# Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

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# Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

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## Abstract

Applying the connection between the parton Wigner distribution and orbital angular momentum (OAM), we investigate the probe of the gluon OAM in hard scattering processes at the planned electron-ion collider. We show that the single longitudinal target-spin asymmetry in the hard diffractive dijet production is very sensitive to the gluon OAM distribution. The associated spin asymmetry leads to a characteristic azimuthal angular correlation of $\sin(\phi_q - \phi_\Delta)$, where $\phi_\Delta$ and $\phi_q$ are the azimuthal angles of the proton momentum transfer and the relative transverse momentum between the quark-antiquark pair. This study will motivate a first measurement of the gluon OAM in the proton spin sum rule.

## I. INTRODUCTION

In the past three decades, we have witnessed significant advances in the understanding of high-energy hadron structure. Great progress has been made in measuring the partonic content of proton spin, as results from SLAC, CERN, DESY, JLab and RHIC have nailed the quark spin contribution to about 30% [1–3]. More recently, RHIC experiments have revealed that the gluon polarization contributes about 40% within the kinematic range of $0.05 \leq x \leq 0.2$ [4], which is an important part of the proton spin sum rule [5]. With the completion of JLab 12 GeV upgrade and implementation of the Electron Ion Collider (EIC), the proton spin structure will be studied to an unprecedented extent with higher precision. Among them, the major focus will be the gluon helicity distribution at smaller $x$, and in particular, the orbital angular momenta (OAM) from the quarks and gluons [6, 7]. The latter play important roles in the partonic structure in nucleon, not only for the proton spin sum rule, but also for the novel phenomena in various high energy scattering processes. It has been shown in [8] that the total angular momentum contributions from the quarks and gluons can be studied through the associated generalized parton distributions (GPDs) [9–11] measured in the hard exclusive processes, such as the Deeply Virtual Compton Scattering (DVCS) [8, 10]. By subtracting the helicity contributions, we will be able to obtain the corresponding OAM contributions from the quarks and gluons.

Recent developments have also unveiled the close connection between the parton OAM and the associated quantum phase space distributions, the so-called Wigner distribution functions [12–16],

$$L_{q,g}(x) = \epsilon_\perp^{\alpha\beta} \left.\frac{\partial}{i\partial\Delta_\perp^\alpha}\right|_{\Delta=0} \int d^2k_\perp \; k_\perp^\beta f_{q,g}(x,\xi,k_\perp,\Delta_\perp) \ , \tag{1}$$

where $f_{q,g}$ represent the quark/gluon Wigner distributions in a longitudinal polarized nucleon, and $\epsilon_\perp^{\alpha\beta}$ represents 2-dimensional Levi-Civita symbol. We focus on the gluon Wigner distribution with light-cone gauge links and the corresponding OAM belongs to the Jaffe-Manohar spin sum rule [17, 18]. The Wigner distributions are also referred to as the generalized transverse momentum dependent parton distributions [19]. This opens a new window to directly access the parton OAM contributions to the proton spin. The goal of this paper is to show that indeed that we can probe the gluon OAM distribution through the hard scattering processes in high energy lepton-nucleon collisions, in particular, at the EIC.

We take the example of the single longitudinal target-spin asymmetries in hard exclusive dijet production in lepton-nucleon collisions [20],

$$\ell + p \to \ell' + q_1 + q_2 + p' \ , \tag{2}$$

where the incoming and outgoing leptons have momenta $l$ and $l'$, proton momenta with $p$ and $p'$, and the final state two jets with momenta $q_1$ and $q_2$, as illustrated in Fig. 1. In high energy experiments at the EIC, the process of (2) is dominated by the gluon distribution from the target nucleon, and in particular, the differential cross section will depend on the gluon Wigner distribution [21]. Because of the relation of Eq. (1), one expects that the single longitudinal target-spin asymmetry of this process will be an ideal probe to the gluon OAM. To show this explicitly, we perform our calculations in a general collinear factorization framework, where the gluon OAM distribution enters at the twist-three level. The spin dependent differential cross section has a characteristic azimuthal angular dependence of

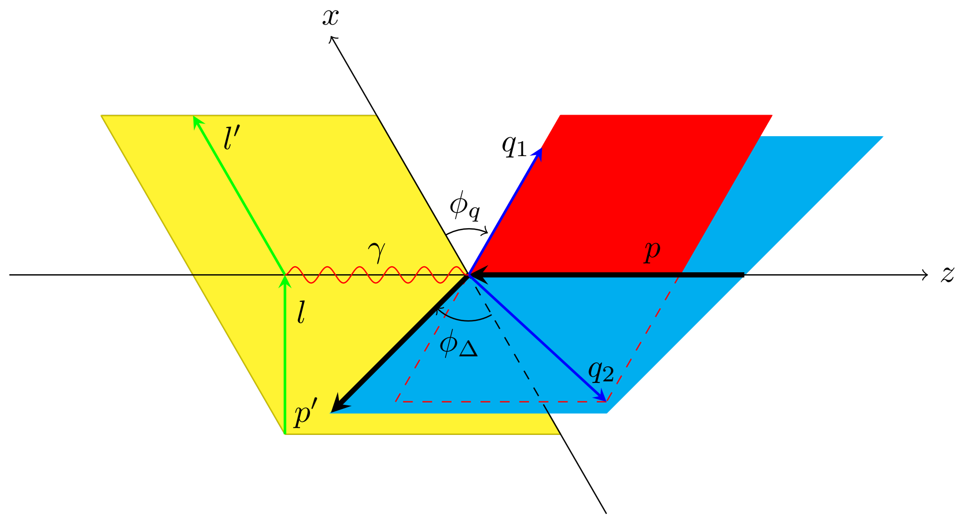


FIG. 1. Hard exclusive dijet production in deep inelastic scattering to probe the gluon orbital angular momentum.

$\sin(\phi_\Delta - \phi_q)$ where $\phi_\Delta$ and $\phi_q$ are the azimuthal angles of the proton momentum transfer and the relative transverse momentum between the quark pair as shown in Fig. 1. With a hermetic detector designed for the EIC, this observable can be well studied in the future, and will help us to finalize the proton spin sum rule, the ultimate goal for hadron physics in past decades.

There has been an argument of strong constraint on the gluon OAM due to the smallness of the Sivers single transverse spin asymmetries in semi-inclusive DIS from COMPASS experiments [22]. However, we would like to emphasize that the Sivers effect does not provide a direct access to the gluon OAM. The goal of this paper is to propose a direct measurement. Our approach and observables are also different from other proposals to measure the parton OAMs [23, 24]. In particular, we focus on the hard scattering processes which can be well studied at the planed EIC. The rest of this paper is organized as follows. In Sec. II, we derive the single longitudinal target-spin asymmetry in hard exclusive dijet production in lepton-nucleon collisions. We take the leading contribution from the gluon OAM distribution in the nucleon. We summarize our results and comment on further developments in Sec. III.

## II. GLUON OAM CONTRIBUTION TO THE SINGLE SPIN ASYMMETRIES

The differential cross section of process (2) can be calculated through the lepton tensor and hadronic tensor,

$$|\mathcal{M}|^2 = L_{\mu\nu} H^{\mu\nu} \ , \tag{3}$$

where the lepton tensor takes a simple form of $L_{\mu\nu} = 2(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l \cdot l')$ due to the fact that the incoming lepton is unpolarized. The main task of our calculation is to evaluate the hadronic tensor, which comes from the Feynman diagrams illustrated in Fig. 2. We adopt the usual kinematics: the incoming photon with momentum $q = l - l'$, $q^2 = -Q^2$, $x_{Bj} =$

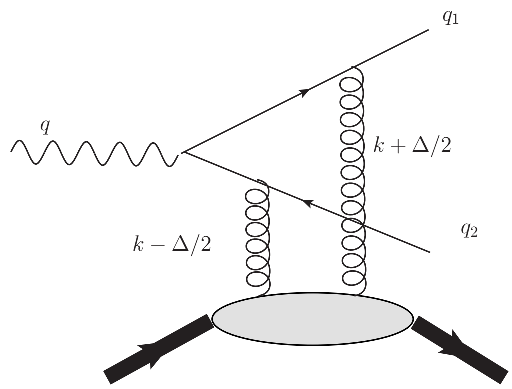


FIG. 2. Generic Feynman diagram to evaluate the single longitudinal spin asymmetry in the hard exclusive dijet production in deep inelastic lepton nucleon scattering processes. All possible gluon attachment has been included in our calculations.

$Q^2/(2q \cdot p)$, $y = q \cdot p/(l \cdot p)$. The quark and antiquark momenta are further parameterized by their longitudinal momentum fractions $z$ and $\bar{z} = 1-z$ as well as their transverse momenta $q_\perp - \Delta_\perp/2$ and $-q_\perp - \Delta_\perp/2$. In addition, for the exclusive processes, we have the following kinematics: $\Delta = p' - p$, $P = (p+p')/2$, $t = \Delta^2$, $(q+p)^2 = W^2$, $(q-\Delta)^2 = (q_1+q_2)^2 = M^2$, and the skewness parameter is defined as $\xi = (p^+ - p'^+)/(p^+ + p'^+)$ with $p^\pm = (p^0 \pm p^z)/\sqrt{2}$, where $q$ and $p$ are chosen to be along the $z$ axis. As shown in Fig. 1, the lepton plane is set as the $x-z$ plane. The quark pair are in one plane with azimuthal angle $\phi_q$ respect to the lepton plane, whereas the recoiled proton is in another plane with momentum transfer $\vec{\Delta}_\perp$ and azimuthal angle $\phi_\Delta$. The spin-average cross section for this process has been calculated in Ref. [20]. In the following, we will compute the single longitudinal target-spin asymmetry. We will show how this asymmetry can be related to the gluon OAM contributions.

Generically, the single longitudinal spin asymmetry in the above process can be evaluated following the usual collinear expansion at the next-to-leading power. We write the scattering amplitude, depicted in Fig. 2, as

$$i\mathcal{A}_f \propto \int dx d^2k_\perp \mathcal{H}(x, \xi, q_\perp, k_\perp, \Delta_\perp)\ x f^g(x, \xi, k_\perp, \Delta_\perp)\ , \tag{4}$$

where $q_\perp$ is the jet transverse momentum defined above, and $k_\perp$ is the gluon transverse momentum entering the hard partonic part of Fig. 2. In this calculation, $q_\perp$ is the same order of $Q$, while the nucleon recoil momentum $\Delta_\perp$ is much smaller than $Q$. In the twist analysis, we expand the scattering amplitude in terms of $k_\perp/q_\perp$ (or $k_\perp/Q$),

$$\mathcal{H}(x, \xi, q_\perp, k_\perp, \Delta_\perp) = \mathcal{H}^{(0)}(x, \xi, q_\perp, 0, \Delta_\perp) + k_\perp^\alpha \frac{\partial}{\partial k_\perp^\alpha} \mathcal{H}(x, \xi, q_\perp, 0, \Delta_\perp) + \cdots\ . \tag{5}$$

For the spin-average cross section, we take the zero-th order expansion of $k_\perp$. As a result, $k_\perp$ is integrated out for the gluon Wigner distribution,

$$\int d^2k_\perp x f^g(x, \xi, k_\perp, \Delta_\perp) = F_g(x, \xi, \Delta_\perp)\ , \tag{6}$$

where $F_g$ is the spin-average gluon GPD. The scattering amplitude can be written as

$$i\mathcal{A}_f^{(0)} \propto \int dx \mathcal{H}^{(0)}(x, \xi, q_\perp, 0, 0)\ x F_g(x, \xi, \Delta_\perp)\ . \tag{7}$$

Because $\Delta_\perp \ll q_\perp$, we have also taken $\Delta_\perp = 0$ in the hard partonic part. This will enter into the spin-average cross section contribution, e.g., Eq. (9) below.

On the other hand, the single longitudinal target-spin asymmetry comes from the next-to-leading power expansion of Eq. (5). Because of the nontrivial correlation between $k_\perp$ and $\Delta_\perp$ in the gluon Wigner distribution due to the gluon orbital motion, this contribution will lead to a novel correlation between $q_\perp$ and $\Delta_\perp$ as mentioned in Introduction,

$$\int d^2k_\perp(\vec{q}_\perp \cdot \vec{k}_\perp)xf^g(x,\xi,k_\perp,\Delta_\perp) = -iS^+(\vec{q}_\perp \times \vec{\Delta}_\perp)xL_g(x,\xi,\Delta_\perp) + \cdots \ , \tag{8}$$

where we have only kept the spin-dependent matrix element in the above equation and $S^+$ represents the longitudinal spin, and we have taken the leading contribution in terms of $(\vec{q}_\perp \cdot \vec{k}_\perp)$ in $\mathcal{H}$. We refer the above $L_g(x,\xi,\Delta_\perp)$ as the gluon OAM distribution, from which we shall be able to obtain the gluon OAM contribution to the proton spin from Eq. (1). According to this result, we only need to measure how the single target-spin asymmetry modulates with $\sin(\phi_q - \phi_\Delta)$— which comes from $(\vec{q}_\perp \times \vec{\Delta}_\perp)$—to extract the gluon OAM density.

The detailed derivations will be presented in a separate publication. Here, we present the main results and demonstrate the sensitivity of the spin asymmetries on the gluon OAM distribution. For the spin-average cross section, we have the following expression [20],

$$\frac{d\sigma}{dydQ^2d\Omega} = \sigma_0\left[(1-y)|A_L|^2 + \frac{1+(1-y)^2}{2}|A_T|^2\right] \ , \tag{9}$$

where $d\Omega$ represents the final hadronic states phase space: $d\Omega = dzdq_\perp^2 d\Delta_\perp^2 d\phi_{q\Delta}$. $\sigma_0$ is defined as

$$\sigma_0 = \frac{\alpha_{em}^2\alpha_s^2 e_q^2}{16\pi^2Q^2yN_c}\frac{4\xi^2 z\bar{z}}{(1-\xi^2)(\vec{q}_\perp^2+\mu^2)^3} \ , \tag{10}$$

where $\mu^2 = z\bar{z}Q^2$, and we have only kept the azimuthal angular symmetric terms in the above result and $\phi_{q\Delta} = \phi_q - \phi_\Delta$. The contributions from the transverse and longitudinal photons are: $|A_L|^2 = 4\beta\left|\mathcal{F}_g + 4\xi^2\bar{\beta}\mathcal{F}_g'\right|^2$, $|A_T|^2 = \bar{\beta}\left(1/(z\bar{z})-2\right)\left|\mathcal{F}_g + 2\xi^2(1-2\beta)\mathcal{F}_g'\right|^2$, where $\beta = \mu^2/(\mu^2+\vec{q}_\perp^2)$. We have defined the following generalized Compton form factors,

$$\begin{aligned}\mathcal{F}_g(\xi,t) &= \int dx \frac{1}{(x+\xi-i\varepsilon)(x-\xi+i\varepsilon)}F_g(x,\xi,t) \ ,\\ \mathcal{F}_g'(\xi,t) &= \int dx \frac{1}{(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}F_g(x,\xi,t) \ .\end{aligned} \tag{11}$$

Following the above procedure, we derive the longitudinal target-spin dependent differential cross section,

$$\frac{d\Delta\sigma}{dydQ^2d\Omega} = \sigma_0\lambda_p\frac{2(\bar{z}-z)(\vec{q}_\perp \times \vec{\Delta}_\perp)}{\vec{q}_\perp^2+\mu^2}\left[(1-y)A_{fL} + \frac{1+(1-y)^2}{2}A_{fT}\right] \ , \tag{12}$$

where $\Delta\sigma = (\sigma(S^+) - \sigma(-S^+))/2$ and $\lambda_p$ represents the longitudinal polarization for the incoming nucleon. The spin-dependence comes from the interferences between the leading-twist and and twist-three amplitudes,

$$\begin{aligned}A_{fL} &= 16\beta\ \mathrm{Im}\left(\left[\mathcal{F}_g^* + 4\xi^2\bar{\beta}\mathcal{F}_g'^*\right]\left[\mathcal{L}_g + 8\xi^2\bar{\beta}\mathcal{L}_g'\right]\right) \ ,\\ A_{fT} &= 2\ \mathrm{Im}\left(\left[\mathcal{F}_g^* + 2\xi^2(1-2\beta)\mathcal{F}_g'^*\right]\left[\mathcal{L}_g + 2\bar{\beta}\left(\frac{1}{z\bar{z}}-2\right)\left(\mathcal{L}_g + 4\xi^2(1-2\beta)\mathcal{L}_g'\right)\right]\right) \ ,\end{aligned} \tag{13}$$

where, again, we have defined the following Compton form factors to simplify the final results,

$$\mathcal{L}_g(\xi,t) = \int dx \frac{x\xi}{(x+\xi-i\varepsilon)^2(x-\xi+i\varepsilon)^2}\; xL_g(x,\xi,t)\;,$$
$$\mathcal{L}'_g(\xi,t) = \int dx \frac{x\xi}{(x+\xi-i\varepsilon)^3(x-\xi+i\varepsilon)^3}\; xL_g(x,\xi,t)\;. \tag{14}$$

The above equations are the main results of our paper. Clearly, because of the pre-factor of Eq. (12), we find that the spin asymmetry is a power correction, which is consistent with our analysis. In addition, it is proportional to $(\vec{q}_\perp \times \vec{\Delta}_\perp)$, so that it has the characteristic azimuthal angular correlation $\sin(\phi_q - \phi_\Delta)$.

In order to observe the above spin asymmetry, we need to distinguish the two final state jets. This can be achieved by identifying the charge of the leading hadron in the jet, or by measuring the heavy quark pair through their decay products. For the latter process, we have to consider the mass effects, which can be straightforwardly taken into account. Similar calculations can be performed for the quark channel contributions, which may play important roles in the large-$x$ region. We will leave that for a future publication.

## III. DISCUSSION AND SUMMARY

As shown above, the gluon OAM contribution to the single longitudinal target-spin asymmetry has the novel azimuthal angular correlation of $\sin(\phi_q - \phi_\Delta)$. Experimentally, we have to identify the azimuthal angles of both $\vec{q}_\perp$ and $\vec{\Delta}_\perp$. In particular, it is challenging to precisely measure $\Delta_\perp$, because majority of the events will have small momentum transfer. Fortunately, the current design for the EIC detector will have excellent coverage to study the $\Delta_\perp$ distribution in the hard diffractive processes, including the proposed measurement of this paper, especially with the Roman Pot device along the beam line of the EIC. With the measurements of these two azimuthal angels, we can form the spin asymmetry,

$$A_{\sin(\phi_q-\phi_\Delta)} = \int d\phi_q d\phi_\Delta\; \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\phi_q d\phi_\Delta} \sin(\phi_q-\phi_\Delta) \Big/ \int d\phi_q d\phi_\Delta\; \frac{d\sigma_\uparrow + d\sigma_\downarrow}{d\phi_q d\phi_\Delta}\;. \tag{15}$$

From the results in the last section, we know that the above asymmetry will be sensitive to the gluon OAM distribution, and has the following kinematic dependence, schematically,

$$A_{\sin(\phi_q-\phi_\Delta)} \propto \frac{(\bar{z}-z)|\vec{q}_\perp||\vec{\Delta}_\perp|}{\vec{q}_\perp^2+\mu^2}\;, \tag{16}$$

where again, it is a twist-three effect. The size of the asymmetry, of course, will depend on how large the gluon OAM is. Therefore, the experimental measurement of this asymmetry will provide direct access to the gluon OAM distribution. The unique angular correlation between the jet transverse momentum and the nucleon's recoiled momentum will help to identify the above asymmetry. We would like to emphasize that even if the asymmetry turns out small, it shall provide a strong constraint on the gluon OAM distribution. We are planing to have model predictions for the typical kinematics at the EIC, and hope this will lead to the first measurement of gluon OAM in the future.

In summary, we have calculated the differential cross section for the hard exclusive electroproduction of quark-antiquark pair. The leading contribution to the single target-spin asymmetry is at order $1/Q$, and crucially depends on the gluon OAM. In the kinematics covered by the EIC, this observable can be well studied, which will provide important information on the gluon OAM distribution in return.

Being a higher-twist effect, the asymmetry defined in Eq. (15) will have contributions from the twist-three multi-parton GPDs, in particular, those associated with three-gluon correlations [17, 18]. For a complete evaluation of the single longitudinal target-spin asymmetries, we need to include these terms as well. We would like to emphasize that the sensitivity to the gluon OAM distribution will remain with the complete calculation. As we have shown above, the gluon OAM contribution leads to the unique angular correlation between the jet transverse momentum and the nucleon's recoiled momentum: $(\vec{q}_\perp \times \vec{\Delta}_\perp)$, which is different from the twist-three effects in the DVCS process (see, e.g., Ref. [25]). The latter process can not have this kind of correlation because there is only one independent transverse momentum due to the fact that the transverse momentum of the final state photon equals in size but opposite in direction to the nucleon's recoiled momentum. Extension to other processes, such as the DVCS, will be also important to build a systematic framework to investigate the comprehensive tomography of partons inside the nucleons. We will address these issues in the future.

We notice that the ZEUS collaboration has recently published their study of the exclusive dijet production in diffractive lepton-nucleon collisions at HERA [26]. These measurements shall provide important guidelines for future experiments at the planed EIC. The experimental observation from Ref. [26] will also stimulate further theoretical developments to understand the QCD dynamics associated with this process, in particular, to consolidate the criteria for the exclusive dijet production, which is essential to our proposal to probe the gluon orbital angular momentum. We anticipate more theoretical investigations along this line.

In this paper, we focus on moderate $x$ range of the gluon OAM distribution. In the large-$x$ region, we will also have quark-channel contributions, which can be used to probe the quark OAM. At small-$x$, on the other hand, we would expect the dipole framework is more appropriate to compute the process (2) [21]. However, the spin asymmetry is much more involved and nontrivial, which will be addressed in an accompanying paper by Hatta, Nakagawa, and the two of us [27].

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