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# Remanent Magnetization: Signature of Many-Body Localization in Quantum Antiferromagnets

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# Remanent magnetization: signature of Many-Body Localization in quantum antiferromagnets

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We study the remanent magnetization in antiferromagnetic, many-body localized quantum spin chains, initialized in a fully magnetized state. Its long time limit is an order parameter for the localization transition, which is readily accessible by standard experimental probes in magnets. We analytically calculate its value in the strong-disorder regime exploiting the explicit construction of quasi-local conserved quantities of the localized phase. We discuss analogies in cold atomic systems.

*Introduction.* The non-equilibrium dynamics in disordered, isolated quantum systems have been subject to theoretical investigations ever since the notion of localization was introduced in [1]. Spin systems in random fields are prototypical models to analyze the disorder-induced breakdown of thermalization: a large number of numerical studies on disordered spin chains [2–13] has provided evidence for a dynamical phase transition between a weak-disorder phase which thermalizes, and a Many-Body Localized (MBL) phase in which excitations do not diffuse, ergodicity is broken and local memory of the initial conditions persists for infinite time [14–16].

Signatures of MBL are found in the properties of individual many-body eigenstates: even highly excited eigenstates exhibit area-law scaling of the bipartite entanglement entropy [4, 7, 8, 17] and Poissonian level statistics [2, 18, 19], both being incompatible with thermalization [20–22]. Novel dynamical properties such as the logarithmic spreading of entanglement have been observed in direct simulations of the time evolution [3, 23]. This phenomenology arises due to an extensive set of conserved operators whose local structure prevents both transport and thermalization [24–29]. While mathematically rigorous results are available for certain localized spin chains [28], the existence of such conserved quantities, and thus of MBL, in dimensions  $d \geq 2$  is debated [30].

The non-equilibrium physics of MBL systems has been probed experimentally in artificial quantum systems made of cold atomic gases [31, 32] and trapped ion systems [33], while an indirect signature via a strong suppression of microwave absorption was found in electron-glasses [34]. However, direct observations of MBL in the solid state are still lacking.

In this work, we propose a readily observable consequence of MBL in quantum magnets: the out-of-equilibrium remanent magnetization that persists after ferromagnetically polarizing an antiferromagnet whose total magnetization is not conserved. As an example, we consider an antiferromagnetic, anisotropic Heisenberg

spin-1/2 chain

$$H = \sum_k \left( h_k \sigma_k^z - \sum_{\alpha=x,y,z} J_\alpha \sigma_k^\alpha \sigma_{k+1}^\alpha \right) \quad (1)$$

subject to random fields  $h_k$  along the easy axis. We assume  $J_z < 0$ , as well as  $J_x \neq J_y$  to ensure the non-conservation of the total magnetization. Such Hamiltonians can be realized, e.g., in Ising compounds with both exchange and dipolar interactions. However, essentially any sufficiently strongly disordered quantum antiferromagnet with non-conserved magnetization should exhibit the same phenomenology as we describe below.

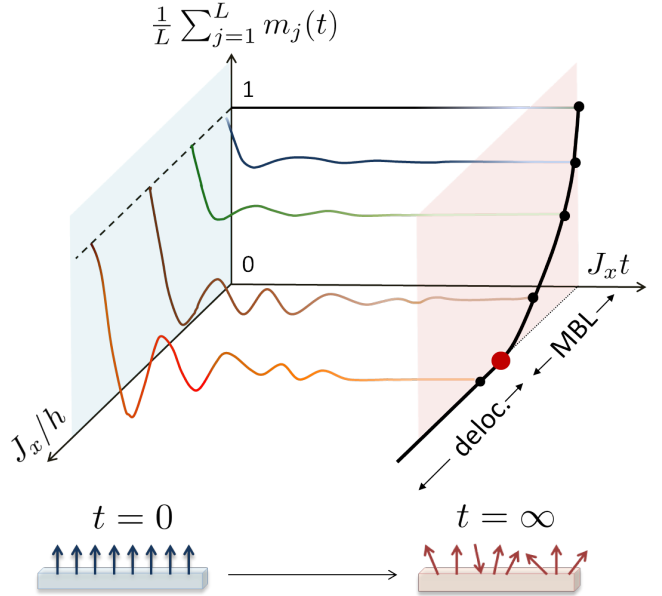


FIG. 1: Relaxation of the total magnetization from a fully polarized initial state. The black curve is the stationary value  $L^{-1} \sum_j \hat{m}_j$ : it vanishes (possibly discontinuously [35, 36]) at the critical point separating the MBL and delocalized phases (red point). It is non-analytic for  $J_x/h \ll 1$ , cf. Eq. (13).

Consider the following protocol: First, the chain is

fully polarized by a strong magnetic field [equilibrated by a (very weak) coupling to a bath]. Then the field is switched off at time  $t = 0$  (cf. Fig. 1). For magnets with spin-phonon coupling much weaker than the spin-spin interactions (see e.g. Ref. [37]), the dynamics is governed by (1) alone over a long intermediate time window, and thus behaves like a closed system. Interchain couplings in quasi 1d magnets are weak and will not modify the short-time phenomenology. On the other hand, the conjectured break-down of MBL at long times due to very rare regions in higher dimensions [30] is expected to be a much less efficient channel of delocalization than that provided by realistic couplings to phonons.

Ergodic spin dynamics would relax the initial magnetization completely. A finite remanent magnetization thus implies non-ergodicity (see also [38, 39] for disorder-free examples), and can serve as an order parameter in the MBL phase of the isolated system. This is a magnetic analogue of the remanent density modulation considered in [40] and measured in recent cold-atom experiments [31, 32]. However, it is experimentally much simpler to access, since it focuses on the total magnetization which can be readily picked up, e.g., by a squid, without requiring scattering measurements or microscopy to resolve spatial patterns.

*Conserved dressed spins.* We consider random fields  $h_k$  uniformly distributed in  $[-h, h]$ , and assume strong anisotropy of the couplings,  $|J_y| \ll |J_x| \ll |J_z|, h$ . For simplicity we set  $J_y = 0$ . For  $J_x = 0$ , the spin chain is classical and trivially localizes dynamically, as the  $\sigma_k^z$  form a complete set of commuting, strictly local conserved operators, and the eigenstates are product states in this basis. Perturbative arguments as in Refs. [15, 16] predict that localization remains intact for small quantum fluctuations  $|J_x| \ll h$ . This comes along with a complete set of mutually commuting conserved operators  $I_k$ . Those are predicted to be *quasi-local* [26, 27, 29], their action decaying exponentially with the distance from a localization center, with a finite correlation length  $\xi$ . The  $I_k$  are dressed (rotated) versions of the spin degrees of freedom. They can be constructed explicitly by applying perturbation theory to the  $\sigma_k^z$  and resumming the divergences associated to sparse local ‘resonant’ regions in the chain where the effect of quantum fluctuations is non-perturbative [27]. Their existence follows as a corollary of Ref. [28], where MBL was shown to occur due to the quasi-locality of the unitary operator  $U$  that diagonalizes the disordered Hamiltonian:  $U$  is proved to be a sequence of small unitary transformations almost everywhere in the chain, up to rare resonant regions. The

inverse  $U^{-1}$  deforms  $\sigma_k^z$  into the conserved quasi-local operator  $I_k = U\sigma_k^z U^{-1}$  which admixes only degrees of freedom in the vicinity of  $k$ .

Below, we explicitly construct the dressed spin operators following the recipe of Ref. [27]. We then use them to calculate analytically the remanent magnetization to low orders in the quantum fluctuations  $|J_x|/h$ . This method is very efficient [56] to describe the asymptotic magnetization in non-resonant regions where the transverse couplings act perturbatively. However, we will see that the dominant effect of weak quantum fluctuations arises from rare resonant regions, where the perturbative expansion in  $J_x$  has to be resummed to infinite order.

*Computation of remanent magnetization.* We consider the dynamical evolution of the fully magnetized initial state  $|\psi_0\rangle$  with density matrix  $|\psi_0\rangle\langle\psi_0| = \prod_i (1 + \sigma_i^z)/2$ , governed by (1). [57] We are interested in the long time remanence of the magnetization, and thus consider the time averaged magnetization at site  $j$ :

$$\hat{m}_j = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt m_j(t); \quad m_j(t) = \langle \psi_0 | \sigma_j^z(t) | \psi_0 \rangle. \quad (2)$$

For  $J_x = 0$ , the local magnetization is trivially conserved,  $m_j(t) = 1$ . For finite  $J_x$ , the non-trivial dynamics of  $\sigma^z(t)$  reduces  $\hat{m}_j$ . In the MBL regime, the time evolution is strongly constrained by the conservation of the dressed spins  $I_k$  with  $|k - j| \lesssim \xi$ . As a consequence, partial memory of the initial order  $\langle \sigma_j^z \rangle = 1$  is retained for arbitrarily long time, resulting in a finite remanence of the site-averaged magnetization  $\hat{m} = L^{-1} \sum_j \hat{m}_j$ .

In the absence of spectral degeneracies, (2) can be expressed via a Lehmann representation as

$$\hat{m}_j = \sum_{\alpha} \langle \psi_0 | P_{\alpha} \sigma_j^z P_{\alpha} | \psi_0 \rangle, \quad (3)$$

where  $P_{\alpha} = |\psi_{\alpha}\rangle\langle\psi_{\alpha}| = \prod_{k=1}^L \left(1 + i_k^{(\alpha)} I_k\right)/2$  projects onto the eigenstate labeled by the quantum numbers  $i_k^{(\alpha)} \in \{\pm 1\}$  of the dressed spins  $I_k$ . Using the operator identity

$$\sum_{\alpha} P_{\alpha} \sigma_j^z P_{\alpha} = \sigma_j^z + \sum_{n=1}^L \sum_{k_n > k_{n-1} \dots > k_1} \prod_{l=1}^n \left(\frac{I_{k_l}}{2}\right) [[[\sigma_j^z, I_{k_1}], I_{k_2}], \dots, I_{k_n}], \quad (4)$$

we obtain (cf. Supplemental Material for details):

$$\hat{m}_j = 1 + \sum_{n=1}^L \sum_{k_n > k_{n-1} \dots > k_1} \text{Tr} \left\{ \prod_{i=1}^n \left(\frac{I_{k_i}}{2}\right) [[[\sigma_j^z, I_{k_1}], I_{k_2}], \dots, I_{k_n}] \prod_{\ell=1}^L \left(\frac{1 + \sigma_{\ell}^z}{2}\right) \right\}, \quad (5)$$

where  $\text{Tr}\{\cdot\}$  denotes the trace, and some fixed ordering among the labels of the operators  $I_k$  is assumed [58].

For the dressed spins  $I_k$  we write the formal expansion

$$I_k = \sigma_k^z + \delta I_k^{(1)} + \delta I_k^{(2)} + \dots \quad (6)$$

with  $\delta I_k^{(n)} = O(J_x^n)$ . At any order  $n$ ,  $\delta I_k^{(n)}$  is uniquely determined by the constraints  $[I_k, H] = 0$  and  $I_k^2 = 1$  [27], see also the Supplemental Material. For the Hamiltonian (1) with  $J_y = 0$ , the first order terms read

$$\begin{aligned} \delta I_k^{(1)} = & \sum_{\rho, \tau = \pm 1} \left( A_{\rho\tau}^{(k)} O_{\rho\tau}^{(k)} - A_{\rho\tau}^{(k-1)} O_{\rho\tau}^{(k-1)} \right) \\ & + \sum_{\rho, \tau = \pm 1} \left( B_{\rho\tau}^{(k)} \Delta_{\rho\tau}^{(k)} + B_{\rho\tau}^{(k-1)} \Delta_{\rho\tau}^{(k-1)} \right), \end{aligned} \quad (7)$$

where we define the local operators

$$\begin{aligned} O_{\rho\tau}^{(k)} &= \frac{1 + \rho \sigma_{k-1}^z}{2} [\sigma_k^+ \sigma_{k+1}^- + \text{h.c.}] \frac{1 + \tau \sigma_{k+2}^z}{2}, \\ \Delta_{\rho\tau}^{(k)} &= \frac{1 + \rho \sigma_{k-1}^z}{2} [\sigma_k^+ \sigma_{k+1}^+ + \text{h.c.}] \frac{1 + \tau \sigma_{k+2}^z}{2}, \end{aligned} \quad (8)$$

and the coefficients

$$\begin{aligned} A_{\rho\tau}^{(k)} &= -\frac{J_x}{h_k - h_{k+1} + J_z(\tau - \rho)}, \\ B_{\rho\tau}^{(k)} &= -\frac{J_x}{h_k + h_{k+1} - J_z(\tau + \rho)}. \end{aligned} \quad (9)$$

At low orders, the sum over multi-indices in Eq. (5) reduces to the few terms involving indices sufficiently close to  $k$ , since other commutators vanish. The lowest order corrections to  $\hat{m}_j$  are given by the terms with  $n = 1, 2$  in (5). Inserting (7) into (4) and (3) we find [59]:

$$\hat{m}_j = 1 - \left( B_{1,1}^{(j)} \right)^2 - \left( B_{1,1}^{(j-1)} \right)^2 + O(J_x^3). \quad (10)$$

The above method is easily extended to higher orders on the majority of sites around which the quantum fluctuations act perturbatively. However, we see that the site average of  $\langle \hat{m}_j \rangle_{\text{dis}}$  is ill-defined for  $|J_z| < h$ . The apparent divergence is due to rare resonances, i.e., realizations of neighboring local fields that give rise to small denominators in Eq. (9). Resonances may also arise at higher order  $n$  in perturbation theory, if two nearly degenerate classical configurations hybridize strongly due to a coupling  $\sim J_x^n$ . The corresponding small denominators render the naive perturbative expansion (6) non-convergent, as a small denominator generated at order  $n$  re-appears repeatedly in higher order terms, giving rise to norm-divergent operator subsequences. However, a defining feature of the MBL phase is that resonances are sparse in space, each of them involving spins only within the typical range of the correlation length  $\xi$ . [27, 28]. Thus, *local* re-summations (or equivalently, exact diagonalizations of resonant subsystems of typical size  $\xi$ ) suffice to cure the divergences of the perturbation theory

(in analogy with the single particle case [1]) and yield a ‘regularized’, norm-convergent operator expansion for the  $I_k$ .

Let us illustrate this re-summation for resonances involving spins on neighboring sites, as those dominate in the strong-disorder limit. Resumming all terms in (6) containing higher powers of the resonant  $J_x$ -coupling is equivalent to determining the operators  $\tilde{I}_i$  (satisfying  $\tilde{I}_i^2 = \mathbb{1}$ ) that are conserved by the reduced Hamiltonian

$$H^{(k)} \equiv \sum_{i=1}^L (h_i \sigma_i^z - J_z \sigma_i^z \sigma_{i+1}^z) - J_x \sigma_k^x \sigma_{k+1}^x, \quad (11)$$

where only the resonant  $J_x$ -coupling is retained. These operators serve as a new basis for the perturbation theory in the remaining, non-resonant  $J_x$ -couplings. They can be constructed explicitly by determining a local rotation that acts non-trivially in the vicinity of the sites  $k, k+1$ , deforming the pair  $\sigma_k^z, \sigma_{k+1}^z$  into two modified, exactly conserved operators  $\tilde{I}_k, \tilde{I}_{k+1}$  (with  $\tilde{I}_i = \sigma_i^z$  for  $i \neq k, k+1$ ), similarly to [28]. For the particular model (1) the resulting  $\tilde{I}_k, \tilde{I}_{k+1}$  are again linear combinations of (8), albeit with modified coefficients (9) given in the Supplemental Material. [60] Inserting the conserved integrals into (3), we find an expression like (10), but with the substitution:

$$B_{\rho\tau}^{(j)} \rightarrow -\frac{J_x}{\left( [h_j + h_{j+1} - J_z(\tau + \rho)]^2 + J_x^2 \right)^{1/2}}, \quad (12)$$

a result one can also derive from exact diagonalization of two neighboring spins in the field of their polarized neighbors, and computing their diminished magnetization. From this we obtain the remanent magnetization

$$\langle \hat{m}_j \rangle_{\text{dis}} = 1 - \frac{\pi |J_x|}{h} \left( 1 + \frac{J_z}{h} \right) + O(J_x^2), \quad (13)$$

which for  $|J_z| < h$  is non-analytic in  $J_x$ . This feature arises due to resonances. The non-analytic cusp at  $J_x = 0$  has the largest magnitude in the limit of vanishing Ising interactions,  $J_z \rightarrow 0$  (recall that  $J_z < 0$ ), as it creates the lowest effective disorder.

*Atomic analogues.* The gauge transformation  $\mathcal{U} = \prod_{j=1}^L \exp(i \frac{\pi}{2} j \sigma_j^x)$  maps the antiferromagnetic chain (1) into its ferromagnetic counterpart with  $J_x \rightarrow J_x$ ,  $J_{y,z} \rightarrow -J_{y,z}$ , and the initial state  $|\psi_0\rangle$  into a Néel state. The order parameter is mapped into the staggered magnetization. Such a quantity has been studied numerically in [41] for disordered, long-range transverse field Ising chains, modeling the ion-trap quantum simulators explored experimentally in [33]. The staggered magnetization is a close analogue of the particle imbalance studied as an experimental probe of many-body localization in cold atoms [31]:

$$\mathcal{I}(t) = \frac{2}{L} \sum_{j=1}^L (-1)^j \langle n_j(t) \rangle. \quad (14)$$

Here  $n_j$  is the occupation number of site  $j$ , after preparation in an initial density wave  $n_j(t=0) = [1 + (-1)^j]/2$ .

A ferromagnetic spin chain with  $J_x = J_y$  is equivalent, via the Jordan-Wigner transformation, to a one-dimensional model of interacting spin-less fermions in a disordered potential. For  $J_z = 0$  it reduces to the non-interacting Anderson model

$$H = -J \sum_{i=1}^{L-1} (c_i^\dagger c_{i+1} + h.c.) + 2 \sum_{i=1}^L h_i n_i \quad (15)$$

for which the imbalance is a sum over single particle contributions, weighted with the occupation probability of eigenstates in the initial state. A standard calculation leads to the remanent imbalance (14) in the form

$$\hat{\mathcal{I}} = \frac{1}{L} \sum_{\alpha=1}^L \left( \sum_{k=1}^L (-1)^k \phi_\alpha^2(k) \right)^2, \quad (16)$$

where  $\phi_\alpha(i)$ , with  $1 \leq \alpha, i \leq L$  are the localized single particle eigenstates of the quadratic Hamiltonian (15). This solvable case is interesting as it can be analyzed deeper into the weak disorder limit.

Fig. 2 shows the imbalance as a function of  $J/h$ , as obtained by exact diagonalization. At small  $J/h$  a linear cusp with the slope derived in (13) (using  $J_z = 0, J_x = J$ ) is seen. For large  $J/h$ ,  $\hat{\mathcal{I}}$  decays algebraically as  $(J/h)^{-2}$ , as derived in the Supplemental Material [61] and verified numerically in Fig. 2. [62]

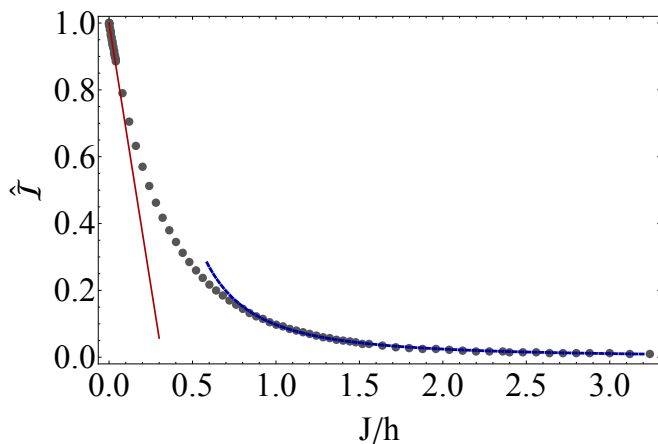


FIG. 2: Dependence of the remanent density imbalance on the hopping strength  $J$  for a chain of non-interacting fermions ( $L = 100, 5 \cdot 10^3$  realizations). The continuous red line is the analytical estimate (13) with  $J_x = J, J_z = 0$ . The blue dashed line is a power law fit  $a + c(J/h)^{-2}$ , with  $a = 0.003, c = 0.101$ .

Let us now discuss the qualitative effects of fermionic interactions. The addition of a term  $U \sum_{i=1}^L n_i n_{i+1}$  (the equivalent of Ising interactions) to the Hamiltonian (15) may have opposite effects, depending on the value of  $J/h$ .

For  $J/h \ll 1$ , the interaction broadens the distribution of the energy denominators, and thus acts as an additional source of disorder, which reduces the deviation of  $\langle \hat{\mathcal{I}} \rangle_{\text{dis}}$  from the classical limit. The same holds in the magnetic analogue as confirmed by Eq. (13). For larger  $J/h > 1$ , the single particle localization length becomes substantial. The dominant effect of interactions is then to mediate (virtual) scattering between single particle states, as discussed in Ref. [15, 16]. One expects that this suppresses the remanent imbalance, as was indeed observed in the experiments of Ref. [31]. At large enough interactions (and finite temperature) delocalization is induced by a proliferation of resonant regions, which leads to the vanishing of the order parameter  $\langle \hat{\mathcal{I}} \rangle_{\text{dis}}$  at a  $U$ -dependent critical hopping  $J^*(U)/h$ . The perturbative arguments in [15, 16, 27] predict that the localized phase is stable for  $U < U^* \propto \delta_\xi / \log(\mathcal{W}/\delta_\xi)$ . Here  $\mathcal{W}$  is the total bandwidth of the non-interacting Hamiltonian (15) and  $\delta_\xi \sim \mathcal{W}/\xi$  the average energy gap between single-particle states localized within the same region of size  $\xi$ , which is assumed to be much larger than the lattice constant,  $\xi \gg a$ . This corresponds to  $J/h \gg 1$ , implying  $\xi \sim (J/h)^2$  and  $\mathcal{W} \approx J$ . However, as was pointed out recently [42], previous studies neglected the phenomenon of spectral diffusion [43, 44], which is expected to reduce the critical interaction strength in the weak disorder limit to  $U^* \propto \delta_\xi (\delta_\xi/\mathcal{W})$  (up to logarithmic corrections).

*Discussion and conclusion.* We have proposed and analyzed the presumably simplest possible protocol for quantum disordered magnets to exhibit the absence of ergodic dynamics, and thus Many-Body Localization in the form of remanent magnetization in initially ferromagnetically polarized antiferromagnets. The present calculation illustrates how our explicit recipe for constructing the conserved quantities allows for analytic predictions for quantities of experimental relevance. It is an analytical alternative to several numerical schemes based on DMRG [45–48] or quantum Monte Carlo [49] that allow one to study properties of specific MBL eigenstates. The simple formula (5) can be applied to the conserved pseudo-spins constructed numerically in Refs. [50–52] for non-perturbative interactions by means of renormalization procedures or diagonalizing flows. It would be interesting to extend this calculation beyond the lowest orders, possibly exploiting the expansion for the conserved quantities in the forward approximation [27, 53], to discuss the behavior of the typical value of the remanent magnetization when approaching the delocalization threshold.

An interesting question concerns the vanishing of the order parameter (16) at criticality. If it is continuous, it might exhibit non-trivial scaling with the system size, potentially reflecting multifractality of critical wave functions. However, numerical studies of the scaling of the entanglement entropy of small subsystems [36, 54] have suggested that at the delocalization transition long-time

averages of typical observables jump discontinuously, implying a discontinuity of the order parameter (remanent magnetization or imbalance). Given the difficulty in accessing numerically the relevant regimes, the prospect of probing these scenariï experimentally is appealing.

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- [56] At high ( $n$ 'th) order of perturbation theory, the method is computationally more efficient than perturbative diag-

- onalization of the exponentially many (in  $n$ ) eigenstates. Resonant regions, instead, require an exact diagonalization anyway, both for the construction of integrals  $I_k$ 's and for eigenstates.
- [57] One may also view this protocol as a quantum quench: First a high energy eigenstate of the Hamiltonian with  $J_x = 0$  is prepared, and then the quantum fluctuations  $J_x \sigma_k^x \sigma_{k+1}^x$  are switched on abruptly at time  $t = 0$ .
  - [58] In the perturbative setting, there is a natural mapping between the set of conserved operators  $I_k$  and the sites  $k$ , since  $I_k$  is a perturbation of  $\sigma_k^z$ .
  - [59] The amplitudes  $A_{\rho\tau}^{(j)}$  do not contribute at this order due to the particular choice of the initial state.
  - [60] In a more general model they involve essentially all products of operators acting on the resonant spot.
  - [61] See Supplemental Material, which includes Ref. [55]
  - [62] Note that in Ref. [31] a different scaling of the form  $1/\xi^2$  was obtained for the same quantity: This discrepancy with our result arises because amplitude fluctuations within a correlation length were neglected in that work.