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Floquet Dynamics of Boundary-Driven Systems at Criticality

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A quantum critical system described at low energy by a conformal field theory (CFT) and subjected to a time-periodic boundary drive displays multiple dynamical regimes depending on the drive frequency. We compute the behavior of quantities including the entanglement entropy and Loschmidt echo, confirming analytic predictions from field theory by exact numerics on the transverse field Ising model, and demonstrate universality by adding non-integrable perturbations. The dynamics naturally separate into three regimes: a slow-driving limit, which has an interpretation as multiple quantum quenches with amplitude corrections from CFT; a fast-driving limit, in which the system behaves as though subject to a single quantum quench; and a crossover regime displaying heating. The universal Floquet dynamics in all regimes can be understood using a combination of boundary CFT and Kibble-Zurek scaling arguments.

Recent years have witnessed substantial progress in understanding the dynamics of periodically driven (Floquet) systems. Such driving has traditionally been used for engineering non-trivial effective Hamiltonians [1–4], but recent research has shown that these dynamics can differ drastically from their static counterparts. Examples include the recently observed Floquet time crystals [5–9], the emergence of topological quasiparticles protected by driving [10, 11], Floquet topological insulators [12–17], and Floquet symmetry-protected topological phases [18–21]. More broadly, periodically driven systems touch on fundamental issues in statistical and condensed-matter physics such as thermalization [22–26] and phase structure [5].

However, relatively little attention has been paid to driven systems at criticality, whose low-energy dynamics are often described by a conformal field theory (CFT). Such quantum critical systems are a natural setting in which to study Floquet dynamics, as many insights into the non-equilibrium dynamics of many-body systems have come from the study of CFTs in 1+1d [27–30]. A naïve expectation is that such a driven critical system would simply heat up. However, in the presence of a boundary drive, the energy injected per cycle is not extensive in system size, and there are multiple possible behaviors in an arbitrarily long period prior to thermalization. Moreover, as CFTs are integrable, it is natural to expect they can escape heating even at low frequencies provided the scaling limit is taken before the long time limit. This opens the door to using scaling theory combined with the analytical toolkit of boundary CFT [31–34] to characterize multiple regimes of universal dynamics in such boundary driven quantum critical points.

In this Letter, we study the dynamics of entanglement entropy $S_l(t)$ and Loschmidt echo $\mathcal{L}(t) = \left|\langle\psi(0)\middle|\psi(t)\rangle\right|^2$ in conformally-invariant quantum critical systems subject to a periodic boundary drive. We find two distinct regimes in which boundary conformal field theory provides an excellent description of the dynamics. For suitably slow drives, the system behaves almost as though subject to a series of independent quantum quenches but with amplitude corrections related to multiple-point correlation functions, while for fast drives, the boundary drive can be averaged out, and the system responds as though subject to a single quench at an averaged value of the field. For intermediate driving frequency, we find universal heating which crosses over from a perturbative regime at weak drive to non-perturbative boundary CFT regime at strong drive. The dynamics in all driving regimes are universal and can be described using field-theoretic tools. We numerically confirm that the dynamics remain robust against adding integrability-breaking interactions up to the finite times that may be simulated.

Model. While our results apply to arbitrary boundary-driven CFTs, for concreteness we will focus on the archetypical transverse-field Ising (TFI) model on the half-line with a time-dependent symmetry-breaking boundary field

$$H = -\sum_{i\geq 0}\left(J\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x + \Gamma\sigma_i^x\sigma_{i+1}^x\right) - h_b(t)\sigma_0^z, \quad (1)$$

with Γ an integrability-breaking perturbation and $h \sim J$ tuned to the critical point. This model has a convenient description in terms of free fermions when $\Gamma = 0$, seen by performing a Jordan-Wigner transformation [35][36] and is thus an ideal numerical test-bed for our model-independent analytical arguments. We initially prepare the system in the ground state at fixed boundary field $h_b(t < 0)$ then quench on a periodic boundary drive, $h_b(t + T) = h_b(t)$, for $t \geq 0$. In equilibrium, the low energy description of this spin chain at criticality is well-understood in terms of gapless left- and right-moving Majorana fields satisfying $\{\eta_{R/L}(x), \eta_{R/L}(y)\} = \delta(x-y)$, with Hamiltonian

$$H = -\frac{iv}{2}\int_0^\infty dx\,(\eta_R\partial_x\eta_R - \eta_L\partial_x\eta_L) - \lambda(t)\sigma_b(0), \quad (2)$$

where we dropped irrelevant terms. Here, v is a non-universal velocity ($v = 2J$ for $\Gamma = 0$) and $\lambda \propto h_b$. In this Majorana formulation, the boundary spin can be represented as $\sigma_b(0) = i(\eta_R + \eta_L)\gamma$ [37], where $\gamma^\dagger = \gamma$ is an ancilla Majorana satisfying $\gamma^2 = 1$ that anticommutes with all fields. In the following, we will assume that the drive is characterized by a single scale $\|h_b(t)\| \sim h_b$ which we take to be much smaller than the single particle bandwidth $h_b \ll \Lambda \equiv 2J = 2$ (setting $J = 1$), for which field theory is a good equilibrium description. The boundary field is a relevant perturbation with scaling dimension $\Delta = \frac{1}{2} < 1$ in the renormalization group (RG) language, with characteristic time scale $t_b \sim |h_b|^{-\nu_b}$, $\nu_b = (1-\Delta)^{-1} = 2$.

There are three energy scales in this problem: the driving frequency $\omega = 2\pi/T$, the bandwidth Λ, and the scale of the boundary perturbation $t_b^{-1} \sim h_b^{\nu_b} \ll \Lambda$. We will now consider various orderings of these scales and argue that essentially all regimes can be understood using a combination of field theory and scaling arguments, even though the drive is continuously injecting energy into the system. While the Hamiltonian (1) for $\Gamma = 0$ can be mapped onto free fermions for numerical convenience [38], we note that our main conclusions follow from general field theory arguments and therefore continue to hold in the non-integrable case. We emphasize that although we choose to focus on the Ising field theory (2) as an example, our field-theoretic arguments are model-independent, so our results carry over immediately to any boundary driven CFT, such as a driven quantum impurity problem with $t_b^{-1} \to T_K$, the Kondo temperature.

Slow driving regime: step drive. We start by considering the slow driving regime $\omega \ll t_b^{-1} \ll \Lambda$ for a step drive starting from the initial field $h_b(t < 0) = -h_b$ with $h_b(t) = +h_b$ for $0 \le t \le T/2$ (Hamiltonian H_1) and $h_b(t) = -h_b$ if $T/2 \le t \le T$ (Hamiltonian H_0) for $t \ge 0$. Intuitively, this drive looks like independent local quenches. Focusing on the Loschmidt echo (return probability) $\mathcal{L} = \left|\langle\psi_0|\psi(t)\rangle\right|^2$ [39], this behavior is best understood by Wick rotating to imaginary time $\tau = it$, where the spin-chain Loschmidt echo can be mapped onto a CFT correlation function. After computing this correlation function, we Wick rotate back to real time to obtain the dynamical echo. In imaginary time, the initial state can be generated by an infinite imaginary time evolution $\lim_{\tau\to\infty} e^{-\tau H_0}|0\rangle \propto |\psi_0\rangle$ from arbitrary initial state $|0\rangle$. In imaginary time, $\exp(-\tau H)$ acts as a projector onto the ground state of H, so for large $T \gg t_b$ we essentially oscillate between the ground states of H_0 and H_1, for which σ_0^z is locked in the direction of the boundary field $\pm h_b$. In the CFT language, a sharp change in boundary conditions can be treated by inserting a boundary-condition changing (BCC) operator [31], as diagrammed in Fig. 1. This means that the Loschmidt echo $\mathcal{L}(NT)$ after N periods of drive corresponds to the $2N$-point correlation function of a BCC operator $\phi_{\rm BCC}$ changing the boundary condition from fixed $\sigma_0^z = \pm 1$ to $\sigma_0^z = \mp 1$.

Analytically continuing to real time, we expect the Loschmidt echo to be a universal function $\mathcal{L}(T/t_b, N)$ in the field theory regime. In the limit $T \gg t_b$, this reduces

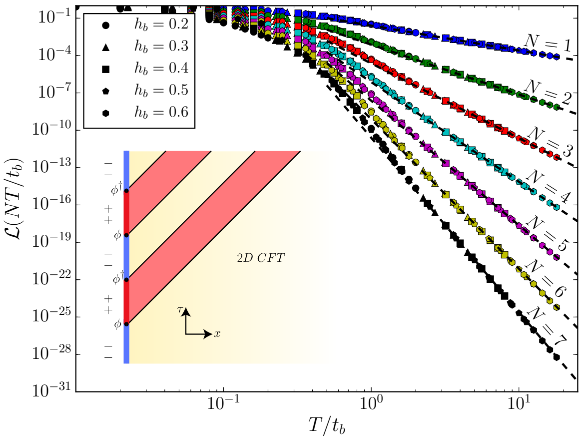

FIG. 1. Slow driving regime $\omega \ll t_b^{-1} \sim h_b^2 \ll \Lambda$ for a step drive alternating between $-h_b$ and $+h_b$ for systems up to $L \sim 3200$ sites ($\Gamma = 0$). For large T, we see clear power-law scaling of the Loschmidt echo with slope $-2N$ as predicted from boundary CFT. The agreement between the CFT $2N$-point function prediction (dashed lines) and numerical data is excellent, where we stress that the only fit parameter is the non-universal offset c_1. Note also the universal collapse of the Loschmidt echo as a function of universal parameter $T/t_b \sim h_b^2 T$. Inset: sketch of the imaginary time picture where the step drive corresponds to inserting BCC operators.

to the $2N$-point function

$$\mathcal{L}(NT) \underset{T\gg t_b}{\sim} \left|\left\langle \prod_{n=0}^{2N-1} \phi_{\rm BCC}(nT/2) \right\rangle\right|^2 = c_N \left(\frac{T}{t_b}\right)^{-\gamma N}, \tag{3}$$

whose form is fixed by scale invariance. The universal exponent $\gamma = 4h_{+-} = 2$ is given by the scaling dimension $h_{+-} = \frac{1}{2}$ of the BCC operator $\phi_{\rm BCC}$ [32, 33]. Other step drives can be dealt with in a similar fashion; for example, a step drive from $h_b = 0$ to $h_b \neq 0$ corresponds to the insertion of a BCC field with scaling dimension $h_{\rm BCC} = \frac{1}{16}$. We emphasize that eq. (3) holds for arbitrary boundary step drives in more general CFTs with the appropriate choice of BCC operator.

Note that although the Loschmidt echo decays exponentially with N, consistent with the independent quenches picture, the fact that the quenches are not fully independent is encoded in the non-trivial N dependence of the coefficients c_N. The ratio $c_N/(c_1)^N$ is universal and can be computed exactly for this specific drive, since the BCC operator $\phi_{\rm BCC}$ corresponds to a chiral fermionic field ψ in the Ising field theory with $2N$-point correlator given by a Pfaffian: $\mathcal{L}(NT) \sim |\langle\psi(0)\psi(T/2)\psi(T)\ldots\rangle|^2 \sim |{\rm Pf}(1/(t_i - t_j))|^2$ with $t_i = 0, T/2, T, \ldots, (N-\frac{1}{2})T$. For step drives in general CFTs, such universal ratios can be computed within the Coulomb gas (bosonization) framework [38]. These analytical expressions are in excellent agreement with numerical simulations for $\Gamma = 0$ (Fig. 1), where the only non-universal fit parameter is c_1. Since these predictions rely solely on field theory, they apply equally well to

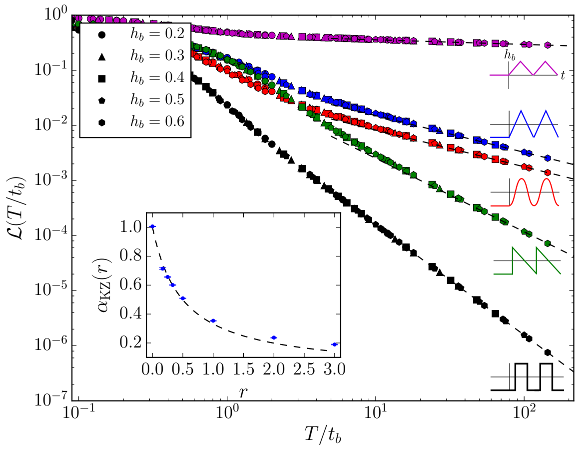

FIG. 2. Loschmidt echo for $\Gamma = 0$ over a single cycle ($N = 1$) in the slow regime for various drive geometries showing renormalized power laws and universal collapse as a function of $T/t_b = Th_b^2$. The dashed lines correspond to the analytic prediction (5) from boundary CFT and KZ arguments. Inset: KZ renormalization factor α_{KZ} of the BCC exponents for a boundary field scaling as $h_b(t) = h_b(\frac{t}{T})^r$ compared to the KZ prediction $\alpha_{\mathrm{KZ}} = (1+\nu_b r)^{-1}$ with $\nu_b = 2$. The dashed line is a fit of the numerical data for small r giving $\nu_b \approx 2.02 \pm 0.08$.

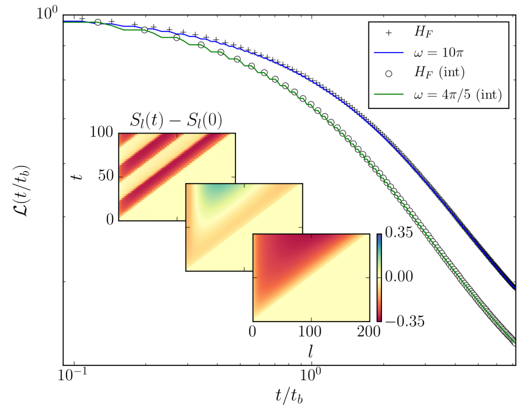

FIG. 3. Fast regime: the Loschmit echo at frequencies $\omega > \Lambda$ for a step drive oscillating between 0 and h_b coincides with the echo after a single local quench with effective field $h_b/2$ with Floquet Hamiltonian H_F (black crosses). This result also holds when interactions are added with $\Gamma = 0.25$ (white circles, green line). Insets: entanglement entropy difference $S_l(t) - S_l(0)$ for $\Gamma = 0$ as the drive frequency crosses over from the intermediate to the fast regime.

the non-integrable case $\Gamma \neq 0$; the interactions $\Gamma\sigma_i^x\sigma_{i+1}^x$ are irrelevant in the RG sense and therefore do not change the universality class. We confirm this numerically by locating the new critical point for $\Gamma \neq 0$ using exact diagonalization, obtaining the ground state using standard density matrix renormalization group (DMRG) techniques [40, 41], and simulating the dynamics of this driven interacting chain using time-evolving block decimation (TEBD) [42]. We find excellent agreement with our field-theoretic argument, as shown in the Supplemental Material [38].

Slow driving regime: general drives. Consider now a more general drive such as $h_b(t > 0) = -h_b\cos(\pi t/T)$ with $h_b(t < 0) = -h_b$. In the large T limit $h_b(t)$ crosses the critical value slowly rather than suddenly, yet the BCC picture suggests that the field should quickly flow to infinity. We find, however, that the vanishing (but finite) crossing speed is strongly relevant, changing the power law entirely (Fig. 2). To understand this difference, we use the concept of Kibble-Zurek (KZ) scaling, which is frequently applied to bulk drives crossing a bulk quantum critical point [43–46] but has not been studied for such boundary drives to our knowledge.

Let us imagine that the drive crosses $h_b = 0$ as a power-law $h_b(t) = h_b|\frac{t}{T}|^r \mathrm{sgn}(t)$ with $r = 1$ in the cosine drive considered above and $r = 0$ for a quench [47]. The effective time scale $t_b(t) \sim [h_b(t)]^{-\nu_b}$ now becomes time-dependent, and we expect the dynamics to be controlled by an emergent time scale

$$t_{\mathrm{KZ}} \sim T^{\frac{r\nu_b}{1+r\nu_b}} h_b^{-\frac{\nu_b}{1+r\nu_b}}, \tag{4}$$

given by $t_{\mathrm{KZ}} \sim t_b(t_{\mathrm{KZ}})$. Though our system is always gapless so that there is no adiabatic limit, it is straightforward to show that this dynamical scale emerges directly from the equations of motion of eq. (2) [48]. It is natural to expect that the slow driving limit $T \gg t_{\mathrm{KZ}}$ should still be described by boundary CFT, suggesting that the Loschmidt echo would scale as (3) with t_b replaced t_{KZ}. We therefore see that the effect of the slow driving amounts to renormalizing the dimension h_{BCC} of the BCC operator by a factor $\alpha_{\mathrm{KZ}} = 1/(1 + r\nu_b)$ with $\nu_b = 2$ in our case. More generally, for a drive where $h_b(t)$ crosses or touches the critical value n times within a single cycle, we predict that the universal exponent γ controlling the exponential decay of the Loschmidt echo is given by

$$\gamma = 2\sum_{i=1}^{n} \frac{h_{\mathrm{BCC}}^i}{1 + r_i\nu_b}, \tag{5}$$

where r_i is the power of $|h_b(t)| \sim |t - t_c^i|^{r_i}$ near the critical time t_c^i. For our model, $h_{\mathrm{BCC}}^i = \frac{1}{2}$ if $h_b(t)$ crosses zero and $h_{\mathrm{BCC}}^i = \frac{1}{16}$ if it touches zero without changing sign [32, 33, 49]. For example, a cosine or triangle drive oscillating between $\pm h_b$ has $n = 2$, $r_1 = r_2 = 1$ so that $\gamma = 2/3$, while a sawtooth drive combines slow ($r_1 = 1$) and fast ($r_2 = 0$) crossings to give $\gamma = 4/3$.

These predictions give good agreement with numerics (Fig. 2) [50]. Furthermore, the only effect of the slow driving is to renormalize the scaling dimensions of the BCC operators while keeping the structure of the $2N$-point function unchanged. In particular, we find that the universal numbers $c_N/(c_1)^N$ in eq. (3) are still given by the boundary CFT predictions for a step drive [38].

Fast driving regime. We now consider the high-frequency regime $t_b^{-1} \ll \Lambda \ll \omega$. This is naïvely outside the regime where field theory results should apply, but we can take advantage of standard Floquet machinery to write a Floquet-Magnus high-frequency expansion for the Floquet Hamiltonian H_F defined by $U(T) = \mathcal{T}\mathrm{e}^{-i\int_0^T dt H(t)} = \mathrm{e}^{-iTH_F}$ [51]. For example, $H_F = \frac{1}{2}(H_0 + H_1) - \frac{i}{4\omega}[H_0, H_1] + \mathcal{O}(\omega^{-2})$ for a step drive. While higher order terms in this expansion are suppressed by powers of ω^{-1} as for any high-frequency Floquet system, we note here that the Floquet Hamiltonian H_F itself corresponds to a CFT subject to an effective boundary field $\overline{h}_b = (1/T)\int_0^T h_b(t)dt$ with higher order terms in the high frequency expansion being RG irrelevant. This is most easily seen using the field theory Hamiltonian (2) where the small parameter controlling the expansion is $v/\omega \ll 1$ with $v \sim \Lambda = 2J$. While the first boundary term has scaling dimension $\Delta = 1/2$ and corresponds to the averaged field $\overline{h}_b$, dimensional analysis immediately implies that terms of order ω^{-n} have scaling dimension of at least $n+1/2$ due to terms such as $\partial^n\eta(0)$ and are thus irrelevant for $n > 0$ [38]. Therefore at late times, the system behaves as though subject to a single local quantum quench with effective boundary field $\overline{h}_b$ (Fig. 3), a problem whose universal dynamics has been studied extensively [52, 53]. We remark that though RG techniques may be in general ill-defined in a Floquet system which, for instance, lacks a notion of ground state, in this high-frequency limit the Floquet evolution is well-controlled by an effective static Hamiltonian. Since our initial state is a conformally invariant ground state and the effective Hamiltonian implements a local quench, the notion of RG flow is well-defined [52] and provides a powerful tool of analysis. Additionally, for the non-interacting (free fermions) case with $\Gamma = 0$ in eq. (1), one may prove that the high-frequency expansion is convergent for $\omega \gtrsim \Lambda$ by bounding the spectral width of the single-particle Hamiltonian [38]. More generally, this effective single quench picture will survive even in the presence of integrability-breaking interactions controlled by Γ up to exponentially long time scales $\tau_{\text{th}} \sim \mathrm{e}^{C\omega/\Lambda}$ [23–26]. We simulated the dynamics of this interacting chain subject to the same drive using TEBD and found excellent agreement with the single effective quench picture even at moderate frequencies (Fig. 3).

Crossover regime. Finally, we discuss the intermediate crossover regime $t_b^{-1} \sim \omega \ll \Lambda$. We focus on a free-to-fixed step drive from $h_b = 0$ to $h_b \neq 0$ with $\Gamma = 0$ for simplicity. In this regime, we expect the system to absorb energy ("heat") via resonant processes within the single-particle bandwidth. This leads to exponential decay of the Loschmidt echo,

$$\mathcal{L}(NT) \underset{t_b^{-1},\omega\ll\Lambda}{\sim} \mathrm{e}^{-N/N_\star(\omega t_b)}, \tag{6}$$

with $N_\star(\omega t_b)$ a universal function (Fig. 4a). For weak drive ($\omega t_b \gg 1$), resonant heating occurs with a rate $\tau^{-1} \sim h_b^2/J$ given by Fermi's golden rule, so that $N_\star \sim \tau/T \sim \omega t_b$. For strong drive ($\omega t_b \ll 1$), we recover the boundary CFT prediction $N_\star \sim -1/(\gamma \log \omega t_b)$. We also find that entanglement entropy of boundary intervals of size ℓ, relative to the ground state entropy, saturates to a volume law behavior $S_\ell \sim \ell$ at long times in the regime $\omega t_b \gg 1$, consistent with heating [54]. At low frequencies, the entropy simply oscillates between ground state values [55] though it may become extensive at much later times. We leave a detailed analysis of the role of interactions in this intermediate regime for future work.

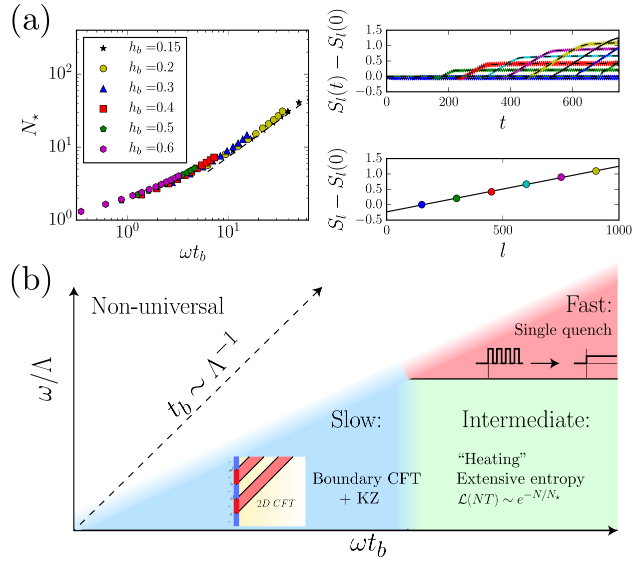

FIG. 4. Intermediate regime $t_b^{-1} \sim \omega \ll \Lambda$ for a step drive from $h_b = 0$ to $h_b \neq 0$. (a) Left panel: universal scaling function $N_\star(\omega t_b)$ characterizing the exponential decay of the Loschmidt echo $\mathcal{L}(NT) \sim \mathrm{e}^{-N/N_\star}$, with the dashed line showing the linear behavior expected from Fermi's golden rule. Right panel: volume-law scaling of the entanglement entropy in the long-time limit for $\omega t_b = 10.5 \gg 1$. An overbar denotes the value of the late-time plateau. (b) Sketch of the three universal driving regimes analyzed in this Letter.

Discussion. We have investigated CFTs subject to a Floquet boundary drive. Despite the naïve expectation that such gapless systems should absorb energy and simply heat up, we have identified three distinct regimes summarized in Fig. 4b in which the system shows universal features that can be understood using tools of field theory and scaling theory. We expect our main conclusions to apply to a broad class of systems, and it will be especially interesting to investigate the consequences of our results for the physics of driven quantum dots and the non-equilibrium signatures of topological edge modes [10]. In general, our results represent an analytically tractable model of a driven gapless system, an active area of research increasingly relevant to experiments.

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the central charge of the CFT and $\tilde{a}$ is a non-universal constant. Pinning the boundary spin causes a universal reduction of the entanglement entropy for $h_b \neq 0$ of $\Delta S = -\log\sqrt{2}$, known as the Affleck-Ludwig entropy [67].

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