Continuous Symmetry Breaking in 1D Long-Range Interacting Quantum Systems

Mohammad F. Maghrebi, Zhe-Xuan Gong, and Alexey V. Gorshkov

Phys. Rev. Lett. **119**, 023001 — Published 11 July 2017

DOI: 10.1103/PhysRevLett.119.023001

Continuous symmetry breaking in 1D long-range interacting quantum systems

Mohammad F. Maghrebi,[1,2,3] Zhe-Xuan Gong,[1,2,4] and Alexey V. Gorshkov[1,2]

[1] *Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA*
[2] *Joint Center for Quantum Information and Computer Science, NIST/University of Maryland, College Park, Maryland 20742, USA*
[3] *Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA*
[4] *Department of Physics, Colorado School of Mines, Golden, Colorado 80401, USA*

Continuous symmetry breaking (CSB) in low-dimensional systems, forbidden by the Mermin-Wagner theorem for short-range interactions, may take place in the presence of slowly decaying long-range interactions. Nevertheless, there is no stringent bound on how slowly interactions should decay to give rise to CSB in 1D quantum systems at zero temperature. Here, we study a long-range interacting spin chain with $U(1)$ symmetry and power-law interactions $V(r) \sim 1/r^{\alpha}$. Using a number of analytical and numerical techniques, we find CSB for α smaller than a critical exponent $\alpha_c(\leq 3)$ that depends on the microscopic parameters of the model. Furthermore, the transition from the gapless XY phase to the gapless CSB phase is mediated by the breaking of conformal and Lorentz symmetries due to long-range interactions, and is described by a universality class akin to, but distinct from, the Berezinskii-Kosterlitz-Thouless transition. Signatures of the CSB phase should be accessible in existing trapped-ion experiments.

Long-range interacting systems have recently attracted great interest as they emerge in numerous setups in atomic, molecular, and optical (AMO) physics [1–15]. The advent of AMO physics has offered the intriguing possibility of simulating many-body systems which have been extensively studied theoretically in condensed matter physics [16–18]. While many properties of long-range interacting systems derive from their short-range counterparts, long-range interactions also give rise to novel phenomena [19–21]. In particular, they can induce spontaneous symmetry breaking in low-dimensional systems, which, for short-range interactions, is forbidden by the Mermin-Wagner theorem [22]. Such possibilities have been studied at finite temperature [21, 23, 24], where stronger versions of the Mermin-Wagner theorem have been proven for long-range interacting spin systems [24]. Despite a number of studies of variable-range models [25–30], this subject is less investigated at zero temperature [31]. As long-range interactions effectively change the dimensionality of the system, the emergence of CSB for sufficiently slowly-decaying interactions is not surprising; however, the equivalents of the stringent bounds at finite temperature [24] are not known at zero temperature. Furthermore, the quantum phase transition from the CSB phase to other 1D quantum phases must be exotic since, at zero temperature, the phases separated by this transition typically occur in different dimensions. With recent advances of the ion-trap quantum simulator in tuning long-range interactions [12, 14, 15], this topic appears to be of immediate experimental relevance.

In this paper, we consider the ferromagnetic XXZ spin-1/2 chain with power-law interactions $V(r) \sim 1/r^{\alpha}$. We find that the continuous $U(1)$ symmetry is spontaneously broken for a sufficiently slow decay of the interaction below a critical value of the exponent $\alpha_c(\leq 3)$ that depends on the microscopic parameters of the model. Exploiting a number of analytical techniques such as spin-wave analysis, bosonization, and renormalization group (RG) theory, as well as a numerical density matrix renormalization group (DMRG) analysis, we explore the phase diagram, and identify the phase transitions between different phases. In particular, we find that the phase transition between the CSB and XY phases is similar to, yet distinct from, the Berezinskii-Kosterlitz-Thouless transition. Signatures of such phases and phase transitions should be accessible in existing trapped-ion experiments.

Model.—Let us consider the long-range interacting XXZ chain

$$H = \sum_{i>j} \frac{1}{|i-j|^{\alpha}} \left(-S_i^x S_j^x - S_i^y S_j^y + J_z S_i^z S_j^z\right), \qquad (1)$$

with $S^{x,y,z} = \sigma^{x,y,z}/2$ where σs are the Pauli matrices. Note that J_z can be either positive or negative, while the S^x-S^x and S^y-S^y interactions are ferromagnetic. This model has a $U(1)$ symmetry with respect to rotations in the x-y plane. To explore the zero-temperature phase diagram of this model, we first bosonize the Hamiltonian [32, 33]. However, with long-range interactions between all pairs of spins, bosonization is rather complicated, at least at a quantitative level. Nevertheless, to capture the essential features of the phase diagram, we can split the Hamiltonian into two parts: the short-range part of the Hamiltonian and the asymptotic long-range interaction terms. In the bosonization language, the spin Hamiltonian can be mapped to one in terms of the bosonic variables ϕ and θ defined in the continuum, which satisfy the commutation relation $[\nabla\phi(x), \theta(y)] = i\pi\delta(x-y)$. Roughly speaking, the field θ gives the spin orientation in the x-y plane, while the gradient of ϕ characterizes the spin component along the z axis; we shall make these definitions more precise below. The short-ranged Hamiltonian can be mapped to the sine-Gordon model [32],

$$H_{\mathrm{SR}} = \frac{u}{2\pi}\int dx\left[\frac{1}{K}(\nabla\phi)^2 + K(\nabla\theta)^2\right] - \frac{2g}{(2\pi a_c)^2}\int dx\cos\left[4\phi(x)\right], \quad (2)$$

with a_c a short-wavelength cutoff, K the so-called Luttinger parameter, u a velocity scale, and g the strength of the cosine interaction term; the values of these parameters have been computed in the Supplemental Material (SM) [34] by including terms up to the next-nearest neighbor in Eq. (1) perturbatively in J_z and $1/2^\alpha$. Higher-order neighbors modify the parameters in Eq. (2), and induce higher-order harmonics which can nevertheless be neglected as they are less relevant in the RG sense.

To find the long-range part of the Hamiltonian, we approximately identify the spin operators in terms of the bosonic fields ϕ and θ as $S_j^\pm \equiv S_j^x \pm iS_j^y \sim e^{\pm i\theta(x_j)}$ and $S_j^z \sim \nabla\phi(x_j)$ where x_j is the position of the spin at site j [32, 33]. More generally, the spin operators can be expanded in a series of harmonics $e^{ip\phi}$; however, we have dropped those with $p \geq 1$ as they give rise to irrelevant terms. With this identification, the long-range S^z-S^z interaction takes the form $\int dxdy\,|x-y|^{-\alpha}\nabla\phi(x)\,\nabla\phi(y)$, which, in momentum space, is proportional to $\int dq|q|^{\alpha+1}|\phi(q)|^2$. We shall restrict ourselves to $\alpha > 1$, that is, the exponent is larger than the spatial dimensionality, so that the Hamiltonian (1) has a well-defined thermodynamic limit. With this assumption, the long-range S^z-S^z interaction is irrelevant compared to the gradient term in ϕ (proportional to $q^2|\phi(q)|^2$) in Eq. (2) and can thus be neglected. On the other hand, the long-range S^x-S^x and S^y-S^y interactions can be cast as

$$H_{\mathrm{LR}} = -g_{\mathrm{LR}}\int' dxdy\,\frac{1}{|x-y|^\alpha}\cos\left[\theta(x)-\theta(y)\right], \quad (3)$$

with g_{LR} the coefficient of long-range interactions. The prime on the integral indicates $|x-y| > \lambda$ with λ a cutoff distance much larger than lattice spacing; in this sense, the long-range interactions can be treated perturbatively. The bosonized Hamiltonian is then approximately the sum of the short- and long-range parts given by Eqs. (2) and (3), respectively. The cosine terms in Eqs. (2) and (3) involve non-commuting fields and thus compete with each other. To determine which one dominates, we shall resort to renormalization group theory.

Quantum phases.—To find the phase diagram, we perform an RG analysis that is perturbative in g and g_{LR}. The quadratic terms in Eq. (2) yield the scaling dimensions $\left[e^{ip\phi}\right] = p^2K/4$ and $\left[e^{ip\theta}\right] = p^2/(4K)$ that characterize scaling properties under spacetime dilations [32, 33]. The RG equations for the interaction coefficients g and g_{LR} then read (space-time rescaled by e^{-dl})

$$\frac{dg}{dl} = (2-4K)\,g\,, \quad \frac{dg_{\mathrm{LR}}}{dl} = \left[3-\alpha-1/(2K)\right]g_{\mathrm{LR}}\,. \quad (4)$$

The RG equations to a higher order, and those for K and u, are given in Eq. (8). Note that the value of K itself also depends on α. In deriving the flow of g_{LR}, we have used the fact that x and y in Eq. (3) are far separated.

Equation (4) gives rise to several phases depending on whether the interaction terms are relevant, and which one is more relevant. When both g and g_{LR} are irrelevant, the cosine terms can be dropped (assuming that they can be treated perturbatively). In this case, one finds an XY-like phase known as the Tomonaga-Luttinger (TL) liquid. In this phase, correlation functions decay algebraically with exponents determined by K [32]. Nevertheless, there is no true $U(1)$ symmetry breaking as $\langle S_i^+S_j^-\rangle \to 0$ for $|i-j| \to \infty$. This phase is described by a conformal field theory with the central charge $c = 1$ and an emergent Lorentz symmetry as long-range interactions are irrelevant. When the local interaction term is relevant, and more relevant than the long-range interaction, the latter can be dropped, while the former gaps out the system. This regime corresponds to an Ising phase, which occurs for a sufficiently large $|J_z|$: An antiferromagnetic (AFM) Ising phase emerges for large and positive, α-dependent, values of J_z, while a ferromagnetic (FM) Ising phase appears for all $J_z < -1$ as shown in the SM [34] via a spin-wave analysis, see Fig. 1. We stress that all the above phases also exist in the absence of long-range interactions; the presence of such terms, however, modifies the boundaries between these phases.

We are mainly interested in a regime where the long-range interaction term is (more) relevant, that is, $\alpha < 3-1/(2K)$. Hence $\alpha \leq 3$ is a necessary condition for the long-range interaction to be relevant. In this regime, assuming that g can be treated peturbatively, one can drop the local cosine term, and the model can be described by the Euclidean action [53]

$$I = \frac{K}{2\pi u}\int d\tau\int dx\left[(\partial_\tau\theta)^2 + u^2(\nabla\theta)^2\right] - g_{\mathrm{LR}}\int d\tau\int'\frac{dxdy}{|x-y|^\alpha}\cos\left[\theta(\tau,x)-\theta(\tau,y)\right], \quad (5)$$

where the $\nabla\phi$ term in Eq. (2), being conjugate to θ, is replaced by the (imaginary) time derivative $\partial_\tau\theta$ up to a prefactor. Since g_{LR} grows under RG, the value of the corresponding cosine term is pinned, i.e., $\theta(x) \approx \theta_0 = \text{const}$. This, in turn, implies a finite expectation value of the spin in the x-y plane, $\langle S_j^+\rangle \sim e^{i\theta_0}$. It thus appears that the ground state breaks a continuous symmetry. To examine the effect of fluctuations, we expand the cosine in Eq. (5) to quadratic order, and combine it with the quadratic terms in Eq. (5) to find $I \sim \int d\omega dq\,\left(\omega^2 + q^2 + |q|^{\alpha-1}\right)|\theta(\omega,q)|^2$, where we have

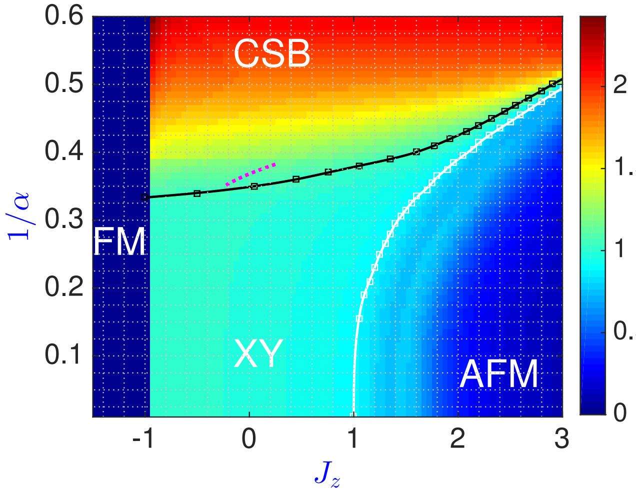

Figure 1: Phase diagram for the Hamiltonian (1) based on a finite-size DMRG calculation of the effective central charge $c_{\rm eff} = 6[S(N_1) - S(N_2)]/[\log(N_1) - \log(N_2)]$ [42]. Here $S(N)$ is the ground-state entanglement entropy for a chain of size N split in two equal halves. We choose $N_1 = 100$ and $N_2 = 110$ in our calculation. The XY phase has conformal symmetry and is identified by $c_{\rm eff} = 1$. The XY-to-CSB phase boundary is numerically obtained by finding the place where $c_{\rm eff}$ starts to increase appreciably (4%) above 1 (the black squares fitted by the black line). The dotted (purple) line is the XY-to-CSB transition line obtained from perturbative field theory calculation in [34]. The XY-to-AFM phase boundary is obtained by finding the place where $c_{\rm eff}$ starts to decrease appreciably (1%) below its value at $J_z = 1$ and $\alpha = \infty$ (the white squares fitted by the white line).

dropped various coefficients for convenience and taken $\theta_0 = 0$ without loss of generality. Clearly, the term proportional to q^2 can be dropped compared to $|q|^{\alpha-1}$ for $\alpha < 3$, thereby long-range interactions are dominant and the conformal and Lorentz symmetries are broken [37, 38]. The long-distance correlation of $S^{\pm}$ is given by

$$\langle S_i^+ S_j^- \rangle \sim \exp\Big[R_{ij}^{-\frac{3-\alpha}{2}}\Big] \to \text{const when } R_{ij} \to \infty, \qquad (6)$$

where $R_{ij} = |x_i - x_j|$. (In the exponent, we have not kept track of a coefficient of order one which depends on α as well as J_z.) Therefore, fluctuations respect the continuous symmetry breaking in this phase, in sharp contrast with the destruction of order in short-range interacting systems [22]. We conclude that CSB may be realized for sufficiently small values of $\alpha (< 3)$. The above findings are consistent with the phase diagram in Fig. 1 obtained numerically using the finite-size DMRG method [39–41] using the central charge. A similar phase diagram can also be obtained from correlation functions, see the SM [34]. It is worth pointing out that the quadratic action, after dropping the q^2 term, is exact in the RG sense; possible higher-order terms that respect the $U(1)$ symmetry are irrelevant. Specifically, the critical dynamic exponent, determining the relative scaling of space and time coordinates, is given exactly by

$$z = \frac{\alpha - 1}{2} < 1\,. \qquad (7)$$

The fact that $z < 1$ indicates that the 'light-cone' characterizing the causal behavior in the CSB phase is sublinear. The response function for this model is studied and is shown to take a universal scaling form [43].

Finally, we remark that an alternative spin-wave analysis ignores vortices [21] and predicts a straight line $\alpha_c = 3$ for the phase boundary between the XY and CSB phases. This also remains true for self-consistent improvements beyond spin-wave analysis [44]. However, the RG equations include the effect of vortices and predict a phase boundary at $3 - \alpha_c - 1/[2K(\alpha_c)] = 0$, see also [27, 29] for similar considerations in related models. For the perturbative evaluation of K as a function of α reported in the SM [34], we find the dotted line in Fig. 1 that captures the qualitative trend of the phase boundary near $J_z = 0$.

Phase transitions.—The ferromagnetic (FM) phase for $J_z < -1$ is connected to the CSB and XY phases at $J_z > -1$ via a first-order transition. The phase transition between the XY and the antiferromagnetic (AFM) phases is the Berezinskii-Kosterlitz-Thouless (BKT) transition, which is well understood for short-range interactions [32, 33]. We are mainly interested in the phase transition from the CSB phase to the XY phase described by Eq. (5).

Next we derive the RG flow beyond Eq. (4). We first consider the RG flow of the parameter K. Since the interaction term in Eq. (5) is nonlocal in space but local in time, we find a renormalization of $(\nabla\theta)^2$, but not $(\partial_\tau\theta)^2$, to first order in $g_{\rm LR}$. This signals breaking of the Lorentz invariance as the space and time coordinates behave differently, which in turn also leads to a renormalization of the velocity u. The RG flow for $g_{\rm LR}$ is given by the second equation in Eq. (4); however, it also receives corrections at the quadratic order in $g_{\rm LR}$: At this order, two vortices from different insertions of $g_{\rm LR}$ can neutralize each other at close distances, while the far-separated vortices form an interaction of the same form as the second line of Eq. (5). Putting the above considerations together, we find the RG equations to first nonzero order

$$\frac{dK}{dl} = A_K\, g_{\rm LR} + \mathcal{O}(g_{\rm LR}^2)\,, \qquad \frac{1}{u}\frac{du}{dl} = \frac{1}{K}\frac{dK}{dl} + \mathcal{O}(g_{\rm LR}^2)\,,$$
$$\frac{dg_{\rm LR}}{dl} = [3 - \alpha - 1/(2K)]\, g_{\rm LR} + B_K\, g_{\rm LR}^2 + \mathcal{O}(g_{\rm LR}^3)\,, \qquad (8)$$

with $A_K, B_K > 0$ depending on the parameter K and the renormalizion scheme, see the SM [34] for details. To find the critical behavior near the fixed point, we expand the above equations in its vicinity defining $\delta = 3 - \alpha - 1/(2K)$. The RG flow equations then read $d\delta/dl = A g_{\rm LR}$ and $dg_{\rm LR}/dl = \delta\, g_{\rm LR} + B g_{\rm LR}^2$—the constants are given by $A = A_K/2K^2$ and $B = B_K$ with the substitution

$K \to 1/[2(3-\alpha)]$. The flow equations are similar, but distinct from, the usual BKT transition: The equation for δ starts at the linear order (as opposed to quadratic) in coupling $g_{\rm LR}$, and the correction to the RG equation for $g_{\rm LR}$ appears at the quadratic (as opposed to cubic) order which should be kept. Indeed, the RG flow for the usual BKT transition is unchanged under reversing the sign of coupling [an example of which is the sine-Gordon model (2), where a change of $g \to -g$ can be simply undone by $\phi \to \phi + \pi/4$], but there is no such requirement for long-range interactions, hence the appearance of lower-order terms in the flow equations. The corresponding RG flow diagram is shown in Fig. 2. The flow trajectory

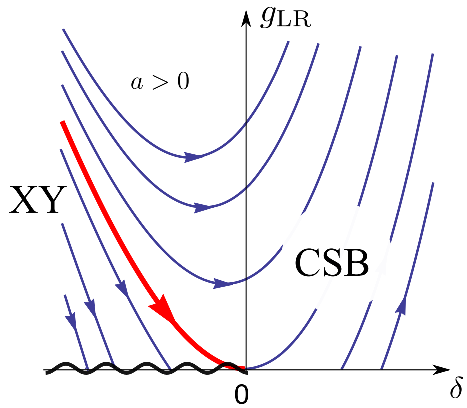

Figure 2: The RG flow in the vicinity of the phase transition, denoted by the thick (red) line, between the CSB phase and the XY phase. We have defined $\delta = 3 - \alpha - 1/(2K)$. The RG flow is given by $g_{\rm LR} \sim \delta^2 + a(1+\delta)$, where the parameter a quantifies the distance from the critical point. For $\delta < 0$, the $a = 0$ contour (denoted by the red line) describes the critical line. The flows with $a > 0$ and those with $a < 0$ and $\delta > 0$ proceed to infinity characterizing the CSB phase. The trajectories with $a < 0$ and $\delta < 0$ flow to the wavy line characterizing the XY phase.

near the transition point has a parabolic form given by $g_{\rm LR} \sim \delta^2 + a(1+\delta)$ with suitably rescaled variables where a parameterizes the distance from the critical trajectory; for $a = 0$, one finds the critical trajectory $g_{\rm LR} \sim \delta^2$. The RG flow and the form of the critical trajectory are distinctly different from the BKT transition, where the trajectories are hyperbolic, and the critical trajectory is a wedge rather than a parabola [45]. We also remark that a similar phase transition arises in a long-range *classical Ising* model with $\alpha = 2$ at finite temperature (as opposed to our XXZ model with variable α at zero temperature) [46–50] where similar flow equations emerge [48]. Nevertheless, this classical phase transition is fundamentally different from the model considered here since the corresponding critical dynamic exponents and dynamics (relaxational vs. quantum) are completely unrelated.

At large distances, the correlation function in the CSB phase approaches a constant; however, at short distances, the system still exhibits power-law decaying correlations predicted by the XY model. The length scale ξ that separates these two regimes diverges near the phase transition as $a \to 0^+$. One finds that $\xi \sim e^{1/\sqrt{a}}$ with a coefficient of order unity ignored in the exponent. This relation is reminiscent of the BKT transition where a should be identified as the distance from the critical temperature, and ξ as the correlation length [45]. In our case, a is simply a parameter that quantifies the distance from the critical trajectory; one can take it, for example, to be the difference of the exponent α from its critical value α_c.

Experimental detection.—Our model Hamiltonian can be realized by optical-dipole-force-induced spin-spin interactions in a trapped ion chain [51]. For $J_z = 0$ and $0.5 < \alpha < 2$, the dynamics of the Hamiltonian, Eq. (1), has already been simulated experimentally, with measurements available for individual spins [14, 15]. In order to observe the continuous CSB phase and related phase transitions, we can experimentally add a tunable-strength magnetic field in the x-y plane. The ground state for a finite-size system can be adiabatically prepared if we ramp down the magnetic field slowly enough compared to the energy gap [12, 52]. For a finite-size chain, the spontaneous symmetry breaking can be simulated by stopping the ramping process at a small but finite residual magnetic field. Then, by measuring spin magnetization along the magnetic field direction, we can confirm the existence of the CSB phase. Indeed, the CSB phase has robust features even in systems of $\lesssim 20$ spins, and can be adiabatically prepared in experimentally accessible time scales; see the SM [34] for details. Finally, the CSB phase is robust against thermal fluctuations for $\alpha \lessapprox 2$ which is well within experimental reach.

Conclusion and outlook.—In this work, we have considered a 1D spin Hamiltonian with long-range interactions, and shown that a phase with continuous symmetry breaking emerges for sufficiently slowly decaying power-law interaction. In particular, we have found a BKT-like transition from the CSB to the XY phase akin to the BKT transition. It is worthwhile confirming the associated universality class employing more powerful numerical techniques. More generally, it is desirable to obtain stringent, and model-independent, bounds on how slowly long-range interactions should decay to give rise to spontaneous symmetry breaking in one-dimensional systems at zero temperature. Furthermore, quantum phase transitions from the CSB phase to other 1D quantum phases are worth exploring. Finally, our approach is not suited to study $\alpha < 1$ when the thermodynamic limit ceases to exist, or for $\alpha = 1$ or 3 where the dispersion acquires a multiplicative logarithm. Such borderline values of the power-law exponent often give rise to new types of phases and phase transitions and would constitute an interesting topic for future investigations.

We acknowledge useful discussions with M. Kardar, M. Foss-Feig, L. Lepori, G. Pupillo, and D. Vodola. This work was supported by ARO, AFOSR, NSF QIS, ARO

MURI, NSF PFC at JQI, and ARL CDQI.

[1] M. Saffman, T. G. Walker, and K. Mølmer, Rev. Mod. Phys. **82**, 2313 (2010).
[2] P. Schauß, M. Cheneau, M. Endres, T. Fukuhara, S. Hild, A. Omran, T. Pohl, C. Gross, S. Kuhr, and I. Bloch, Nature (London) **491**, 87 (2012).
[3] O. Firstenberg, T. Peyronel, Q.-Y. Liang, A. V. Gorshkov, M. D. Lukin, and V. Vuletic, Nature (London) **502**, 71 (2013).
[4] B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. R. A. Hazzard, A. M. Rey, D. S. Jin, and J. Ye, Nature (London) **501**, 521 (2013).
[5] K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, Phys. Rev. Lett. **108**, 210401 (2012).
[6] M. Lu, N. Q. Burdick, and B. L. Lev, Phys. Rev. Lett. **108**, 215301 (2012).
[7] L. Childress, M. V. Gurudev Dutt, J. M. Taylor, A. S. Zibrov, F. Jelezko, J. Wrachtrup, P. R. Hemmer, and M. D. Lukin, Science **314**, 281 (2006).
[8] G. Balasubramanian, P. Neumann, D. Twitchen, M. Markham, R. Kolesov, N. Mizuochi, J. Isoya, J. Achard, J. Beck, J. Tissler, V. Jacques, P. R. Hemmer, F. Jelezko, and J. Wrachtrup, Nature Mater. **8**, 383 (2009).
[9] J. R. Weber, W. F. Koehl, J. B. Varley, A. Janotti, B. B. Buckley, C. G. Van de Walle, and D. D. Awschalom, Proc. Natl. Acad. Sci. **107**, 8513 (2010).
[10] F. Dolde, I. Jakobi, B. Naydenov, N. Zhao, S. Pezzagna, C. Trautmann, J. Meijer, P. Neumann, F. Jelezko, and J. Wrachtrup, Nature Phys. **9**, 139 (2013).
[11] S. Gopalakrishnan, B. L. Lev, and P. M. Goldbart, Phys. Rev. Lett. **107**, 277201 (2011).
[12] R. Islam, C. Senko, W. C. Campbell, S. Korenblit, J. Smith, A. Lee, E. E. Edwards, C.-C. J. Wang, J. K. Freericks, and C. Monroe, Science **340**, 583 (2013).
[13] J. W. Britton, B. C. Sawyer, A. C. Keith, C. C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Nature (London) **484**, 489 (2012).
[14] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feig, S. Michalakis, A. V. Gorshkov, and C. Monroe, Nature (London) **511**, 198 (2014).
[15] P. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt, and C. F. Roos, Nature (London) **511**, 202 (2014).
[16] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[17] I. Bloch, J. Dalibard, and S. Nascimbène, Nature Physics **8**, 267 (2012).
[18] M. Lewenstein, A. Sanpera, and V. Ahufinger, *Ultracold Atoms in Optical Lattices: Simulating quantum many-body systems* (Oxford University Press, 2012).
[19] B. Spivak and S. A. Kivelson, Phys. Rev. B **70**, 155114 (2004).
[20] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009).
[21] D. Peter, S. Müller, S. Wessel, and H. P. Büchler, Phys. Rev. Lett. **109**, 025303 (2012).
[22] N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966).
[23] J. R. de Sousa, EPJ B **43**, 93 (2005).
[24] P. Bruno, Phys. Rev. Lett. **87**, 137203 (2001).
[25] E. Yusuf, A. Joshi, and K. Yang, Phys. Rev. B **69**, 144412 (2004).
[26] N. Laflorencie, I. Affleck, and M. Berciu, Journal of Statistical Mechanics: Theory and Experiment **2005**, P12001.
[27] M. A. Cazalilla, F. Sols, and F. Guinea, Phys. Rev. Lett. **97**, 076401 (2006).
[28] A. M. Lobos, A. Iucci, M. Müller, and T. Giamarchi, Phys. Rev. B **80**, 214515 (2009).
[29] A. M. Lobos, M. Tezuka, and A. M. García-García, Phys. Rev. B **88**, 134506 (2013).
[30] M. Tezuka, A. M. García-García, and M. A. Cazalilla, Phys. Rev. A **90**, 053618 (2014).
[31] It is generically believed that the CSB is forbidden in zero-temperature 1D systems. An exception is when the order parameter commutes with the Hamiltonian as in the isotropic ferromagnetic point.
[32] T. Giamarchi, *Quantum physics in one dimension* (Oxford University Press, 2004).
[33] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, 2011).
[34] See Supplemental Material at [URL will be inserted by publisher] for details omitted from the main text, which includes Refs. [35, 36].
[35] B. Neyenhuis et al., arXiv:1608.00681 (2016) .
[36] Z.-X. Gong, M. F. Maghrebi, A. Hu, M. Foss-Feig, P. Richerme, C. Monroe, and A. V. Gorshkov, Phys. Rev. B **93**, 205115 (2016).
[37] D. Vodola, L. Lepori, E. Ercolessi, A. V. Gorshkov, and G. Pupillo, Phys. Rev. Lett. **113**, 156402 (2014).
[38] D. Vodola, L. Lepori, E. Ercolessi, and G. Pupillo, New Journal of Physics **18**, 015001 (2016).
[39] U. Schollwöck, Annals of Physics **326**, 96 (2011).
[40] G. M. Crosswhite, A. C. Doherty, and G. Vidal, Phys. Rev. B **78**, 035116 (2008).
[41] Our DMRG code is largely based on the open source MPS project at http://sourceforge.net/projects/openmps/. A maximum bond dimension of 200 is used. The $1/r^\alpha$ interaction is represented as a matrix product operator by fitting the power law to a sum of exponentials [40].
[42] P. Calabrese and J. Cardy, J. Stat. Mech. **2004**, P06002 (2004).
[43] M. F. Maghrebi, Z.-X. Gong, M. Foss-Feig, and A. V. Gorshkov, Phys. Rev. B **93**, 125128 (2016).
[44] P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
[45] J. Cardy, *Scaling and renormalization in statistical physics*, Vol. 5 (Cambridge university press, 1996).
[46] D. J. Thouless, Phys. Rev. **187**, 732 (1969).
[47] M. E. Fisher, S.-k. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972).
[48] J. M. Kosterlitz, Phys. Rev. Lett. **37**, 1577 (1976).
[49] A. Dutta and J. K. Bhattacharjee, Phys. Rev. B **64**, 184106 (2001).
[50] M. Knap, A. Kantian, T. Giamarchi, I. Bloch, M. D. Lukin, and E. Demler, Phys. Rev. Lett. **111**, 147205 (2013).
[51] X.-L. Deng, D. Porras, and J. I. Cirac, Phys. Rev. A **72**, 063407 (2005).
[52] K. Kim, M.-S. Chang, S. Korenblit, R. Islam, E. E. Edwards, J. K. Freericks, G.-D. Lin, L.-M. Duan, and C. Monroe, Nature **465**, 590 (2010).

[53] A similar action appears in a somewhat different model [27] with long-range interactions along the imaginary-time direction. Consequently, the RG flow equations (8) are almost identical to [27] with the exception of the quadratic term in coupling.