Chiral Shock Waves

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We study the shock waves in relativistic chiral matter. We argue that the conventional Rankine-Hugoinot relations are modified due to the presence of chiral transport phenomena. We show that the entropy discontinuity in a weak shock wave is quadratic in the pressure discontinuity when the effect of chiral transport becomes sufficiently large. We also show that rarefaction shock waves, which do not exist in usual nonchiral fluids, can appear in chiral matter. The direction of shock wave propagation is found to be completely determined by the direction of the vorticity and the chirality of fermions. These features are exemplified by shock propagation in dense neutrino matter in the hydrodynamic regime.

Introduction.—Recently, relativistic chiral matter has attracted great interest both theoretically and experimentally. Chiral matter is considered to be realized in a wide range of systems from the electroweak plasmas in the early Universe [1, 2], quark-gluon plasmas in heavy ion collisions [3, 4], Weyl (semi)metals [5–8], and electron plasmas in neutron stars [9–11] to neutrino media in core-collapse supernova explosions [12, 13]. A remarkable property of chiral matter is the presence of unusual transport phenomena related to quantum anomalies in field theory [14, 15], called the chiral magnetic effect (CME) in a magnetic field [4, 5, 16, 17] and chiral vortical effect (CVE) in a vorticity [18–21]. These chiral transport phenomena lead to new types of collective modes, such as the chiral magnetic wave [22, 23], chiral vortical wave [24], chiral Alfvén wave [25], chiral heat wave [26], and the chiral plasma instability [1, 2, 27]; see also Refs. [28, 29] for recent related works.

In this paper, we study the shock propagation in relativistic chiral matter. We first argue that the so-called Rankine-Hugoniot relations, or the jump conditions at the shock front [30], must be modified by the presence of chiral transport phenomena. This in turn leads to modifications of the basic properties of weak shock waves in chiral matter. Our main findings are summarized as follows:

- The dependence of entropy discontinuity at the shock front on the corresponding pressure discontinuity is quadratic, $\Delta S \propto (\Delta p)^2$ [see Eq. (43)], when the effect of chiral transport becomes sufficiently large. This should be contrasted with the behavior $\Delta S \propto (\Delta p)^3$ in nonchiral matter [30].

- Rarefaction shock waves can appear in chiral matter for a sufficiently large vorticity $\omega \gg \omega_{\rm c}$, where $\omega_{\rm c}$ is defined in Eq. (42). This should be contrasted with the fact that rarefaction shock waves are usually prohibited in nonchiral matter [30, 31].

- For a given chirality of fermions, the direction of shock wave propagation is completely determined by the direction of the vorticity.

We exemplify these features by studying shock waves in dense charge neutral chiral matter in the hydrodynamic regime. Such a situation is realized, e.g., by neutrino media at the core of supernovae [12]. These qualitatively new aspects of shock waves in chiral matter may have possible relevance, e.g., to the dynamics of supernovae. Although our argument does in part depend on this particular case of chiral matter, we expect that the qualitative features of our result is more generic. We discuss the possible applications of our arguments and results to other systems in the conclusion.

Chiral hydrodynamics.—Our starting point is the relativistic chiral hydrodynamics [20]. With keeping a specific application to the neutrino media in supernovae [12] in mind, we consider chiral hydrodynamics for charge neutral chiral matter. Our argument can be extended to charged chiral matter in an external electromagnetic fields in a straightforward manner. For simplicity, we will here ignore the dissipative effects.

The equations of relativistic chiral hydrodynamics for a single (right- or left-handed) chiral fermion are given by energy-momentum conservation and particle number conservation as

$$\partial_\mu T^{\mu\nu} = 0, \tag{1}$$
$$\partial_\mu j^\mu = 0. \tag{2}$$

Here the energy-momentum tensor $T^{\mu\nu}$ and particle number current j^μ are given in the Landau-Lifshitz frame by [20][1]

$$T^{\mu\nu} = hu^\mu u^\nu - pg^{\mu\nu}, \tag{3}$$
$$j^\mu = nu^\mu + \xi\omega^\mu, \tag{4}$$

[1] We use the metric $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ in this paper.

where $h = \epsilon + p$ is the enthalpy density, n is the particle number density, $u^\mu = \gamma(1, \boldsymbol{v})$ is the fluid velocity with $\gamma = (1 - \boldsymbol{v}^2)^{-1/2}$, and $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu \partial_\lambda u_\rho$ is the fluid vorticity.

What is different in the neutral chiral hydrodynamics from the conventional nonchiral hydrodynamics [30] is the presence of the CVE proportional to ω^μ in Eq. (4) [18–21]. The transport coefficient ξ takes the form of [20, 21, 33]

$$\xi = C\mu^2 \left(1 - \frac{2}{3}\frac{n\mu}{h}\right) + DT^2 \left(1 - \frac{2n\mu}{h}\right), \tag{5}$$

where μ is the chemical potential and T is the temperature. The coefficients C and D are related to those of the chiral anomaly and mixed gauge-gravitational anomaly as [20, 21, 34, 35]

$$C = \pm\frac{1}{4\pi^2}, \qquad D = \pm\frac{1}{12}, \tag{6}$$

for right- and left-handed chiral fermions, respectively.

In the following, we will focus on the regime $\mu \gg T$ for demonstration. This is relevant to, e.g., the dense neutrino matter at the core of supernovae [12]. Note however that our argument itself is not limited to this regime and is applicable to other regimes as well. For a relativistic gas of noninteracting fermions (which is a reasonable assumption for a neutrino gas), the expressions of n, p, ϵ, and the entropy S are given by

$$n = \frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6}, \tag{7}$$

$$p = \frac{\epsilon}{3} = \frac{\mu^4}{24\pi^2} + \frac{\mu^2 T^2}{12}, \tag{8}$$

$$S = \frac{\pi^2 T}{\mu}, \tag{9}$$

to the leading corrections in $T/\mu \ll 1$. From Eqs. (7) and (8), the transport coefficient ξ in Eq. (5) in the regime $\mu \gg T$ reduces to

$$\xi \approx \frac{1}{3}C\mu^2 + \left(\frac{2\pi^2}{3}C - D\right)T^2. \tag{10}$$

Shock waves in relativistic nonchiral matter.—We first revisit the properties of shock waves in relativistic nonchiral matter [30] before we analyze shock waves in chiral matter. Consider a relativistic gas of particles moving along the x axis towards the positive x direction and that there is a surface of discontinuity perpendicular to the direction of propagation of the gas. This surface of discontinuity divides the three dimensional space into two regions, side 1 and side 2. The sides are defined in such a way that the gas moves from side 1 to side 2.

Imposing continuity in particle number flux, energy and momentum flux, $j_1^x = j_2^x$, $T_1^{xx} = T_2^{xx}$, and $T_1^{0x} = T_2^{0x}$, we have the following three equations relating the two sides [30],

$$\frac{v_1\gamma_1}{V_1} = \frac{v_2\gamma_2}{V_2}, \tag{11}$$

$$h_1 v_1^2\gamma_1^2 + p_1 = h_2 v_2^2 \gamma_2^2 + p_2, \tag{12}$$

$$h_1 v_1 \gamma_1^2 = h_2 v_2 \gamma_2^2, \tag{13}$$

where the subscripts 1 and 2 stand for the sides 1 and 2 and V is the volume per particle, $V \equiv 1/n$. Equations (11)–(13) constitute the Rankine-Hugoniot relations for shock propagation in nonchiral matter.

By solving Eqs. (12) and (13) in terms of v_1 and v_2, we have [30]

$$v_1 = \sqrt{\frac{(p_2 - p_1)(\epsilon_2 + p_1)}{(\epsilon_2 - \epsilon_1)(\epsilon_1 + p_2)}}, \tag{14}$$

$$v_2 = \sqrt{\frac{(p_2 - p_1)(\epsilon_1 + p_2)}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + p_1)}}. \tag{15}$$

Substituting these expressions into Eq. (11), we obtain the pressure-volume relation [30]:

$$h_1^2 V_1^2 - h_2^2 V_2^2 + (p_2 - p_1)(h_1 V_1^2 + h_2 V_2^2) = 0. \tag{16}$$

For given p_1 and V_1, it provides the relation between p_2 and V_2 under the equation of state $p = p(\epsilon)$.

Let us consider the case of weak shock waves. For weak shock waves, we have $\epsilon_2 \to \epsilon_1$ and $p_2 \to p_1$, etc. If we expand the expression for the speed on side 1 given in Eq. (14) in $\Delta\epsilon/h_1 \ll 1$ and $\Delta p/h_1 \ll 1$, with $\Delta\epsilon \equiv \epsilon_2 - \epsilon_1$ and $\Delta p \equiv p_2 - p_1$, we find that

$$\begin{aligned}
(v_1)^2 &= \lim_{2\to 1} \frac{\Delta p}{\Delta\epsilon}\frac{\epsilon_2 + p_1}{\epsilon_1 + p_2} \\
&= \left.\frac{dp}{d\epsilon}\right|_1 \lim_{2\to 1}\left(1 + \frac{\Delta\epsilon}{h_1} - \frac{\Delta p}{h_1} + \cdots\right) \\
&= \left.\frac{dp}{d\epsilon}\right|_1 \left[1 + \frac{\Delta\epsilon}{h_1}\left(1 - \left.\frac{dp}{d\epsilon}\right|_1\right) + \cdots\right] \\
&= (c_{s1})^2 \left[1 + \frac{\Delta\epsilon}{h_1}\left(1 - (c_{s1})^2\right) + \cdots\right],
\end{aligned} \tag{17}$$

where c_{s1} is the speed of sound on side 1, $(c_{s1})^2 = (dp/d\epsilon)_1$. Similarly we find that

$$(v_2)^2 = (c_{s2})^2 \left[1 - \frac{\Delta\epsilon}{h_1}\left(1 - (c_{s2})^2\right) + \cdots\right], \tag{18}$$

where c_{s2} is the speed of sound on side 2, $(c_{s2})^2 = (dp/d\epsilon)_2$. In order to simplify our discussion below, we assume the equations of states on the two sides to be the same, where $c_{s1} = c_{s2}$.

As can be seen from Eqs. (14) and (15), $(v_1)^2 > (v_2)^2$ when $\epsilon_2 > \epsilon_1$, and $(v_1)^2 < (v_2)^2$ when $\epsilon_2 < \epsilon_1$. The former is known as a compression shock wave and the latter is known as a rarefaction shock wave. Remember also that, for compression shock waves, we have $p_2 > p_1$, and for rarefaction shock waves, we have $p_1 > p_2$. This is ensured by the fact that

$$c_s^2 = \lim_{2\to 1} \frac{p_2 - p_1}{\epsilon_2 - \epsilon_1} > 0 \,. \tag{19}$$

Although the equations of relativistic hydrodynamics allow for the existence of both compression shock waves and rarefaction shock waves, only compression shock waves are consistent with the second law of thermodynamics, but rarefaction shock waves are not.

In order to see it, we expand the adiabatic of Eq. (16) using

$$\Delta H = T\Delta S + V_1 \Delta p + \frac{1}{2}\frac{\partial V}{\partial p}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^3 + \cdots \,, \tag{20}$$

$$\Delta V = \frac{\partial V}{\partial p}\bigg|_1 \Delta p + \frac{1}{2}\frac{\partial^2 V}{\partial p^2}\bigg|_1 (\Delta p)^2 + \frac{1}{6}\frac{\partial^3 V}{\partial p^3}\bigg|_1 (\Delta p)^3 + \frac{\partial V}{\partial S}\bigg|_1 \Delta S + \cdots \,, \tag{21}$$

the former of which can be derived by using the thermodynamic relation $V = \partial H/\partial p|_S$. Here $H = hV$ is the enthalpy, $S = sV$ is the entropy, and we defined $\Delta H = H_2 - H_1$, $\Delta S = S_2 - S_1$, and $\Delta V = V_2 - V_1$. After substituting Eqs. (20) and (21) into Eq. (16), one finds [30]

$$\Delta S = \frac{1}{12 H_1 T}\frac{\partial^2 (HV)}{\partial p^2}\bigg|_1 (\Delta p)^3 + O\big((\Delta p)^4\big) \,. \tag{22}$$

It can be seen by plotting the adiabatic for any reasonable equation of state that $\partial^2 (HV)/\partial p^2 > 0$; for ultrarelativistic gases at low T, see the explicit calculation of the coefficient of the term $(\Delta p)^3$ in Eq. (41) below. The second law of thermodynamics requires that $S_2 > S_1$, which then requires that $p_2 > p_1$. This corresponds to compression shock waves explained earlier. Hence we see that, in nonchiral relativistic fluids, second law of thermodynamics allows for the existence of compression shock waves alone and that there are no rarefaction shock waves. This is the so-called Zemplén's theorem (see also [31]).

Chiral shock waves.—Let us now consider shock waves in chiral matter in the presence of a vorticity ω^μ. The important point here is that the vorticity induces the CVE in chiral matter, leading to the modifications of the Rankine-Hugoniot relations. As a result, basic properties of shock waves are qualitatively modified compared with those in nonchiral matter above.

We again consider a situation where a shock wave is moving along the x axis towards the positive x direction. For simplicity, we consider the case with a nonzero constant vorticity in the x direction,

$$\omega_x = \frac{1}{2} u_t (\partial_z u_y - \partial_y u_z) \equiv \omega \,, \tag{23}$$

with $\omega_y = \omega_z = 0$, but the extension of our analysis to more general cases should be straightforward. Note that we are in a system where we cannot go to a frame where $v_y = v_z = 0$ at all points, unlike the case of shock waves in nonchiral matter discussed earlier.

In what follows, we consider the regime $|\omega|\rho \ll 1$, where $\rho = \sqrt{y^2 + z^2}$ is the distance from the axis of the vorticity. Assuming that $\partial_y v_x = \partial_z v_x = 0$, we can solve Eq. (23) for $v_\perp \equiv \sqrt{v_y^2 + v_z^2}$ to find

$$v_\perp = \omega\rho(1 - v_x^2) + O\big((\omega\rho)^2\big) \,. \tag{24}$$

Now let us look at how the jump conditions at the shock front are modified in the presence of the vorticity. Continuity of particle flux and energy-momentum flux across a surface of discontinuity, $j_1^x = j_2^x$, $T_1^{xx} = T_2^{xx}$, $T_1^{0x} = T_2^{0x}$, $T_1^{yx} = T_2^{yx}$, and $T_1^{zx} = T_2^{zx}$, reads

$$\frac{v_1^x \gamma_1}{V_1} + \xi_1 \omega_1 = \frac{v_2^x \gamma_2}{V_2} + \xi_2 \omega_2 \equiv j \,, \tag{25}$$

$$h_1 (v_1^x)^2 \gamma_1^2 + p_1 = h_2 (v_2^x)^2 \gamma_2^2 + p_2 , \tag{26}$$

$$h_1 v_1^x \gamma_1^2 = h_2 v_2^x \gamma_2^2 , \tag{27}$$

$$h_1 v_1^x v_1^y \gamma_1^2 = h_2 v_2^x v_2^y \gamma_2^2 , \tag{28}$$

$$h_1 v_1^x v_1^z \gamma_1^2 = h_2 v_2^x v_2^z \gamma_2^2 . \tag{29}$$

The modification, compared with Eqs. (11)–(13) in nonchiral matter, is the presence of the CVE in Eq. (25).

Notice first that Eqs. (28) and (29) require that $v_1^\perp = v_2^\perp$. From Eq. (24), we then must have $\omega_1[1 - (v_1^x)^2] = \omega_2[1 - (v_2^x)^2]$. Below we will be interested in weak shock waves, for which $v_2^x \to v_1^x$. In this limit

$$\Delta\omega \equiv \omega_2 - \omega_1 = \omega_1 \frac{(v_2^x)^2 - (v_1^x)^2}{1 - c_{s1}^2} \,, \tag{30}$$

up to terms that are suppressed by $|(v_1^x)^2 - (v_2^x)^2| \ll 1$. To analyze the weak shock wave, only Eqs. (25)–(27) are thus relevant, which constitute the modified Rankine-Hugoniot relations for shock propagation in chiral matter.

These equations may also be viewed as the interface conditions between the two sides of chiral matter in a *global* rotation $\boldsymbol{\Omega} = \Omega\hat{\boldsymbol{x}}$ under the replacement $\omega \to \Omega$.

From Eqs. (25) and (26), we have

$$(h_1 V_1^2 - h_2 V_2^2) j^2 - 2(h_1 V_1^2 \omega_1 \xi_1 - h_2 V_2^2 \omega_2 \xi_2) j + p_1 - p_2 + (h_1 \omega_1^2 \xi_1^2 V_1^2 - h_2 \omega_2^2 \xi_2^2 V_2^2) = 0 \,. \tag{31}$$

This can be solved for j as

$$j = \frac{(h_1 V_1^2 \xi_1 \omega_1 - h_2 V_2^2 \xi_2 \omega_2) \pm \sqrt{\mathcal{D}}}{h_1 V_1^2 - h_2 V_2^2} \,, \tag{32}$$

with

$$\mathcal{D} \equiv (p_2 - p_1)(h_1 V_1^2 - h_2 V_2^2) + h_1 h_2 V_1^2 V_2^2 (\xi_1 \omega_1 - \xi_2 \omega_2)^2 . \tag{33}$$

In order for j to be real, we must have $\mathcal{D} \geq 0$.

Note that we must have $|\omega_{1,2}| \ll \mu_{1,2}$ for hydrodynamics to make sense, where $\mu_{1,2}$ is the chemical potential of side 1 or 2. Since we are interested in the regime $\frac{4\pi}{3}\rho^3 \gg V$, the limit $|\omega_{1,2}|\rho \ll 1$ is within the applicability of hydrodynamics. In this limit, Eqs. (26) and (27) can be solved to obtain the expressions for v_1^x and v_2^x which invariably turn out to be identical to the expressions in Eqs. (14) and (15) up to corrections suppressed by powers of $\rho\omega_{1,2}$. Hence, we can use the leading-order expression for the velocities in the expansion of $\rho\omega_{1,2}$ in Eq. (25) to obtain the relation on the two sides as

$$\frac{v_1}{V_1\sqrt{1-v_1^2}} - \frac{v_2}{V_2\sqrt{1-v_2^2}} = -(\omega_1\xi_1 - \omega_2\xi_2)\,. \quad (34)$$

Here $v_{1,2}$ are given in Eqs. (14) and (15). Recalling that $\mu_{1,2}$ and $T_{1,2}$ can be written in terms of $p_{1,2}$ and $V_{1,2}$ from Eqs. (7) and (8), both sides of Eq. (34) can be expressed in terms of $p_{1,2}$ and $V_{1,2}$ alone, for a given equation of state $p = p(\epsilon)$. This provides the pressure-volume relation of the two sides for chiral matter in a constant vorticity.

Let us now consider how the relation (22) is modified for weak shock waves in chiral matter. The modified relation will be obtained by expanding Eq. (34) in terms of $\Delta S = S_2 - S_1$ and $\Delta p = p_2 - p_1$ across the surface of discontinuity. The expansion of the left-hand side of Eq. (34), but ignoring the right-hand side, would lead to the result (22) for nonchiral matter. Here we need to expand the right-hand side in ΔS and Δp as well.

For this purpose, we first solve $p = p(\mu, T)$ and $S = S(\mu, T)$ in Eqs. (8) and (9) for μ and T in terms of p and S, by treating $S \ll 1$ as a perturbation for $\mu \gg T$. The results to the leading correction in $S \ll 1$ are given by

$$\mu(p, S) = \left(1 - \frac{S^2}{2\pi^2}\right)(24\pi^2 p)^{1/4}\,, \quad (35)$$

$$T(p, S) = \frac{S}{\pi^2}(24\pi^2 p)^{1/4}\,. \quad (36)$$

From these equations, $\Delta\mu \equiv \mu_2 - \mu_1$ and $\Delta T \equiv T_2 - T_1$ can be expanded in Δp and ΔS as

$$\Delta\mu = \frac{6\pi^2}{\mu^3}\Delta p - T\Delta S\,, \quad (37)$$

$$\Delta T = \frac{6\pi^2 T}{\mu^4}\Delta p + \frac{\mu}{\pi^2}\Delta S\,. \quad (38)$$

We then expand $\Delta\xi \equiv \xi_2 - \xi_1$ in ΔS and Δp. When $\mu \gg T$, using Eqs. (6), (10), (37), and (38), we have

$$\Delta\xi = \lambda\left[\frac{1}{\mu^2}\Delta p - \frac{6\pi^2}{\mu^6}(\Delta p)^2 + \cdots\right]\,, \quad (39)$$

where

$$\lambda \equiv 4\pi^2 C = \pm 1, \quad (40)$$

for right- and left-handed chiral matter, respectively. In Eq. (39), "$\cdots$" stands for higher order terms in Δp and terms of quadratic and higher order in ΔS. Note here that the coefficient of ΔS in Eq. (39) is proportional to $2C/3 - 2D/\pi^2$ and vanishes identically regardless of the chirality.

Collecting the leading terms in the expansion of Eqs. (34) in ΔS and Δp using Eqs. (20), (21), (30), and (39), we arrive at

$$\Delta S = \frac{216\pi^6}{\mu_1^{11}T_1}(\Delta p)^3 - \omega_1\lambda\frac{36\sqrt{2}\pi^4}{\mu_1^8 T_1}(\Delta p)^2 + \cdots\,, \quad (41)$$

where "$\cdots$" includes terms that are higher order in Δp. Note that the first term on the right-hand side of Eq. (41) corresponds to the result (22) for nonchiral matter. The correction in the second term due to the vorticity becomes negligible and reduces to the result (22) when $|\omega_1| \ll \omega_c$ for a given Δp, where

$$\omega_c = 3\sqrt{2}\pi^2\frac{\Delta p}{\mu_1^3}\,. \quad (42)$$

Let us now consider the case with $|\omega_1| \gg \omega_c$. However, ω_1 still needs to be small enough such that $|\omega_1|\rho \ll 1$ remains valid. In this case, Eq. (41) is approximately

$$\Delta S \approx -\omega_1\lambda\frac{36\sqrt{2}\pi^4}{\mu_1^8 T_1}(\Delta p)^2\,. \quad (43)$$

This is our main result.

Recall that the second law of thermodynamics requires that $\Delta S > 0$. From Eq. (43), this allows for shock waves only when $\omega_1\lambda < 0$. According to our conventions here, a positive vorticity is assumed to be pointing opposite to the direction of the shock wave propagation. Hence, for right-handed fermions (where $\lambda = 1$), we must have $\omega_1 < 0$ for shock wave propagation and vice versa. In particular, for neutrino matter (where $\lambda = -1$), we must have $\omega_1 > 0$ to satisfy $\Delta S > 0$. The other remarkable feature of our result is that the sign of ΔS is independent of the sign of Δp. Hence, for $\omega_1\lambda < 0$, both compression and rarefaction shock waves are realizable in contrast with nonchiral shock waves which can only be of the compression type.

Discussion.—In this paper, we have explored the shock propagation in relativistic chiral matter. We have seen that the conventional Rankine-Hugoniot relations are modified due to the presence of chiral transport phenomena. In particular, we have shown that rarefaction shock waves can appear in chiral matter. Also we have shown that the existence of a shock wave itself is dependent on the chirality of the fermions involved and the direction of shock wave propagation. In this sense, the shock wave found in this paper is *chiral*, similar to the other chiral waves [22–26]. It would be interesting to study possible phenomenological consequences of the chiral shock wave, e.g., in the dynamics of supernovae. In fact, the mean free path of neutrinos, $l_{\mathrm{mfp}} \lesssim 1$ m at the core of supernovae (with matter density $\rho \gtrsim 10^{13}$ g/cm^3), is much smaller than the typical size of the core, $l_{\mathrm{core}} \sim 100$ km. Hence, chiral hydrodynamics is applicable to neutrino matter at least in such a system [12].

Although we limit ourselves to dense and cold charge neutral chiral matter in this paper, our argument should be applicable to hot and/or dense charged chiral matter in an external magnetic field as well. In this case, the chiral magnetic effect modifies the Rankine-Hugoniot relations in a way qualitatively similar to what we found in this paper. Such a new type of shock wave may be relevant to the electroweak plasmas in the early Universe.

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Note added.—After this work was being completed, M. N. Chernodub informed the authors that he also obtained the results [36] that have some overlap with ours.

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[31] Precisely speaking, rarefaction shock waves can appear in nonchiral matter when $\partial^2(HV)/\partial p^2|_S < 0$ (or $\partial^2 V/\partial p^2|_S < 0$ in the nonrelativistic limit). However, this condition is usually not met except very particular situations, e.g., near a gas-liquid critical point [32]. On the other hand, the presence of chiral rarefaction shock waves found in this paper does *not* depend on the sign of $\partial^2(HV)/\partial p^2|_S$, providing a novel realization of rarefaction shock waves.
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