Dynamically Correcting a CNOT Gate for any Systematic Logical Error

F. A. Calderon-Vargas and J. P. Kestner

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Dynamically correcting a CNOT gate for any systematic logical error

F. A. Calderon-Vargas* and J. P. Kestner
Department of Physics, University of Maryland Baltimore County, Baltimore, MD 21250, USA

We derive a set of composite pulse sequences that generate CNOT gates and correct all systematic errors within the logical subspace to arbitrary order. These sequences are applicable for any two-qubit interaction Hamiltonian, and make no assumptions about the underlying noise mechanism except that it is constant on the timescale of the operation. We do assume access to high-fidelity single-qubit gates, so single-qubit gate imperfections eventually limit the achievable fidelity. However, since single-qubit gates generally have much higher fidelities than two-qubit gates in practice, these pulse sequences offer useful dynamical correction for a wide range of coupled qubit systems.

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Scalable fault tolerant quantum computing requires gate operations with errors below the quantum error correction threshold, with lower errors allowing more efficient scaling. A significant source of error in many settings is coherent, systematic gate error. In particular, maximally entangling two-qubit operations tend to have significantly worse fidelities than single-qubit operations, for instance due to weak interactions requiring a long gate time in the presence of low-frequency noise. Such unknown systematic errors may be compensated using composite pulse sequences [1–4], where a desired operation is replaced by a set of imperfect pulses in such a way that the systematic errors inherent in each pulse cancel with each other. A feature of composite sequences is that their analytical construction allows great generality, and they have been applied in NMR [5], trapped ions [6], nitrogen-vacancy centers in diamond [7], electron spins in semiconductor quantum dots [8], quantum optics [9], atom interferometry [10], etc.

However, most of the progress with this approach has been made on single-qubit gates, and there is no general two-qubit sequence able to produce an entangling gate corrected to leading order in every possible systematic error. In fact, there is a "no-go" theorem for black-box dynamical correction [11], requiring detailed knowledge of the relationships between the errors of each component of a composite sequence. Some sequences circumvent this by correcting only a limited subset of errors using an isomorphism between the SU(2) generators and a subgroup of the SU(4) generators [12, 13]. Alternatively, the term isolation approach [14] recovers the generality of the error suppression, but by a factor of the inverse of the number of pulses rather than nulling first or higher order terms in the noise, and that requires many more pulses to achieve the same level of performance as an order-by-order approach at small error rates.

Another approach is to use optimal control theory to numerically optimize performance for a specific system [15–17]. This has been highly successful, particularly in NMR [18], and the resulting pulses, though they rarely provide much physical insight, tend to be much shorter, lower power, and produce higher gate fidelities at the cost of system-independence. One must be able to write down the full master equation in order to do the numerics, and unknown correlations between the noise and the control cannot be handled in an open-loop process. Such numerical techniques also can be complicated by control constraints that introduce local traps into the search landscape [19]. We shall focus on the complementary analytical approach. Thus, the prime use for our results will be in cases where one has limited knowledge of the underlying physical noise and control mechanisms, highly constrained control, or both, especially in an open-loop setting.

In this letter, we present a family of general composite pulse sequences that generate CNOT gates compensated for all systematic errors to arbitrary order using as a building block any single imperfect entangling gate, while only assuming access to high-fidelity single-qubit operations. The assumption of negligible errors in the local gates opens a loophole in the no-go theorem [11], similar in spirit to the exploitation of a "robust operating point" in Ref. [20]. The purpose of only employing a single nonlocal gate is to guarantee that all systematic errors entering the sequence are identical, though unknown. This makes our treatment highly compatible and modular, being impervious to the details of the qubit system and the method by which the gates are performed. Thus, unlike the application of standard local dynamical decoupling sequences during entanglement [21], our approach does not require local noise nor instantaneous pulses. Each part of the sequence may be performed in whatever way is experimentally convenient; the modular construction does not assume any particular form. This feature allows it to be used in conjunction with optimal control theory or other dynamical correction methods if desired, for instance using numerically shaped pulses to optimally produce the components of the sequence or else using our sequence to inform the trial pulse input to a numerical search algorithm, or even provide a second boost to the two-qubit gate fidelity after an initial improvement using particular experimental techniques such as active cancellation tones [22].

The pulse sequences are composed using repetitions of the nonlocal gate $(\theta)_{ZZ} = \exp\left[-i\frac{\theta}{2}\sigma_{ZZ}\right]$, which, regardless of the form of the two-qubit interaction Hamiltonian, can be generated by one or more applications of the evolution operator [23, 24] along with appropriate single-qubit gates obtained from Cartan decomposition [25]. The family of composite pulse sequences is formed by the sequential application of $(\theta)_{ZZ}$ interleaved with local π-rotations $\sigma_{ij} \equiv \sigma_i \otimes \sigma_j$, with $i, j = \{I, X, Y, Z\}$. In this letter we first introduce various composite sequences capable of correcting various subsets of systematic errors and then we nest those sequences to form a completely general sequence that corrects any systematic error and generates a first-order error-free CNOT gate using no more than 120 nonlocal gates. This sequence can itself be nested to correct errors up to any order.

As a starting point, we represent the error in a noisy realization of the building block $(\theta)_{ZZ}$ by expanding it to first order as

$$(\theta)_{ZZ} = \exp\left[-i\frac{\theta}{2}\sigma_{ZZ}\right]\left(I + i\sum_{i,j=\{I,X,Y,Z\}}\delta_{ij}\sigma_{ij}\right), \tag{1}$$

where δ_{ij} is hereafter referred to as the error term in the ij error channel.

As a warm-up, consider the simplest possible sequence, formed by inserting a local π-rotation, σ_{ab}, hereafter referred to as an echo pulse, in between two applications of the noisy entangling operation, $(\theta)_{ZZ}$. To generate an entangling operation, one should choose a σ_{ab} that commutes with σ_{ZZ} since the anticommuting alternative would simply produce a purely local operation. The result, up to first order in the errors, is

$$\mathcal{U}_{ab}^{(2)}\left[(\theta)_{ZZ}\right] = (\theta)_{ZZ}\,\sigma_{ab}\,(\theta)_{ZZ}\,\sigma_{ab} = \exp\left[-i\frac{2\theta}{2}\sigma_{ZZ}\right] \times\left\{I + i\sum_{i,j}\delta_{ij}\left(\sigma_{ab}\sigma_{ij}\sigma_{ab} + (\theta)^{\dagger}_{ZZ}\,\sigma_{ij}\,(\theta)_{ZZ}\right)\right\}, \tag{2}$$

where the bracketed term on the lhs indicates the nonlocal rotation used to build the sequence. Hence, this sequence corrects all error channels that simultaneously commute with the entangling operation and anticommute with the echo pulse, $[\sigma_{ij}, \sigma_{ZZ}] = \{\sigma_{ij}, \sigma_{ab}\} = 0$. In fact, this is clearly true not just to first order, but to all orders. This simple sequence is already known [26] and has appeared in both theory [14] and experimental work [27, 28]. We call this a length-2 sequence, where by "length-n" we mean a sequence having n applications of the noisy entangling operation.

Furthermore, by placing this length-2 pulse sequence (2) inside a second length-2 sequence that uses an echo pulse that anticommutes with the first pulse, one produces a length-4 sequence that exactly cancels all error channels that commute with σ_{ZZ}, excluding the coupling error δ^{ZZ} itself. (Note that it is thus not actually necessary for the controlled part of the building block to be strictly a ZZ rotation when using the length-4 sequence, as long as the other generators in the exponent commute with σ_{ZZ}.) Choosing $\theta = \pi/4$ for length-2 or $\theta = \pi/8$ for length-4, the result of the sequence is locally equivalent to a CNOT. Then it is straightforward to apply the two-qubit variant of the BB1 pulse sequence [12] to also cancel the ZZ error channel to second order.

Now we turn our attention to canceling, to leading order, the error channels that do not commute with σ_{ZZ}. By analogy with the previous case, we focus our attention on a sequence of the form

$$\begin{aligned}\sigma^{(n)}_{\text{echo}}\,(\theta)_{ZZ}\,\sigma^{(n)}_{\text{echo}}\sigma^{(n-1)}_{\text{echo}}\,(\theta)_{ZZ}\,\sigma^{(n-1)}_{\text{echo}}\ldots\sigma^{(1)}_{\text{echo}}\,(\theta)_{ZZ}\,\sigma^{(1)}_{\text{echo}} \\ = \exp\left[-i\frac{\theta}{2}\left(\sum_{l=1}^{n}\xi_l\right)\sigma_{ZZ}\right] \\ \times\left(I + i{\sum_{i,j}}'\delta_{ij}\sigma_{ij}\sum_{m=1}^{n}\zeta_m^{ij}\exp\left[-i\theta\left(\sum_{l=1}^{m-1}\xi_l\right)\sigma_{ZZ}\right]\right),\end{aligned} \tag{3}$$

where $\sigma^{(m)}_{\text{echo}}$ denotes local π-rotations of the form σ_{ab}, the primed sum indicates that we only include error channels that anticommute with σ_{ZZ}, and

$$\xi_l \equiv \begin{cases}+1 & \text{if } \left[\sigma^{(l)}_{\text{echo}}, \sigma_{ZZ}\right] = 0 \\ -1 & \text{if } \{\sigma^{(l)}_{\text{echo}}, \sigma_{ZZ}\} = 0\end{cases}, \tag{4}$$

$$\zeta_m^{ij} \equiv \begin{cases}+1 & \text{if } \left[\sigma^{(m)}_{\text{echo}}, \sigma_{ij}\right] = 0 \\ -1 & \text{if } \{\sigma^{(m)}_{\text{echo}}, \sigma_{ij}\} = 0\end{cases}. \tag{5}$$

Setting the real and imaginary part of the error term in Eq. (3) to equal zero requires the two equations

$$\begin{aligned}\zeta_1^{ij} + \sum_{m=2}^{n}\zeta_m^{ij}\cos\left[\sum_{l=1}^{m-1}\xi_l\theta\right] = 0 \\ \sum_{m=2}^{n}\zeta_m^{ij}\sin\left[\sum_{l=1}^{m-1}\xi_l\theta\right] = 0\end{aligned} \tag{6}$$

to hold for each error channel corrected. For lengths $n = 3$ and $n = 4$, we have found solutions that cancel some but not all error terms. With $n = 5$, though, by choosing echo pulses such that all $\xi_l = 1$, $\zeta_2^{ij} = \zeta_4^{ij}$, $\zeta_1^{ij} = \zeta_5^{ij}$, and using Chebyshev's recursive formula for cosine and sine of multiple angles we simplify Eq. (6) to the single condition

$$\zeta_3^{ij} + \zeta_4^{ij}2\cos\theta + \zeta_5^{ij}\left(4\cos^2\theta - 2\right) = 0. \tag{7}$$

There are real solutions for θ that satisfy this equation for all error terms as long as the echo pulses are taken such that either $\zeta_3^{ij} = \mp 1$ and $\zeta_4^{ij} = \zeta_5^{ij} = \pm 1$ or $\zeta_5^{ij} = \mp 1$ and $\zeta_3^{ij} = \zeta_4^{ij} = \pm 1$; we proceed with the former choice since it gives the smaller value of θ, and hence, presumably, the faster implementation. This solution is

$\theta_0 = \arccos\left[\frac{1}{4}\left(\sqrt{13}-1\right)\right] \approx 0.27\pi$. The corresponding $\sigma^{(l)}_{\text{echo}}$ in Eq. (3) are $\sigma^{(1,2,4,5)}_{\text{echo}} = I$ and $\sigma^{(3)}_{\text{echo}} = \sigma_{ZZ}$. Therefore, we have found a length-5 sequence that corrects all anticommuting error channels to first order,

$$\begin{aligned}\mathcal{U}^{(5)}\left[(\theta_0)_{ZZ}\right] &= (\theta_0)_{ZZ}\,(\theta_0)_{ZZ}\,\sigma_{ZZ}\,(\theta_0)_{ZZ}\,\sigma_{ZZ}\,(\theta_0)_{ZZ}\,(\theta_0)_{ZZ} \\ &= \exp\left[-i\frac{5\theta_0}{2}\sigma_{ZZ}\right]\left(I+\mathcal{O}\left(\delta^2_{\text{anticomm}}\right)\right),\end{aligned} \tag{8}$$

where the above neglects commuting errors.

Now we combine the length-5 sequence (8) above with the length-2 sequence (2) that addresses commuting errors to correct both types of errors at the same time. For instance, nesting a single length-2 sequence, with a total rotation angle equal to θ_0, within the length-5 pulse sequence produces a length-10 sequence, $\mathcal{U}^{(10)}\left[(\theta_0/2)_{ZZ}\right] = \mathcal{U}^{(5)}\left[\mathcal{U}^{(2)}_{ab}\left[(\theta_0/2)_{ZZ}\right]\right]$, with three remaining error terms to leading order: δ_{ZZ}, δ_{ab}, and $\delta_{ZZ\cdot ab}$. Of course, for a specific physical system, if one can arrange for those terms to be negligible, one need not go further. But one may remove all non-ZZ error channels with a length-20 sequence,

$$\begin{aligned}\mathcal{U}^{(20)}\left[\left(\frac{\theta_0}{4}\right)_{ZZ}\right] =& \mathcal{U}^{(5)}\left[\mathcal{U}^{(2)}_{XX}\left[\mathcal{U}^{(2)}_{ZI}\left[\left(\frac{\theta_0}{4}\right)_{ZZ}\right]\right]\right] \\ =& \exp\left[-i\frac{5\theta_0}{2}\sigma_{ZZ}\right]\left(I+\mathcal{O}\left(\delta^2_{\text{non-ZZ}}\right)\right).\end{aligned} \tag{9}$$

It is worth noting that the order in which we nest the length-2 and length-5 sequences can be interchanged in the combined sequences discussed above, giving us correcting sequences that require a smaller number of local gates (a difference of 4 and 8 local operations, for the length-10 and length-20 sequences, respectively), but with the caveat that the error correction performance suffers due to the asymmetry that the length-2 sequence cancels errors to all orders while the length-5 only to first order, and it is clearly better to repeatedly invoke the more accurate sequence.

While the nonlocal $5\theta_0$ rotation is not equivalent to a CNOT, two applications of this gate with appropriate single-qubit operations can generate a CNOT gate,

$$\begin{aligned}\text{CNOT} =& A_1 \exp\left[-i\frac{\psi}{2}\sigma_{XI}\right]\mathcal{U}^{(k)}\exp\left[-i\frac{\phi}{2}\sigma_{XI}\right] \\ &\times \mathcal{U}^{(k)}\exp\left[-i\frac{\psi}{2}\sigma_{XI}\right]A_2,\end{aligned} \tag{10}$$

where $k = 5$, 10, or 20 and the local gates are given by $A_1 = \exp\left[\frac{-i\pi(\sigma_X-\sigma_Y)}{2\sqrt{2}}\right]\otimes\exp\left[\frac{-i5\pi(\sigma_X+\sigma_Y-\sigma_Z)}{3\sqrt{3}}\right]$, $A_2 = \sigma_X\otimes\exp\left[\frac{-i\pi\sigma_Y}{4}\right]$, $\psi = 2\arctan\left[\frac{\sqrt{-57+16\sqrt{13}}}{4-\sqrt{13}+2\sqrt{-7+2\sqrt{13}}}\right]$, and $\phi = -2\arccos\left[-\frac{1}{2\sqrt{-14+4\sqrt{13}}}\right]$.

As a simple example, we consider qubits with an XYZ coupling perturbed by random local terms, $H = \alpha\left(\sigma_{ZZ}+\Delta_X\sigma_{XX}+\Delta_Y\sigma_{YY}\right) + \sum_{j=\{X,Y,Z\}}\left(B_{j,1}\sigma_{jI}+B_{j,2}\sigma_{Ij}\right)$. This model encompasses both Ising and isotropic Heisenberg couplings, both of which are realized in a broad variety of physical settings. For instance, one concrete realization would be electron spins in GaAs lateral quantum dots coupled via Heisenberg exchange in the presence of noisy magnetic fields due to nuclear spin fluctuations and motion of the spin in an applied magnetic field gradient [29]. Figure 1 shows the average infidelity, defined as in Ref. [30], of the uncorrected CNOT formed in the standard way via two $\sqrt{\text{SWAP}}$ gates along with local operations [31]. The infidelity is averaged over the noise by independently sampling the six random variables over a normal distribution of standard deviation σ, with the average being taken over 2000 samples for each value of σ. The average infidelity of the corrected sequence of Eq. (10) with $k = 20$ is also plotted, showing impressive reductions in the error with relatively low overhead. The sequence has 40 coupling pulses for a total interaction time of $5\theta_0/\alpha$, only about three times longer than the total naive interaction time of $\pi/2\alpha$. In the supplemental material, we present one more example of gate fidelity improvement by applying our length-5 sequence, Eq. (10) with $k = 5$, to the cross resonance gate between transmon qubits [22].

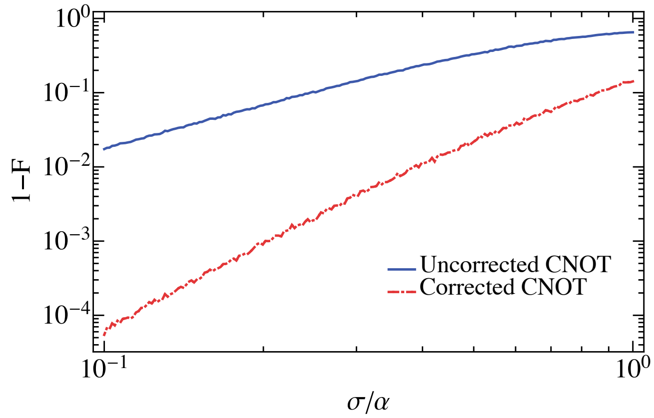

FIG. 1. (Color online.) Infidelity vs noise strength for uncorrected (solid) and corrected (dashed) CNOT gates for Heisenberg-coupled spins in a random magnetic field.

Returning to our program, we still have to deal with δ_{ZZ}. BB1 [12, 32]is a well-known remedy for such errors, but it cannot be applied as the last step here because, after constructing a CNOT with $\mathcal{U}^{(k)}$ (10), δ_{ZZ} is no longer just the coefficient of the ZZ error channel, but appears in several error channels. Thus one must remove the δ_{ZZ} term in $\mathcal{U}^{(k)}$ before it gets mixed into other channels. Furthermore, BB1 cannot be applied to $\mathcal{U}^{(k)}$ because BB1 requires application of nonlocal π rotations whose error is proportional to the error in the nonlocal $5\theta_0$ rotation, and this is not possible since θ_0 is not a rational multiple of π.

Nonetheless, the δ_{ZZ} error in $\mathcal{U}^{(k)}$ can be compensated using a new composite pulse sequence that is similar in principle to BB1, where we perform the same nonlocal rotation repeatedly about different axes that are tilted from σ_{ZZ} toward σ_{ZX} by means of inserting single-qubit rotations around σ_{IY}. We found numerically that at least five different nonlocal axes are needed to cancel δ_{ZZ} to first order. The sequence has the form (for $k=5$, 10, or 20)

$$\mathcal{U}^{(6k)} = \left(\prod_{j=4}^{1} \exp\left[i\frac{\psi_j}{2}\sigma_{IY}\right]\left[\mathcal{U}^{(k)}\right]^{n_j}\exp\left[-i\frac{\psi_j}{2}\sigma_{IY}\right]\right)\mathcal{U}^{(k)}, \tag{11}$$

where the product is in descending order to reflect the time-ordering of the operators, $1+\sum n_j = 6$, and the values of the parameters depend on what desired operation we are targeting and are obtained from a numerical minimization of an objective function containing the magnitude of the first order error terms of $\mathcal{U}^{(6k)}$ as well as the distance between the local invariants [33] of $\mathcal{U}^{(6k)}$ and those of the target operation. When targeting a CNOT equivalent, we obtain a solution $\mathcal{U}^{(6k)}_{\text{CNOT}}$ with an intrinsic infidelity (i.e., in the absence of noise) of $\sim 10^{-12}$ given by $n_1 = n_2 = n_3 = 1$, $n_4 = 2$, $\psi_1 \approx 1.13527$, $\psi_2 \approx -0.40553$, $\psi_3 \approx -1.84186$, and $\psi_4 \approx 0.19175$. The local operations that transform that solution into CNOT are given by

$$\text{CNOT} = A_1 \exp\left[-i\frac{\phi_1}{2}\sigma_{IY}\right]\mathcal{U}^{(6k)}_{\text{CNOT}}\exp\left[-i\frac{\phi_2}{2}\sigma_{IY}\right]A_2, \tag{12}$$

where A_1 and A_2 are the same single-qubit operations as in Eq. (10), $\phi_1 \approx -1.60782$, and $\phi_2 \approx 0.23403$ (see Supplemental Material).

Alternatively, rather than numerically targeting a CNOT, one can search for parameters such that $\mathcal{U}^{(6k)}$ instead yields a corrected rotation equivalent to $(5\theta_0/k)_{ZZ}$. In that way the output of the sequence would be a leading-order corrected version of the basic two-qubit input rotation and thus, in principle, one could correct errors to arbitrarily higher order by nesting the above sequence within itself. Taking $n_1 = n_3 = n_4 = 1$ and $n_2 = 2$ in Eq. (11), we do find such numerical solutions with intrinsic infidelities ranging from 10^{-12} to 10^{-14}. For $k = \{5, 10, 20\}$, respectively, we find $\psi_1 \approx \{-0.18359, -0.10304, -0.05223\}$, $\psi_2 \approx \{-3.06178, -3.12993, -3.13844\}$, $\psi_3 \approx \{-2.01932, -2.58384, -2.86285\}$, and $\psi_4 \approx \{1.75080, 0.84439, 0.41865\}$. The necessary local operations to be applied before nesting are given by

$$\exp\left[-i\frac{5\theta_0/k}{2}\sigma_{ZZ}\right] = \sigma_{XI}^{m_k}\exp\left[-i\frac{\beta_k}{2}\sigma_{IY}\right] \times \mathcal{U}^{(6k)}_{5\theta_0/k}\sigma_{XI}^{m_k}\exp\left[-i\frac{\gamma_k}{2}\sigma_{IY}\right], \tag{13}$$

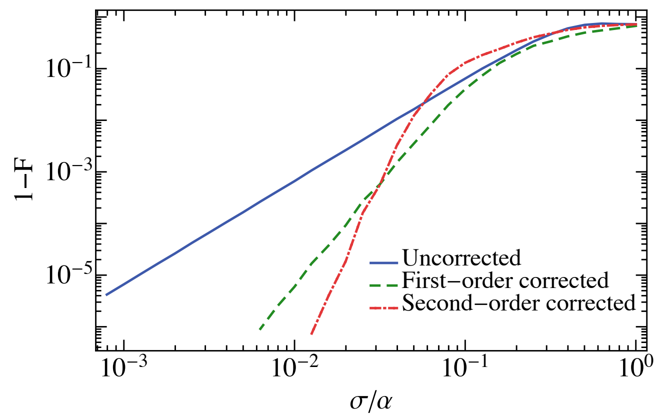

FIG. 2. (Color online.) Infidelity vs noise strength for uncorrected (solid) and corrected CNOT gates to first (dashed) and second (dot-dashed) order, using an Ising Hamiltonian with general SU(4) errors.

where $m_5 = m_{10} = 1$, $m_{20} = 0$, $\beta_5 \approx 3.11104$, $\gamma_5 \approx -2.11735$, $\beta_{10} \approx 2.29085$, $\gamma_{10} \approx -1.85051$, $\beta_{20} \approx -1.21618$, and $\gamma_{20} \approx 1.43078$ (see Supplemental Material).

We show the efficacy of the full composite pulse sequence by again applying it to a Hamiltonian with an XYZ coupling, but this time with random fluctuations on every one of the 15 SU(4) generators, $H = \alpha\sigma_{ZZ} + \sum \delta_{ij}\sigma_{ij}$. (In the context of our prior example of exchange coupled spins in semiconductor dots, some of the undesired nonlocal terms could, for example, come from a Dzyaloshinskii-Moriya interaction [34, 35].) Using the length-120 sequence (Eq. (12) with $k=20$), we form a CNOT that is dynamically corrected against this completely general error to first order. The infidelities are shown for comparison in Fig. 2. We also show the infidelity for a second order corrected CNOT gate, generated by nesting the sequence from Eq. (13) into Eq. (12). Again, there are 15 stochastic noise variables, but for the purposes of plotting infidelity we have averaged each one independently over the same normal distribution of standard deviation σ. In practice, the rather long full composite sequence would only be needed in a worst-case scenario where appreciable systematic error is present in all channels. However, qubits in isotopically enriched silicon (using either gate-defined quantum dots [36] or phosphorus donors [37]) present long enough coherence times compared to qubit operation times to feasibly implement a sequence of this length.

Finally, though we have assumed ideal single-qubit gates throughout (as in Ref. [38]), we now discuss the effect imperfect single-qubit gates would have on the above composite sequences. We characterize the effect by assigning to each local gate a random systematic perturbation of the form $\exp[-i\sum\Delta_i\sigma_i]\otimes\exp[-i\sum\Delta_j\sigma_j]$ and numerically averaging over noise realizations. Unsurprisingly, we find that the average infidelity due to imperfect single-qubit gates increases with the sequence length, from three times the average single-qubit infidelity for the length-2 sequence up to 80 times the local gate infidelity for the length-120 sequence, which has 121 local gates (see Supplemental Material for a detailed discussion on the effect of imperfect single-qubit gates with both systematic and random errors). Thus, we expect the various sequences to be useful when the single-qubit gate fidelities are at least an order of magnitude greater than the two-qubit gate fidelities. However, even in cases where that condition does not hold, the very existence of these sequences reduces the task of raising two-qubit gate fidelity to the considerably simpler task of raising single-qubit gate fidelity.

In summary, we have introduced a family of composite pulse sequences capable of correcting any systematic logical error that could appear when generating an entangling gate from any Hamiltonian. We have shown how to use these sequences to generate CNOT gates that are error-free to arbitrary order. No knowledge of the underlying noise mechanisms is assumed, other than that they are quasistatic on the time scale of the operations. This sort of black-box approach is generally forbidden [11], but we have explicitly shown here that it is permitted in the case of access to ideal single-qubit operations. The generality of the composite pulse sequences we have presented above makes them a powerful tool for robust generation of entanglement and quantum computing in the presence of systematic error.

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* f.calderon@umbc.edu

[1] M. H. Levitt, Prog. Nucl. Magn. Reson. Spectrosc. **18**, 61 (1986).
[2] J. A. Jones, Prog. Nucl. Magn. Reson. Spectrosc. **59**, 91 (2011).
[3] J. T. Merrill and K. R. Brown, in *Quantum Inf. Comput. Chem.* (John Wiley & Sons, Inc., 2014) pp. 241–294, 1203.6392.
[4] D. A. Lidar and T. A. Brun, eds., *Quantum Error Correction* (Cambridge University Press, 2013).
[5] L. M. K. Vandersypen and I. L. Chuang, Rev. Mod. Phys. **76**, 1037 (2005).
[6] E. Mount, C. Kabytayev, S. Crain, R. Harper, S.-Y. Baek, G. Vrijsen, S. T. Flammia, K. R. Brown, P. Maunz, and J. Kim, Phys. Rev. A **92**, 060301 (2015).
[7] C. A. Ryan, J. S. Hodges, and D. G. Cory, Phys. Rev. Lett. **105**, 200402 (2010).
[8] K. Eng, T. D. Ladd, A. Smith, M. G. Borselli, A. A. Kiselev, B. H. Fong, K. S. Holabird, T. M. Hazard, B. Huang, P. W. Deelman, I. Milosavljevic, A. E. Schmitz, R. S. Ross, M. F. Gyure, and A. T. Hunter, Science Advances **1** (2015), 10.1126/sciadv.1500214.
[9] G. T. Genov, D. Schraft, T. Halfmann, and N. V. Vitanov, Phys. Rev. Lett. **113**, 043001 (2014).
[10] P. Berg, S. Abend, G. Tackmann, C. Schubert, E. Giese, W. P. Schleich, F. A. Narducci, W. Ertmer, and E. M. Rasel, Phys. Rev. Lett. **114**, 063002 (2015).
[11] K. Khodjasteh and L. Viola, Phys. Rev. A **80**, 032314 (2009).
[12] J. A. Jones, Phys. Rev. A **67**, 012317 (2003).
[13] Y. Tomita, J. T. Merrill, and K. R. Brown, New J. Phys. **12**, 015002 (2010).
[14] C. D. Hill, Phys. Rev. Lett. **98**, 180501 (2007).
[15] J. P. Palao and R. Kosloff, Phys. Rev. Lett. **89**, 188301 (2002).
[16] N. Khaneja, T. Reiss, C. Kehlet, T. Schulte-Herbrüggen, and S. J. Glaser, J. Magn. Reson. **172**, 296 (2005).
[17] S. Machnes, U. Sander, S. J. Glaser, P. de Fouquières, A. Gruslys, S. Schirmer, and T. Schulte-Herbrüggen, Phys. Rev. A **84**, 022305 (2011).
[18] D. Lu, H. Li, D.-A. Trottier, J. Li, A. Brodutch, A. P. Krismanich, A. Ghavami, G. I. Dmitrienko, G. Long, J. Baugh, and R. Laflamme, Phys. Rev. Lett. **114**, 140505 (2015).
[19] G. Riviello, K. M. Tibbetts, C. Brif, R. Long, R.-B. Wu, T.-S. Ho, and H. Rabitz, Phys. Rev. A **91**, 043401 (2015).
[20] W. M. Witzel, I. Montaño, R. P. Muller, and M. S. Carroll, Phys. Rev. B **92**, 081407 (2015).
[21] A. D'Arrigo, G. Falci, and E. Paladino, Phys. Rev. A **94**, 022303 (2016).
[22] S. Sheldon, E. Magesan, J. M. Chow, and J. M. Gambetta, Phys. Rev. A **93**, 060302 (2016).
[23] J. Zhang, J. Vala, S. Sastry, and K. B. Whaley, Phys. Rev. A **67**, 042313 (2003).
[24] M. R. Geller, E. J. Pritchett, A. Galiautdinov, and J. M. Martinis, Phys. Rev. A **81**, 012320 (2010).
[25] B. Kraus and J. I. Cirac, Phys. Rev. A **63**, 062309 (2001).
[26] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).
[27] M. D. Shulman, O. E. Dial, S. P. Harvey, H. Bluhm, V. Umansky, and A. Yacoby, Science **336**, 202 (2012).
[28] J. M. Chow, J. M. Gambetta, A. W. Cross, S. T. Merkel, C. Rigetti, and M. Steffen, New Journal of Physics **15**, 115012 (2013).
[29] R. Brunner, Y.-S. Shin, T. Obata, M. Pioro-Ladrière, T. Kubo, K. Yoshida, T. Taniyama, Y. Tokura, and S. Tarucha, Phys. Rev. Lett. **107**, 146801 (2011).
[30] R. Cabrera and W. Baylis, Phys. Lett. A **368**, 25 (2007).
[31] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[32] S. Wimperis, J. Magn. Reson. **109**, 221 (1994).
[33] Y. Makhlin, Quant. Inf. Proc. **1**, 243 (2002).
[34] I. Dzyaloshinsky, Journal of Physics and Chemistry of Solids **4**, 241 (1958).
[35] T. Moriya, Phys. Rev. **120**, 91 (1960).
[36] M. Veldhorst, C. H. Yang, J. C. C. Hwang, W. Huang, J. P. Dehollain, J. T. Muhonen, S. Simmons, A. Laucht, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, Nature **526**, 410 (2015).
[37] R. Kalra, A. Laucht, C. D. Hill, and A. Morello, Phys. Rev. X **4**, 021044 (2014).
[38] X. Wang, E. Barnes, and S. Das Sarma, NPJ Quantum Inf. **1**, 15003 (2015).
[39] See Supplemental Material [url], which includes Ref. [40].
[40] S. Sheldon, L. S. Bishop, E. Magesan, S. Filipp, J. M. Chow, and J. M. Gambetta, Phys. Rev. A **93**, 012301

(2016).