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Enhancing Near-Field Radiative Heat Transfer with Si-based Metasurfaces

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We demonstrate in this work that the use of metasurfaces provides a viable strategy to largely tune and enhance near-field radiative heat transfer between extended structures. In particular, using a rigorous coupled wave analysis, we predict that Si-based metasurfaces featuring two-dimensional periodic arrays of holes can exhibit a room-temperature near-field radiative heat conductance much larger than any unstructured material to date. We show that this enhancement, which takes place in a broad range of separations, relies on the possibility to largely tune the properties of the surface plasmon polaritons that dominate the radiative heat transfer in the near-field regime.

Thermal radiation is one of the most ubiquitous physical phenomena. In recent years, there has been a renewed interest in this topic due to the confirmation of the prediction that radiative heat transfer can be drastically enhanced for bodies separated by small gaps [1, 2]. This occurs when the gap is smaller than the thermal wavelength (9.6 μm at room temperature) and it is due to the contribution of evanescent waves that dominate the near-field regime. The fact that this near-field radiative heat transfer (NFRHT) can overcome the far-field limit set by the Stefan-Boltzmann law has now been verified in a variety of experiments exploring different materials, geometrical shapes, and gaps ranging from micrometers to a few nanometers [3−17]. These experiments have also triggered off the hope that NFRHT could have an impact in different thermal technologies [18] such as thermophotovoltaics [19], heat-assisted magnetic recording [20, 21], scanning thermal microscopy [22−24], nanolithography [25], thermal management [26, 27] or coherent thermal sources [28, 29].

In this context, the question on the fundamental limits of NFRHT is attracting a lot of attention [30]. So far, the largest NFRHT enhancements in extended structures have been reported for polar dielectrics (SiC, SiO\textsubscript{2}, SiN, etc.), in which the NFRHT is dominated by surface phonon polaritons (SPhPs) [31, 32]. There has not been any report of an extended structure that has heat transfer coefficient exceeding that between two planar polar dielectric surfaces, and that includes metamaterials like hyperbolic ones [33, 34]. In an attempt to tune NFRHT, several calculations of NFRHT between periodic metallic nanostructures in both 1D [35-38] and 2D [39] have been reported. These calculations have shown some degree of tunability and a NFRHT enhancement over the corresponding material without nanostructuring. However, the reported NFRHT in these structures is still smaller than in the case of parallel plates made of polar dielectrics. There have also been theoretical studies of the NFRHT between photonic crystals and periodic metamaterials made of dielectrics [40−42] that show how the radiative properties can be enhanced with respect to the bulk counterpart. However, the resulting NFRHTs are again much smaller than in planar polar dielectrics.

In this Letter we show that metasurfaces of doped Si (see Fig. 1) can be used to boost NFRHT. Making use of a rigorous coupled wave analysis, we demonstrate that one can design Si metasurfaces that not only exhibit a room-temperature NFRHT much larger than that of bulk Si or other proposed periodic structures [39, 41, 42], but they also outperform the best unstructured polar dielectric (SiO\textsubscript{2}). By appropriately choosing the geometrical parameters of the metasurfaces, the enhancement over polar dielectrics occurs over a broad range of separations (from 13 nm to 2 μm). The underlying physical mechanisms responsible of this striking behavior are the existence of broad-band surface-plasmon polaritons (SPPs) in doped Si, and the ability to tune via nanostructuring the dispersion relation of these SPPs that dominate NFRHT in our structure. The predictions of this work show the great potential of metasurfaces for the field of NFRHT and can be tested with recent advances to measure NFRHT in parallel extended structures [16].

The system that we consider consists of two identical metasurfaces formed by 2D periodic arrays of square holes drilled in a doped Si layer, see Fig. 1. The metasurfaces are deposited on semi-infinite planar substrates. The geometrical parameters of the metasurfaces are the lattice constant \(a\), the distance between holes \(p\), the gap size \(d\), and the thickness of the metasurfaces \(\tau\), which is equal to the depth of the holes in the structure. We define the filling factor of this structure as \(f = (a - p)^2/a^2\), which describes the fraction of vacuum in the structure \((f = 0 \text{ means no holes}, \text{while } f = 1 \text{ means no Si})\). We focus on the analysis of the heat transfer coefficient, i.e., the linear radiative thermal conductance per unit of area, at room temperature (300 K). The dielectric function of doped Si is described within a Drude model [43]: \(\epsilon_{\text{Si}}(\omega) = \epsilon_{\infty} - \omega_p^2/(\omega^2 + i\gamma\omega)\), where \(\epsilon_{\infty} = 11.7\), \(\omega_p = 711\) meV is the plasma frequency, and \(\gamma = 61.5\) meV is the damping. These values correspond to a doping level of
$10^{20}$ cm$^{-3}$. The choice of this material and the doping level were motivated by the possibility to sustain SPPs at frequencies that can be thermally excited at room temperature. This is not possible for very high doping levels, while for very low ones the SPPs are not very confined and give a modest contribution to the NFRHT. Our main idea is that by introducing holes in the Si layers, one can reduce the losses and the effective plasma frequency, and give a modest contribution to the NFRHT. Our main focus is to test the idea that by introducing holes in the Si layers, one can reduce the losses and the effective plasma frequency. This way, these modes could be more easily occupied at room temperature leading to an enhancement of the NFRHT.

To test this idea we have combined the framework of fluctuational electrodynamics (FE) [2] with a rigorous coupled wave analysis (RCWA) [44] to compute NFRHT between periodic systems. In the frame of FE, the heat transfer coefficient (HTC) for two arbitrary periodic multilayer structures is given by [45]

$$h(d, T) = \frac{\int_0^\infty d\omega}{2\pi} \frac{\partial \Theta(\omega, T)}{\partial T} \int \frac{d\mathbf{k}_\parallel}{(2\pi)^2} T(\omega, \mathbf{k}_\parallel),$$  \hspace{1cm} (1)

where $\Theta(\omega, T) = h\omega/\exp[h\omega/k_B T] - 1$ is the mean energy of Planck oscillators at temperature $T$, $\omega$ is the angular frequency, $\mathbf{k}_\parallel = (k_x, k_y)$ is the wave vector parallel to the surface planes, and $T(\omega, \mathbf{k}_\parallel)$ is the sum over polarizations of the transmission probability of the electromagnetic waves. The second integral in Eq. (1) is carried out over all possible directions of $\mathbf{k}_\parallel$ and it includes the contribution of both propagating waves with $k_\parallel < \omega/c$ and evanescent waves with $k_\parallel > \omega/c$, where $k_\parallel$ is the modulus of $\mathbf{k}_\parallel$ and $c$ is the velocity of light in vacuum.

Within the RCWA approach, we express the fields in our periodic system as a sum of plane waves using Bloch theorem. Thus, the transmission function above can be obtained by combining scattering matrices of the different interfaces in reciprocal space. In particular, if we place the coordinates origin at metasurface 1, the transmission coefficient can be expressed as [45]

$$T(\omega, \mathbf{k}_\parallel) = \text{Tr} \left\{ \hat{D} \hat{W}_1 \hat{D}^\dagger \hat{W}_2 \right\},$$  \hspace{1cm} (2)

where

$$\hat{D} = (1 - \hat{S}_1 \hat{S}_2)^{-1},$$  \hspace{1cm} (3)
$$\hat{W}_1 = \hat{\Sigma}^{pw}_{-1} - \hat{S}_1 \hat{\Sigma}^{pw}_{-1} \hat{S}_1^\dagger + \hat{S}_1 \hat{\Sigma}^{ew}_{-1} - \hat{\Sigma}^{ew}_{-1} \hat{S}_1^\dagger,$$  \hspace{1cm} (4)
$$\hat{W}_2 = \hat{\Sigma}^{pw}_{+1} - \hat{S}_2 \hat{\Sigma}^{pw}_{+1} \hat{S}_2^\dagger + \hat{S}_2 \hat{\Sigma}^{ew}_{+1} - \hat{\Sigma}^{ew}_{+1} \hat{S}_2^\dagger.$$  \hspace{1cm} (5)

Here, $\hat{S}_1 = \hat{R}_1$ and $\hat{S}_2 = e^{ik_zd} \hat{R}_2 e^{ik_zd}$, where $\hat{R}_1$ and $\hat{R}_2$ are the reflection matrices of the two vacuum–metasurface interfaces and $k_z^2 = \omega^2/c^2 - k_\parallel^2$. These matrices were computed with the scattering-matrix approach of Ref. [44]. Moreover, the matrix $\Sigma^{pw}_{-1(+1)}$ is a projector into the propagating (evanescent) sector. All these matrices are $2N_g \times 2N_g$ matrices, where $N_g$ is the number of reciprocal lattice vectors included in the plane-wave expansions. On the other hand, the $\mathbf{k}$-integral in Eq. (1) must be calculated in the interval $(-\pi/a, \pi/a)$ for both $k_x$ and $k_y$. A key point in our method is the use of the so-called fast Fourier factorization when dealing with the Fourier transform of two discontinuous functions in the Maxwell equations [44, 46]. This factorization solves the known convergence problems of the RCWA approach.

Let us start the discussion of the results by illustrating the main finding of our work. For simplicity, we first assume that the Si layer thickness is infinite (no substrate). In Fig. 2(a) we show the room-temperature HTC as a function of the gap size for two metasurfaces with $a = 50$ nm and $f = 0.9$. This result is compared with the HTC for two doped-Si and two SiO$_2$ parallel plates. Notice that the NFRHT between the Si metasurfaces is more than an order of magnitude larger than the corresponding result for Si plates for a broad range of separations. More importantly, the Si metasurfaces also exhibit a higher HTC than the silica plates in a broad distance range (from 13 nm to 2 microns), an enhancement that reaches a factor 3 for gap sizes of about 100 nm. Let us emphasize that our structure exhibits a super-Planckian radiative heat transfer in that range of gaps (see arrow on the right). We also show in Fig. 2(a) the contribution of evanescent TM-polarized waves to the HTC of the doped-Si metasurfaces. Notice that this contribution dominates the NFRHT for separations below 1 $\mu$m, which is a first hint that surface modes are responsible for the enhancement in this regime.

To get further insight into the role of the nanostructure, we show in Fig. 2(b) the spectral HTC of the metasurfaces for a gap $d = 20$ nm, a lattice constant $a = 50$ nm, and for different filling factors. This spectral
FIG. 2. (Color online) (a) Room-temperature heat transfer coefficient (HTC) as a function of the gap size for doped-Si metasurfaces with \(a = 50\) nm and \(f = 0.9\) (black dashed line). For comparison, we show the results for the Si metasurfaces computed with effective medium theory (orange line), SiO\(_2\) parallel plates (blue line), and doped-Si parallel plates (red line). The green line shows the contribution of evanescent TM-polarized waves to the HTC of the Si metasurfaces and the arrow on the right indicates the blackbody limit, 6.1 W/(m\(^2\)K). (b) Spectral HTC as a function of the photon energy for a gap of \(d = 20\) nm and a lattice constant \(a = 50\) nm. The different lines correspond to different filling factors.

HTC is defined as the HTC per unit of frequency or photon energy. As one can see, the maximum of the spectral HTC is redshifted upon increasing the size of the holes from 0.2 eV for \(f = 0\) up to around 0.05 eV for \(f = 0.98\). Notice also that the HTC (the integral of these spectral functions) also increases drastically with the filling factor reaching a maximum at \(f \approx 0.98\). These results illustrate the high tunability of NFRHT in metasurfaces.

The origin of the redshift in the spectral HTC can be understood with an analysis of the frequency and parallel wave vector dependence of the transmission \(T(\omega, k_\parallel)\). Such a dependence is displayed in Fig. 3 for p-polarized waves, which dominate the NFRHT. In particular, we show the transmission along the \(x\)-direction \([k_\parallel = (k_x, 0)]\), see Fig. 1, for \(d = 20\) nm, \(a = 50\) nm, and different filling factors. As one can see in Fig. 3(a) for the case of two Si parallel plates \((f = 0)\), the transmission maxima resemble the dispersion relation of a surface mode. As shown below, it corresponds to a cavity SPP mode that emerges from the hybridization of the SPP modes of the two vacuum-Si interfaces. Notice that the transmission maxima lie to the right of the light line (or dispersion relation of light in vacuum), which indicates that these modes correspond to evanescent waves (both in vacuum and inside Si). As the filling factor increases, we find that the transmission maxima redshift, see Fig. 3(b-c), which is consistent with our observation above about the spectral HTC. Indeed, the frequency at which the maxima of the spectral HTC occur, see Fig. 2(b), corresponds exactly to the position at which the transmission maxima fold back towards the light line. The reason is that in that frequency region the transmission is not only maximum, but also \(k_\parallel\) takes the largest value, maximizing thus the density of photonic modes.

To confirm that cavity SPPs are indeed responsible for the NFRHT in our structure, we have analyzed their dispersion relation. For this purpose, we have made used of an effective medium theory (EMT) [47]. Within this theory our metasurfaces can be modeled as uniaxial materials with a diagonal permittivity tensor: \(\hat{\epsilon} = \text{diag}(\epsilon_\text{o}, \epsilon_\text{e})\), where the subindex o and e denote the ordinary and extraordinary optical axis, respectively.

The components of the dielectric tensor are given by [47]

\[
\epsilon_\text{o} = \epsilon_\text{Si}(1 - f) + 1 + f, \quad \epsilon_\text{e} = \frac{\epsilon_\text{Si}(1 + f)}{1 + 1 - f^2}, \quad \epsilon = f + (1 - f)\epsilon_\text{Si}. \tag{6}
\]
In such an system, light propagates along the optical axes with perpendicular components of the wave vector given by $k_{x,0}^2 = \epsilon_0 \omega^2 / c^2 - k_y^2$ and $k_{z,0}^2 = \epsilon_0 \omega^2 / c^2 - k_z^2/\epsilon_0/\epsilon_r$.

Within this approximation, the SPP dispersion relation is given by the solution of the following equation [48]

$$e^{ik_zd} = \pm \left( \frac{k_{x,0}^2 - \epsilon_0 k_z^2}{k_{x,0}^2 + \epsilon_0 k_z^2} \right). \quad (7)$$

In the electrostatic limit ($k_0 \gg \omega/c$), this equation leads to the following dispersion relation for the cavity SPPs

$$k_{\text{SPP}}(\omega) = \frac{1}{d} \ln \left( \frac{\epsilon_0(\omega) - \sqrt{\epsilon_0(\omega)/\epsilon_0(\omega)}}{\epsilon_0(\omega) + \sqrt{\epsilon_0(\omega)/\epsilon_0(\omega)}} \right). \quad (8)$$

The exact dispersion relations obtained from Eq. (7) are shown in Fig. 3 as red dashed lines. These dispersion relations nicely coincide with the transmission maxima for the whole range of filling factors, which unambiguously demonstrates that cavity SPPs dominate the NFRHT in our system. Moreover, this shows that NFRHT is drastically enhanced upon increasing the filling factor because the surfaces modes shift to lower frequencies, which increases their thermal occupation at room temperature. This is illustrated in Fig. 3 where we show the frequency dependence of the factor $\partial \Theta(\omega,T) / \partial T$ that determines the occupation of these surface modes.

The enhancement of NFRHT in our metasurfaces over polar dielectrics like SiO$_2$ can be understood as follows. As we show in Fig. 3(d), the transmission between two silica plates is dominated by SPhPs [31], whose dispersion relation is given by Eq. (8) with $\epsilon_0 = \epsilon_r = \epsilon_{\text{SiO}_2}$ [13]. Although these modes exhibit larger $k_0$-values than the SPPs in our Si metasurfaces and therefore larger photonic density of states, they are restricted to rather narrow frequency regions corresponding to the two Reststrahlen bands in this material. Thus, the larger extension in frequency of the SPPs in the Si structures is one of the key factors that leads to a higher NFRHT.

The fact that the EMT nicely describes the position of the transmission maxima raises the question of whether this theory can also accurately describe the NFRHT. This is actually not the case because EMT assumes that the geometrical features are much smaller than the relevant physical length scales of the problem. In our case, where there is a considerable damping, the natural lateral scale is set by the propagation length of the cavity SPP wavelengths, $1/(2 \Im{k_{\text{SPP}}})$. For frequencies close to the folding back of the dispersion relation, which are the ones that dominate the spectral HTC, this propagation length can obtained from Eq. (8). Thus for instance, for the structure analyzed in Fig. 2(a), Eq. (8) predicts that the SPP propagation length for the frequency of the spectral HTC maximum becomes of the order of the lattice parameter for $d \approx 100$ nm. Thus, the EMT is expected to fail below this gap size. To confirm this idea, we have computed the HTC in this structure within the EMT using the formalism for anisotropic planar systems of Ref. [48] and the result is shown in Fig. 2(a). As one can see, the EMT fails for gaps below the SPP propagation length. This analysis illustrates the need of an exact approach to accurately predict the NFRHT in these metasurfaces. Besides, the agreement between EMT results and the exact calculations for large gaps is an evidence of the validity of our approach.

Let us stress that our periodic structures truly behave as metasurfaces, which can be understood as follows. From Eq. (8) we can estimate the penetration depth of the cavity SPPs, which in the electrostatic limit is given by $1/(2 \Re{k_{\text{SPP}}})$. Thus, we see that this penetration depth diminishes as the gap is reduced, which implies that the NFRHT is dominated by the surface of the periodic structures. Thus for instance, for $d = 20$ nm, $a = 50$ nm, and $f = 0.9$ the penetration depth estimated from Eq. (8) is about 27 nm for the frequency of the spectral HTC maximum. Thus, one expects that all periodic structures with Si layers thicker than this penetration depth behave in the same way. To test this idea we have computed the HTC as a function of the thickness of the periodic Si layers, $\tau$, assuming that the substrate underneath is also made of (unstructured) doped Si. In Fig. 4 we show the results for this thickness dependence for the case mentioned above. Notice that when the layer thickness becomes larger than the gap, which is comparable to the SPP penetration depth, the HTC quickly tends to the result for a semi-infinite structure. This behavior is illustrated in the inset of Fig. 4 with the corresponding spectral HTCs. Thus, we can conclude that our periodic structures effectively behave as true metasurfaces as long as their thickness is larger than the gap size.

In summary, we have proposed a novel mechanism to further enhance NFRHT with the use of Si metasurfaces, which is based on the broad spectral bandwidth and high tunability of the SPPs that dominate NFRHT in these structures. We have shown that these metamaterials can...
exhibit room-temperature near-field radiative heat conductances higher than any existent or proposed structure. The fabrication of these metasurfaces is feasible with the state-of-the-art nanolithography [49] and our predictions could be tested with the recent developments in the measurement of NFRHT in parallel extended structures [16].

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