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## Ultrafast Excited-State Dynamics of $V_3O_5$ as a Signature of Photoinduced Insulator-Metal Phase Transition

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The ultrafast elastic light scattering technique is applied to reveal strong nonlinearity of  $V_3O_5$  associated with photoinduced insulator-metal phase transition. Observation of time-domain relaxation dynamics suggests several stages of structural transition. We discuss the nonequilibrium processes in  $V_3O_5$  in terms of photoinduced melting of polaronic Wigner crystal, coalescence of V-O octahedra and photogeneration of acoustical phonons in the low-T and high-T phases of  $V_3O_5$ . A molecular dynamics computation supports experimentally observed stages of  $V_3O_5$  relaxation dynamics.

Vanadium oxides show exotic optical, electronic, magnetic and structural properties, including metal-insulator phase transitions due to specific behavior of d-electrons [1]. Among these oxides, V<sub>3</sub>O<sub>5</sub> undergoes an insulatorto-metal transition (IMT) at highest temperature  $T_c \sim$ 430 K [2]. V<sub>3</sub>O<sub>5</sub> is the first member of the Magnéli series  $V_nO_{2n-1}$ ,  $n=3, 4, \dots 9$  [3], and is the only other known vanadium oxide, besides VO<sub>2</sub> [4], which shows a phase transition above room temperature. Its initial monoclinic symmetry does not change through the transition point, except for a slight variation of lattice parameters [5, 6]. With  $T_c$  above room temperature,  $V_3O_5$  is a very promising candidate for new optoelectronic applications. It has been shown that IMT can be triggered by various degrees of freedom such as temperature [2, 7], electric field [8] and pressure [9, 10]. However, the evidence of IMT in V<sub>3</sub>O<sub>5</sub> induced by light has not been demonstrated.

In this Letter, we report on the observation of excited state dynamics of in  $V_3O_5$ . Applying ultrafast light scattering technique we show that the complete formation of new phase occurs within 1 ps. The phase transition dynamics depends on the level of optical excitation and shows several stages, interpreted as order-disorder structural transition triggered by photoinduced melting of polaronic Wigner crystal. A photoacoustic strain can produce coherent oscillations of the light scattering signal. The ultrafast structural dynamics was modeled by molecular dynamics computation.

High quality 120 nm thick  $V_3O_5$  film was grown on  $SiO_2$  substrate by reactive DC magnetron sputtering from vanadium target with 99.95% purity. The sample was characterized by X-ray diffraction (XRD). All sharp XRD peaks [Fig. 1(a)] are assigned to the low-T monoclinic phase of  $V_3O_5$ , according to Ref.[11].

Ultrafast pump-probe scattering measurements were performed with an ultrafast scatterometer in reflection geometry described elsewhere [12]. Amplified 130 fs pulses with central wavelength  $\lambda{=}800$  nm are split into a pump beam and frequency-doubled probe beam ( $\lambda{=}400$  nm). Scattered light was collected within the front hemisphere by an elliptical mirror and focused on an silicon photodetector. Temperature T of the sample

was controlled by Peltier module. The penetration depth of insulating  $V_3O_5$  was measured as 30 nm for  $\lambda$ =800 nm and 25 nm for  $\lambda$ =400 nm. Therefore a relaxation dynamics was monitored for a uniformly excited sample within the penetration depth of the probe.

Figure 1(b) shows the temperature dependence of the sample resistivity  $\rho$ , static scattering signal, and transmittance. From 415 K to 435 K these dependencies show a sharp change due to IMT, defining a transition temperature  $T_c \sim 425$  K, which agrees with the known value for V<sub>3</sub>O<sub>5</sub> [13]. Moreover, a noticeable dip in transmittance signal was observed at  $\sim 360$  K. This behavior shows that the monoclinic structure starts preparing for the IMT at  $\sim 65$  K below  $T_c$ , which is consistent with the literature for this compound [14, 15].

Figure 1(c) shows the transient light scattering  $\Delta I_s(t)/I_s(0)$  measured with a pump fluence  $F=14~\rm mJ/cm^2$  at different sample temperatures, ranging from 293 K to 470 K. For every temperature the signal is composed of two distinct processes: a noticeable decrease of the signal within  $\sim$ 2 ps, followed by its relatively small change during several picoseconds afterwards. Moreover, pronounced oscillations appear in the metallic phase of V<sub>3</sub>O<sub>5</sub> at  $T=470~\rm K$ . Additional details are provided in the Supplemental Material[16].

The rapid change of scattering signal magnitude  $|\Delta I_s(1ps)/I_s(0)|$  measured at 1 ps delay at different temperatures [Fig. 1(d)] shows a noticeable decrease at 360 K, followed by a relatively small rise until ~420 K. This behavior is similar to that of the transmittance [inset in Fig. 1(b)], showing that the Nonlinear Optical (NLO) properties of V<sub>3</sub>O<sub>5</sub> have a specific feature at 360 K. Within the interval from 420 K to 450 K the signal shows a sharp drop due to thermally induced IMT. However, the metallic V<sub>3</sub>O<sub>5</sub> also shows well-resolved transient NLO response with nearly constant signal above 450 K.

The evolution of transient scattering on relatively long 2 ns timescale [Fig. 2(a)] and quantitative analysis of the heat deposited to the film allows estimating a rise of the film temperature upon photoexcitation. Here we use the heat capacity and the latent heat [200 cal/mole (3594 J/kg)] of  $V_3O_5$  obtained in Refs. 17 and 18, respec-

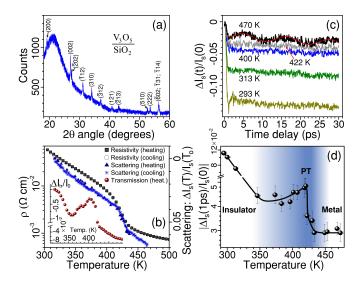


FIG. 1. (a) The XRD pattern of V<sub>3</sub>O<sub>5</sub>/SiO<sub>2</sub> film. (b) Resistivity  $\rho$  and relative change  $\Delta I_s(T)/I_s(T_0)$  of static scattering intensity  $I_s(T)$  as a function of sample temperature,  $\Delta I_s(T) = I_s(T) - I_s(T_0)$ . Inset shows the relative change of transmittance  $\Delta I_{tr}(T)/I_0$ , where  $\Delta I_{tr}(T) = I_{tr}(T) - I_0$ ,  $I_{tr}(T)$  is transmitted and  $I_0$  is incident intensity of light;  $\lambda$ =800 nm. (c) Differential scattering  $\Delta I_s(t)/I_s(0)$  versus time t at different temperatures of the sample; F=14 mJ/cm².  $\Delta I_s(t) = I_s(t) - I_s(0)$ . Dashed line is the fit by Eq.(1) at T=470 K. (d) Photoinduced change of scattering signal at 1 ps delay versus temperature.

tively. Above  $\sim 270 \text{ K}$  the heat capacity increases linearly from 688 J/(K kg) to 760 J/(K kg), as the temperature changes from 293 K to 350 K. For 120 nm  $V_3O_5$  film the measurements of the reflection coefficient (R=11%) and transmission (T=2.4%) yield the absorption coefficient of 0.866. For reliable estimation of  $V_3O_5$  temperature in insulating phase we assume that the only photon energy of 1.55 eV- $E_g$  is transferred to the lattice. Since the band gap of  $V_3O_5$  is  $E_q=0.6$  eV [7], the estimation shows that at least 60% of absorbed energy is transferred to the lattice after electron-phonon relaxation. Taking into account a density of the film  $\rho=4.69$  g/cm<sup>3</sup> [19], simple calorimetric calculations show that the fluence of 12 mJ/cm<sup>2</sup> is enough to heat entire film from room temperature  $T_0 = 293 K$  to  $T_c = 425 K$  and to produce complete thermal phase transition. At  $F=14 \text{ mJ/cm}^2$ , the film temperature exceeds the  $T_c$  point on  $\sim 30$  K, and at  $F=24 \text{ mJ/cm}^2 \text{ on } \sim 125 \text{ K}.$ 

The 2 ns timescale is sufficiently long to convert the energy of photoexcited electrons into the heat via electron-phonon and phonon-phonon scattering. The photoin-duced dynamics in Figs. 2(a)–2(c) at F=14 , 16, 20 and 24 mJ/cm² is accompanied by significant increase of the temperature well above  $T_c$ . Moreover, the relative change of the scattering signal for these excitations is the same. As it shown in Fig. 2(a) for F=14 and 24 mJ/cm², there is no any noticeable relaxation which

can be observed within 2 ns timescale (except of acoustic oscillations at  $F{=}24~\rm mJ/cm^2$ ). Therefore it can be argued that the V<sub>3</sub>O<sub>5</sub> film at  $F{\,\geqslant}14~\rm mJ/cm^2$  is already in metallic state within observed 2 ns timescale, at least due to thermally induced phase transition. The only transient dynamics which can be observed on shorter timescale is the relaxation within  ${\sim}1~\rm ps$  [see the signal evolution for  $F{=}16$  and 20 mJ/cm<sup>2</sup> in Figs. 2(a), 2(b)]. From 1 ps and up to maximally observed 2 ns timescale, the signal is constant and corresponds to the same level of  ${\Delta}I_s(t)/I_s(0){=}0.1$  for any excitation fluence above 16 mJ/cm<sup>2</sup>. This yields the conclusion that the optical excitation with  $F{\,\geqslant}16~\rm mJ/cm^2$  produces complete I-M PT in V<sub>3</sub>O<sub>5</sub> film on the timescale as short as  ${\sim}1~\rm ps$ .

New features appear as the pump fluence reaches  $24 \text{ mJ/cm}^2$ . Figure 2(a) shows oscillations of the signal within 500 ps with a period of 120 ps. Additional measurements of this NLO dynamics at  $F=28 \text{ mJ/cm}^2$  show that the transient scattering response can be rationalized as a three-step process, labeled as T1, T2 and T3 in Fig. 2(e). Initial process T1 is associated with the generation of photoexcited metallic state within 1 ps. This is followed by excitation of coherent acoustic phonons (process T2), which decays after  $\sim$ 50 ps into mode T3. The acoustic mode T3 corresponds to the oscillations shown in the inset of Fig. 2(a).

The pronounced oscillations within  $\sim 50$  ps [process T2 in Fig. 2(e)] compare well with oscillations which appear in the metallic phase of V<sub>3</sub>O<sub>5</sub> at T=470 K [Fig. 1(c)] and manifest the excited metallic state. The transient scattering signal in Fig. 2(e) within  $\sim 50$  ps can be described by equation

$$\frac{\Delta I_s(t)}{I_s(0)} = A_0 + A_1 \sin[2\pi f(t - t_c)] + A_2 e^{\frac{-t}{\tau_R}}$$
 (1)

where  $A_0$ ,  $A_1$  and  $A_2$  are fitting amplitude constants, f is the vibrational frequency,  $t_c$  is the time of complete electron-phonon relaxation during the IMT and  $\tau_R$  is the relaxation time of process T2. The exponential term accounts for the exponential decay background. Acoustic mode T2 after  $\sim 50$  ps decays into mode T3 with the signal  $\Delta I_s(t)/I_0(0) = A_0 + A_1 \sin(2\pi ft)$ . Fitting of  $\Delta I_s(t)/I_s(0)$  to two oscillatory components gives a period of 15 ps (f=67 GHz) for the mode T2, and a period of 120 ps (f=8.3 GHz) for the mode T3.

According to Thomsen et al. [20], the absorption of the pump pulse with  $\lambda{=}800$  nm excites a longitudinal acoustic wave within the penetration depth of the film  $\zeta{=}30$  nm. This wave reflects several times from the boundary of photoexcited metallic phase of  $V_3O_5$  within  $\zeta$  thickness, resulting in the oscillations of the  $\Delta I_s(t)/I_s(0)$  signal. Speed of sound  $v_s$  and period of oscillations  $T_S$  are related as  $v_s = 2\zeta/T_S$  [20]. Using the period of oscillations of  $T_S{=}15$  ps, the speed of sound comes out to be  $v_s{=}4{\times}10^3$  m/s. After  ${\sim}50$  ps the acoustic

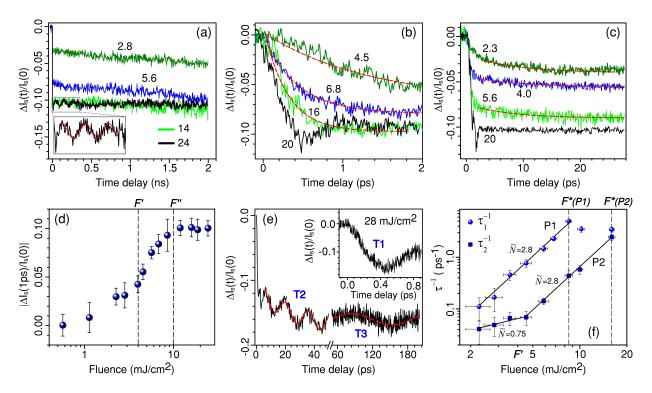


FIG. 2. Transient differential scattering signal  $\Delta I_s(t)/I_s(0)$  within (a) 2 ns (b) 2 ps and (c) 27 ps time scale. Numbers specify the laser pump fluence in mJ/cm<sup>2</sup>. Dashed lines are the fit by Eq.(2) for the process P1 in (b) and P2 in (c). (d) The magnitude  $|\Delta I_s(1ps)/I_s(0)|$  of scattering signal measured at 1 ps delay. (e) Differential signal  $\Delta I_s(t)/I_s(0)$  at F=28 mJ/cm<sup>2</sup>. Dashed lines are fits by Eq.(1). (f) Characteristic relaxation rates for the processes P1 and P2. Solid lines are fits by Eq.(3).

pulse penetrates further into the film volume and reflects already from the film boundary at the substrate, producing lower frequency oscillations with  $T_S$ =120 ps. For this case, the relation between  $v_s$  and period  $T_S$  is  $v_s = 4d/T_S$  [21–23], where d is the film thickness. The measured period of oscillations of  $T_S$ =120 ps and d=120 nm also give the value of sound speed  $v_s$ =4×10<sup>3</sup> m/s.

The light-induced IMT monitored within 2 ps [Fig. 2(b)] and 27 ps time scales [Fig. 2(c)] at  $F < 20 \text{ mJ/cm}^2$  shows two distinctive relaxation components which can be approximated by the function

$$\frac{\Delta I_s(t)}{I_s(0)} = A_0 + A_1 \exp(-t/\tau_1) + A_2 \exp(-t/\tau_2)$$
 (2)

where  $\tau_1$  and  $\tau_2$  are characteristic relaxation times. The relaxation rates  $\tau_1^{-1}$  and  $\tau_2^{-1}$  obtained are plotted in Fig. 2(f) and assigned to two relaxation processes designated as P1 and P2. In a log – log graph the relaxation rates show linear dependencies on pump fluence. The P2 process shows two components separated by fluence  $F{=}4.5~\mathrm{mJ/cm^2}$ . The relaxation rate of each process can be fitted as

$$\tau^{-1} = \gamma F^{\widetilde{N}} \tag{3}$$

where  $\gamma$  and  $\widetilde{N}$  are fitting constants. Although the phase transition of V<sub>3</sub>O<sub>5</sub> shows features of a first-order transition [7, 14], the lattice transformation occurs within a

relatively broad temperature range and almost without a hysteresis, altering electrical and optical properties of the material below and above  $T_c$  [Fig. 1(b)]. As shown in Ref.[13], such critical behavior of  $V_3O_5$  is likely originated from an order-disorder phase transition, which is second-order. In the case of light-induced transition, increasing pump fluence reduces the activation energy  $E_a$  of each process involved in the IMT. The relaxation rate is proportional to the probability of the process and can be written as

$$\tau^{-1} = \tau_0^{-1} \exp(-E_a/k_B T) \tag{4}$$

where  $\tau_0$  is the shortest relaxation time, and  $k_B$  is Boltzmann constant. Equations (2) and (3) yield the dependence of the energy  $E_a$  on pump fluence

$$E_a = -\widetilde{N}k_B T \ln(F/F^*) \tag{5}$$

where  $F^*$  is the pump fluence at which the shortest relaxation time  $\tau_0$  is detected.  $F^*(P1)=8.5 \text{ mJ/cm}^2$  and  $F^*(P2)=16 \text{ mJ/cm}^2$  for the processes P1 and P2. The fitting constant  $\tilde{N}$  defines the slope of  $\tau_1^{-1}$  and  $\tau_2^{-1}$  in Fig. 2(f) and can be interpreted as the constant which characterizes the number of atoms and number of degrees of freedom involved in the IMT process. Thus,  $\tilde{N}=0.75$  for the first component of P2, and  $\tilde{N}=2.8$  for P1 and for the second component of P2. The fact that slopes

of  $\tau_1^{-1}(F)$  and  $\tau_2^{-1}(F)$  for  $F > 4.5 \text{ mJ/cm}^2$  are equal indicates that P1 and P2 represent similar relaxation processes in V<sub>3</sub>O<sub>5</sub>. On the other hand, P1 and P2 could also represent different stages of a single pathway of IMT, and this is a subject for further investigations. As the pump fluence further increases to 16 mJ/cm<sup>2</sup>, processes P1 and P2 merge, and complete IMT occurs in less than 1 ps.

The processes P1 and P2 both compare with the magnitude of scattering signal  $|\Delta I_s(1ps)/I_s(0)|$  measured at 1 ps delay for a series of pump fluences F [Fig. 2(d)]. The monotonic rise of  $|\Delta I_s(1ps)/I_s(0)|$  changes at F'=4 mJ/cm<sup>2</sup> and at F''=10 mJ/cm<sup>2</sup>. It is interesting to note that the fluence F'' is fairly close to the upper limit  $F^*(P1)$  for process P1, and the fluence F' is close to the fluence where the slope  $\widetilde{N}$  of  $\tau_2^{-1}(F)$  changes  $[F=4.5 \text{ mJ/cm}^2 \text{ in Fig. 2(f)}]$ . The different slope of  $\tau_2^{-1}(F)$  below and above  $F=4.5 \text{ mJ/cm}^2$  indicates different relaxation dynamics in the process P2. Overall, the NLO response of  $V_3O_5$  undergoes sharp qualitative change starting from F'=4 mJ/cm<sup>2</sup>. Therefore, this fluence was assigned as the threshold for the IMT.

Although strong inter-electron correlations play a crucial role in stabilizing the insulating phase, the IMT of  $V_3O_5$  is not considered as a pure Mott-Hubbard type transition [7]. In the insulating phase, the  $V_3O_5$  unit cell consists of four independent V-O octahedrons, referenced as V(11), V(12), V(21), and V(22) [19]. The V(11) octahedron hosts a  $V^{4+}$  ion. V(12), V(21), and V(22) all host  $V^{3+}$  ions. The holes are localized within the V(11) octahedron [19] and can be considered as small polarons [7, 10]. As a result, V(11) is the smallest and the most distorted octahedron. In this system the IMT can be induced by altering the spatial segregation of the charges [7].

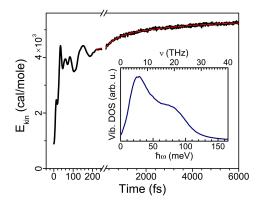


FIG. 3. Kinetic energy relaxation for  $V_3O_5$  lattice. A double exponential fit (dashed line) yields two relaxation times:  $\tau_1{=}405$  fs and  $\tau_2{=}3.1{\times}10^3$  fs. Inset: vibrational density of states of all  $V_3O_5$  atoms.

The low-T insulating phase of  $V_3O_5$  originates from strong electron correlations. It is very likely that the correlations result in Wigner crystallization of small polarons [7, 10]. This process significantly decreases con-

ductivity and results in ordering of ions of different valence in the monoclinic lattice [space group P2/c, Supplementary Fig. 2] with unit cell parameters: a=9.859 Å, b=5.0416 Å, c=6.991 Å and angle  $\beta$ =109.478<sup>0</sup> [5].

The concept of polaronic Wigner crystal is very important for understanding of photoinduced IMT in  $V_3O_5$ . In this scenario, photogeneration of a dense electron-hole plasma produces screening of electron correlations to the level when Coulomb interaction cannot provide segregation and localization of charges. As a result, the polaronic crystal melts into noncorrelated polaronic states. Additional charge transfer may occur between  $V^{3+}$  and  $V^{4+}$  ions. This process significantly increases the mobility of polarons, resulting in lattice instability followed by structural transition. It is very likely that the photoinduced melting of polaronic crystal is sufficiently fast and occurs on a femtosecond timescale as a part of the process P1 [Figs. 2(b), 2(f)].

According to the Refs.[19, 24], the IMT phase transition of  $\rm V_3O_5$  occurs as a coalescence of V(11) and V(12) octahedra into V(1), hosting a mixture of V<sup>3+</sup> and V<sup>4+</sup> ions, and also as a coalescence of V(21) and V(22) octrahedra into V(2), hosting only V<sup>3+</sup> ions. It is likely that for the IMT triggered by femtosecond light pulses, these two coalescence processes take place at different rates.

In order to gain deeper insight into the ultrafast structural dynamics of  $V_3O_5$ , we modeled lattice relaxation by the semi-classical computational method of molecular dynamics (MD) using QuantumWise software package [25]. Figure 3 shows the adiabatic relaxation of kinetic energy for a  $V_3O_5$  cluster of 16,384 atoms at T=298 K. The computation of MD applied ReaxFF reactive force field simulation method, which does not include correlation effects, but retains nearly the accuracy of quantum mechanical calculations [26]. Therefore, MD of V<sub>3</sub>O<sub>5</sub> can be considered as a fairly close approximation of lattice relaxation of real V<sub>3</sub>O<sub>5</sub> lattice obtained from XRD experiment [5] to some other structure when correlation effects are suppressed by photoexcitation. While the metallic phase of V<sub>3</sub>O<sub>5</sub> is still a correlated metal, here we assume that these correlations are much weaker as compared to the case of insulating  $V_3O_5$ .

The relaxation dynamics of kinetic energy  $E_{kin}$  in Fig. 3 reveals significant instability of the V<sub>3</sub>O<sub>5</sub> lattice. Thus,  $E_{kin}$  increases by 3.5 kcal/mole in less than 100 fs, followed by further nearly exponential growth with two distinct characteristic relaxation times:  $\tau_1$ =405 fs and  $\tau_2$ =3.1×10<sup>3</sup> fs. These values compare well with experimentally observed relaxation times of initial drop ( $\tau_1$ =405 fs, process P1) and posterior picosecond relaxation ( $\tau_2$ =4.4×10<sup>3</sup> fs, process P2) of the scattering signal at F=6.8 mJ/cm<sup>2</sup> [see Fig. 2(f)]. Thus, computed relaxation dynamics strongly supports the presence of experimentally observed processes P1 and P2 [Figs. 2(b), 2(c), 2(f)]. These two processes could be associated with coalescence of vanadium octahedra. The initial rise of  $E_{kin}$ 

within 100 fs can be associated with melting of polaronic crystal in the absence of electron correlations.

The inset in Fig. 3 shows the vibrational density of states (DOS) calculated for all atoms of the  $\rm V_3O_5$  cluster. MD computation shows a broad resonance with maximum at  $\sim\!6.5$  THz. However, in the present study lattice oscillations at THz frequencies were not resolved.

In summary, the ultrafast scattering data reveal noticeable transient nonlinearity of  $V_3O_5$  associated with IMT. The phase transition dynamics can be interpreted in terms of an order-disorder structural transition triggered by photoinduced screening of electron correlations followed by melting of polaronic Wigner crystal and coalescence of V-O octahedra. Noticeable photoacoustic response of  $V_3O_5$  was revealed at temperatures above and also below  $T_c$ . The molecular dynamics computation supports the model for the experimentally observed relaxation dynamics.

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