain $\mu_0 H_c \sim 10^{-4}T$ or 1 Gauss, comparable with the earth's magnetic field. In a magnetically screened ($\mu-$ metal) environment we expect SmB_6 to become fully diamagnetic with magnetic susceptibililty $\chi = -1/4\pi$.

A microscopic model for the development of composite order in a Kondo lattice was studied by Coleman, Miranda and Tsvelik[25, 36] (CMT) and recently revisited by Baskaran[37]. This model allows us to pursue the microscopic consequences of the failed-superconductivity hypothesis. In a conventional Kondo lattice the local moments fractionalize into charged Dirac fermions; the CMT model considers an alternative fractionalization into *Majorana fermions*. In the corresponding mean-field theory, spin $1/2$ local moments $\mathbf{S}$ are represented as a bilinear $\mathbf{S} = -\frac{i}{2}\hat{\boldsymbol{\eta}} \times \hat{\boldsymbol{\eta}}$, where $\hat{\boldsymbol{\eta}} = (\hat{\eta}_x, \hat{\eta}_y, \hat{\eta}_z)$ is a triplet of Majorana fermions. In this representation, the Kondo interaction factorizes as follows:

$$\begin{aligned} H_K[i] &= J_K(\hat{\psi}^\dagger_{i\alpha}\boldsymbol{\sigma}_{\alpha\beta}\hat{\psi}_{i\beta}) \cdot \mathbf{S}_i \\ &\rightarrow \left[\hat{\psi}^\dagger_{i\alpha}(\boldsymbol{\sigma}_{\alpha\beta} \cdot \hat{\boldsymbol{\eta}}_i)\mathcal{V}_{i\beta} + \mathrm{H.c}\right] + \mathcal{V}^\dagger_i \mathcal{V}_i/J_K, \end{aligned} \quad (11)$$

where J_K is the Kondo interaction strength, $c^\dagger_{i\gamma}$ creates a conduction electron and $[\mathcal{V}_i]_\beta = -\frac{J_K}{2}\langle(\boldsymbol{\sigma}_{\beta\gamma} \cdot \boldsymbol{\eta}_i)c_{i\gamma}\rangle$ is a two-component spinor. $\mathcal{V}_j$ determines the composite order via the equation $\vec{\Psi}(\mathbf{x}) = \mathcal{V}^T i\sigma_2\vec{\sigma}\mathcal{V}$. We have extended the model to include spin-orbit coupling by incorporating a 'p-wave' form factor into the definition of the conduction Wannier states c_i, derived from the angular momentum difference $|\Delta l| = 1$ between the heavy f and light d electrons[33, 38]. Our mean-field calculations confirm that even in the presence of the spin-orbit coupling, the ground-state energy is independent of the orientation of the composite order parameter $\vec{\Psi}$, so the system remains isotropic[33].

In the CMT model, the conduction electrons, represented by four degenerate Majorana bands, hybridize with the three neutral Majorana fermions, gapping all but one of them which is left behind to form a gapless Majorana Fermi sea [Fig 2 (a)]. This unique feature provides an appealing explanation of the robust linear specific heat $C_v = \gamma T$ observed in this material. The neutrality of the Majorana Fermi sea strictly eliminates the *DC* conductivity, but the current and spin matrix elements are actually proportional to energy, which will lead to a quasiparticle AC conductivity of the form

$$\mathrm{Re}[\sigma(\omega)] = \frac{\sigma_0}{1+\omega^2\tau^2}\omega^2, \quad (12)$$

where τ is the relaxation rate. The analogous matrix element effect also suppresses the Koringa spin relaxation rate, giving rise in the clean limit to a T^3 NMR relaxation rate[36]. When we include the spin-orbit coupling, we find that an additional topological Majorana surface state develops, reminiscent of the surface states of superluid He-3. This interesting state is protected by the crystal mirror symmetry and decouples from the gapless bulk band[33]. Thus the insulating state retains some of the surface conductivity of a topological Kondo insulator[7, 39].

Perhaps the most puzzling aspect of SmB_6 is the reported observation of 3D bulk quantum oscillations. An approximate treatment of the effect of a magnetic field on the Majorana Fermi surface can be made by initially ignoring the skyrmion fluid background. The dispersion of the Majorana band in a field can then be calculated by projecting the Hamiltonian into the low-lying Majorana band.

$$\epsilon^{\mathrm{M}}_{\mathbf{k},\mathbf{A}} = \langle\phi^{\mathrm{M}}_{\mathbf{k}}|H(\mathbf{k},\mathbf{A})|\phi^{\mathrm{M}}_{\mathbf{k}}\rangle = \frac{1}{2}(\epsilon^{\mathrm{e}}_{\mathbf{k}-e\mathbf{A}} + \epsilon^{\mathrm{h}}_{\mathbf{k}+e\mathbf{A}}), \quad (13)$$

where $\epsilon^{\mathrm{e}}_{\mathbf{k-eA}}$ and $\epsilon^{\mathrm{h}}_{\mathbf{k+eA}}$ are the dispersion for electrons and holes. Although the scattering off the triplet condensate mixes the electron and hole components of the field, giving rise to a neutral quasiparticles for which current operator $J_\alpha = \partial\epsilon^{\mathrm{M}}_{\mathbf{k},\mathbf{A}}/\partial A_\alpha|_{\mathbf{A=0}} = 0$ vanishes, this cancellation does not extend to the second derivative of the energy $\partial^2\epsilon^{\mathrm{M}}_{\mathbf{k},\mathbf{A}}/\partial {A_\alpha}^2|_{\mathbf{A=0}} \neq 0$ which is responsible for the diamagnetic response. This is a consequence of the broken gauge-invariant environment provided by the Skyrme insulator. In Fig. 2(c), we show the density of states of the Majorana band in a magnetic field, demonstrating a discrete Landau quantization with broadened Landau levels. Since quantum oscillations originate from the discretization of the density of states into Landau levels, we anticipate that a Majorana Fermi surface does give rise to quantum oscillations. Moreover since the Majorana Fermi surface originates predominantly from the conduction electron band, it has a small effective mass, in accordance with quantum oscillation experiments[15, 16].

We note that triplet odd frequency pairing is expected to be highly prone to disorder. Weakly disordered samples may revert to a topological Kondo insulating phase in a majority of the sample, accounting for the marked sample dependence, and the discrepancies between samples grown by different crystal growth techniques. Nevertheless, we expect that small patches of failed superconductivity will still lead to enhanced diamagnetism in a screened ($\mu-$ metal) environment.

Our results also set the stage for a broader consideration of failed superconductivity in other strongly correlated materials. There are several known Kondo insulators with marked linear specific heat coefficients, including $Ce_3Bi_4Pt_3$[40], $CeRu_4Sn_6$[41] and $CeOs_4As_{12}$[42] which might fall into this class. We end by noting that Skyrme insulators may also be relevant in an astrophysical context such as color superconductivity in white dwarf or neutron stars[43, 44].

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